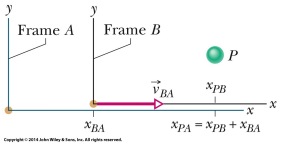
Relative Motion in One Dimension

- Measurements of position and velocity depend on the reference frame of the observer.
- Notation: x_{BA} (of, by)

Means position of frame B as measured by an observer in frame A (or with respect to an observer in frame A)

$$x_{AB} = -x_{BA}$$

Frame B moves past frame A while both observe P.



- Position of particle P in frames A and B are related by:

$$x_{PA} = x_{PB} + x_{BA}$$

- Taking the derivative, we see velocities are related by:

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA})$$

$$\therefore v_{PA} = v_{PB} + v_{BA}$$

- Non-accelerating reference frames have $a_{BA} = 0$ (*i.e.* v_{BA} is constant). Therefore

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}) = \frac{d}{dt}(v_{PB})$$

$$\therefore a_{PA} = a_{PB}$$

- Symmetry: $x_{AB} = -x_{BA}, \quad \therefore v_{AB} = -v_{BA}$

Example

A rower can row a boat at 4 km/h in still water. A river has a current of 2 km/h.

(a) How long does it take to row 2 km downstream?
(b) How long does it take to row the 2 km back to the starting point?
(c) What happens if the current is faster than 4 km/h?

$$v_{BS} = v_{BW} + v_{WS}$$

B: boat
S: shore
W: water

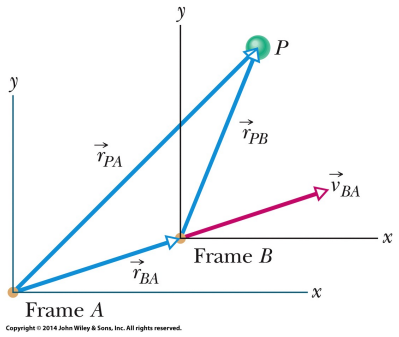
Relative Motion in Two Dimensions

- Frames A and B are both observing the motion of P

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$



Relative Motion in Two Dimensions

Notes:

1. There is complete symmetry between frames A and B:

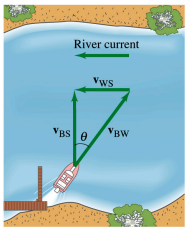
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \quad \therefore \vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA}$$
$$\text{But } \vec{v}_{BA} = -\vec{v}_{AB} \quad \therefore \vec{v}_{PB} = \vec{v}_{PA} + \vec{v}_{AB}$$

2. A non-accelerating frame is called an **inertial frame**.
   - Any frame moving at a constant velocity with respect to an inertial frame is also an inertial frame.
   - All inertial frames measure the same acceleration for any object P.

Problem 4.83

A rower can row a boat at 4 km/h in still water. A river has a current of 2 km/h.

(a) At what angle θ should the boat be pointed to reach the bank directly opposite?
(b) How long does it take to cross 4 km?
(c) At what angle should the boat be pointed to cross the river in the least time, and what is that time?



$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$

B: boat
S: shore
W: water

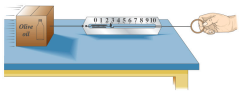
Chapter 5

Dynamics I (Force and Motion)

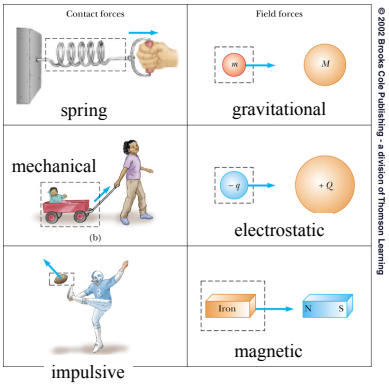
Force

Intuitive (operational) definition: push or pull on an object

- may or may not give rise to motion
- force has magnitude and direction, and is therefore a vector
- can measure by a spring scale, for example



Contact (direct) and Field (action at a distance) Forces



What is the connection between force and motion?

Aristotle (384-322 BC)

- force required to keep an object moving horizontally
  remove force ⟹ object slows and stops
- natural state is at rest
  (preferred frame – earth as centre of universe)
  - larger force implies larger speed
    (examples: swimming, driving a car)

Motion takes place in a viscous medium

Galileo (1564-1642)

- experiment at Pisa:
  speed of falling bodies increases at the same rate, independent of mass
- idealization: imagine no viscous medium (*e.g.* air)
  Just as natural for an object to be in (horizontal) motion at constant speed as it is to be at rest!
- if no force is applied to a moving object, it will continue moving at constant speed in a straight line
  Interpretation: viscosity, or friction, exerts a force
- to push an object with constant speed, the propulsive force is balanced by the force due to friction
  ⟹ no **net** force

Implications: No preferred frame.
Earth no longer centre of universe.
Postulate that earth goes around the sun?