Midterm exam

- Thursday, February 11, 7:00-9:00 p.m.
- Bring calculator (no text or image storage), pencil
- Turn off or silence cell phones
- Bring student card. Put face up on desk.
- Last names
  - A-D: go to 111 Armes
  - E-O: go to 208 Armes
  - P-Z: go to 205 Armes
- Paper A: answers in #1-18 of column 1
- Paper B: answers in #41-58 of column 2

Chapter 7

Work and Energy

Energy Concepts

An alternative approach to $\vec{F} = m\vec{a}$ that focuses on energy.

Kinetic energy: $K = \frac{1}{2}mv^2$

- Energy association with motion
- A scalar quantity
- Depends on speed, but not direction
- For 3D motion, $v^2 = v_x^2 + v_y^2 + v_z^2$

Work: $W = \vec{F} \cdot \Delta\vec{r}$ for a constant force $\vec{F}$, displacement $\Delta\vec{r}$.

- Represents energy transferred to an object by a force
- Only the component of the force in the direction of displacement contributes

Work-Energy theorem:

$$\Delta K = W \quad \therefore \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \vec{F} \cdot \Delta\vec{r}$$

Constant acceleration in 1D

$$v_f^2 = v_i^2 + 2ad; \quad d \equiv \Delta x = x_f - x_i$$ displacement d

$$\times\tfrac{1}{2}m: \quad \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + \underbrace{ma}_{F}d$$

$F = ma$ is a constant force in the direction of motion

$$\therefore K_f = K_i + W$$

Alternatively, look at change: $\Delta K \equiv K_f - K_i = W$

This is the **Work-Energy** theorem.

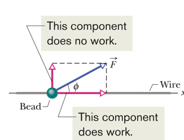
Constant acceleration in 1D

$$\Delta K = W$$

- $W > 0 \Rightarrow v_f^2 > v_i^2$ increase in kinetic energy
  - F and d in same direction
  - energy transferred to object
- $W < 0 \Rightarrow v_f^2 < v_i^2$ decrease in kinetic energy
  - F and d in opposite directions
  - energy transferred from object

Constrained motion in 2D

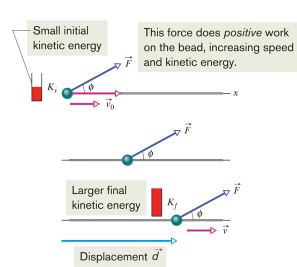
Suppose constant force acts at a constant angle ϕ to the objects path. For example, consider a bead constrained on a wire:



$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + F_x d$$

But $W = F_x d = (F\cos\phi)d$

$$= \vec{F} \cdot \vec{d}$$



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Several forces acting on an object

If there are several forces acting on an object, we use the <u>net force</u>:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \ldots = \sum_i \vec{F}_i$$

Therefore
$$\begin{aligned} W &= \vec{F}_{\text{net}} \cdot \vec{d} \\ &= (\textstyle\sum_i \vec{F}) \cdot \vec{d} \\ &= \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \ldots \\ &= W_1 + W_2 + \ldots \end{aligned}$$

Problem 7-58

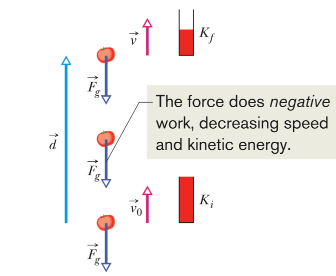
Consider a crate of mass 50 kg pulled across a horizontal floor by a force of magnitude 210 N, applied at an angle 20° above the horizontal, for a distance of 3 m. What is the work done by

(a) the applied force;
(b) the normal force;
(c) gravity;
(d) What is the total work?

Work done by gravity

For gravity, $F_g = -mg$, taking y as positive upward.

$$\therefore W_g = -mg\Delta y$$



Example

Consider an object of mass m raised at constant velocity through a vertical displacement $\vec{d}$: