April 16, 2016 (9:00 am – 12:00 noon) FINAL EXAMINATION

COURSE NO.: PHYS 1050 (Physics 1: Mechanics) FORMULA SHEET: Page 1 of 2

TIME: 3 hours EXAMINER: P. G. Blunden

Mathematics

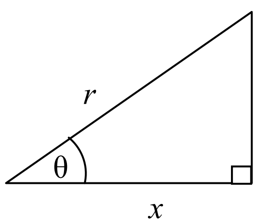
Quadratic equation:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry:

2π rad = 360° $x^2 + y^2 = r^2$



$$\sin\theta = y/r$$
$$\cos\theta = x/r$$
$$\tan\theta = y/x$$

Vectors:

$$a \cdot b = ab\cos\theta$$
$$= a_x b_x + a_y b_y + a_z b_z$$

Calculus:

$$\frac{d}{dt}\left(t^n\right) = nt^{n-1} \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

Constants and Units

k = 10^3, μ = 10^{-6}, n = 10^{-9}
g = 9.80 m/s^2 c = 3.00×10^8 m/s
1 N = 1 kg m/s^2 1 J = 1 kg m^2/s^2
1 W = 1 J/s

Translational Kinematics

One dimension:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Three dimensions:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

Constant acceleration in one dimension:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

Projectile motion ($a_y = -g$):

$$v_{0x} = v_0 \cos\theta_0$$
$$v_{0y} = v_0 \sin\theta_0$$
$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$v_y = v_{0y} - gt$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

Uniform circular motion:

$$a = \frac{v^2}{r} \qquad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{1}{f} \quad \text{period}$$

Relative Motion

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$
$$\vec{v}_{AB} = -\vec{v}_{BA}$$
(AB means A relative to B, etc.)

Particle Dynamics

$\vec{F} = m\vec{a}$ $W = mg$ weight

$f_s \le \mu_s F_N$ static friction
$f_k = \mu_k F_N$ kinetic friction

Kinetic Energy, Work, and Potential Energy

$K = \tfrac{1}{2}mv^2$ kinetic energy
$W = \vec{F} \cdot \vec{d} = \vec{F} \cdot \Delta\vec{x}$ work by constant force
$\Delta K = K_f - K_i = W$

$W = \int_{x_i}^{x_f} F(x)\,dx$ work by variable force
$W = -mg\Delta y$ work by gravitational force

$F_s = -kx$ spring force (Hooke's Law)
$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$ work by spring force

$\Delta U = -W = -\int_{x_i}^{x_f} F(x)\,dx$ potential energy

$$F(x) = -\frac{dU(x)}{dx}$$

$U(y) = mgy$ gravitational PE
$U(x) = \tfrac{1}{2}kx^2$ spring PE

$E_{\text{mec}} = K + U$ mechanical energy

$\Delta E_{\text{mec}} = \Delta K + \Delta U = W_{\text{NC}}$ work by NC forces

$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$ for conservative forces

$P = \dfrac{dW}{dt} = \vec{F} \cdot \vec{v}$ power supplied by force $\vec{F}$

Momentum and Collisions

$\vec{p} = m\vec{v} \qquad \vec{F} = \dfrac{d\vec{p}}{dt}$

$$\Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt = \vec{F}_{avg}\Delta t$$

$\vec{r}_{\text{com}} = \dfrac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$

$\vec{r}_{\text{com}} = \dfrac{1}{M}\int \vec{r}\,dm$ continuous body

$\vec{P} = \sum_{i=1}^{n} \vec{p}_i = M\vec{v}_{\text{com}} \qquad \vec{F}_{\text{ext}} = \dfrac{d\vec{P}}{dt}$

Two-body collisions:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Totally inelastic collision:

$$\vec{v}_f = \vec{v}_{\text{com}} = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}$$

Elastic collision in 1D with target at rest ($v_{2i} = 0$):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Rotational Kinematics

$\omega = \dfrac{d\theta}{dt} = \dfrac{v_t}{r}$ (tangential velocity v_t)

$\alpha = \dfrac{d\omega}{dt} = \dfrac{a_t}{r}$ (tangential acceleration a_t)

$a_r = \dfrac{v^2}{r} = \omega^2 r$ (radial acceleration a_r)

Constant angular acceleration α:

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Rotational Dynamics

$I = \sum_{i=1}^{n} m_i r_i^2$ rotational inertia

$I = \int r^2\,dm$ continuous body

$I = I_{\text{com}} + Mh^2$ parallel axis theorem

$I = \frac{1}{12}ML^2$ (thin rod, about centre)

$I = \frac{1}{3}ML^2$ (thin rod, about end)

$$\tau = F_t r = F r_\perp = Fr\sin\phi$$

$$\tau_{\text{ext}} = I\alpha$$

$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad W = \int_{\theta_i}^{\theta_f} \tau\,d\theta$

$\ell = rp_\perp = (mr^2)\omega \qquad \dfrac{d\ell}{dt} = \tau$ single particle

$L = I\omega \qquad \dfrac{dL}{dt} = \tau_{\text{ext}}$ rigid body

$L_i = L_f$ isolated system, $\tau_{\text{ext}} = 0$

Rolling without slipping:

$v_{\text{com}} = \omega R \qquad a_{\text{com}} = \alpha R$ magnitude only

$K = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{\text{com}}^2(1+\beta)$

$$I = \beta m R^2 \qquad \beta = \begin{cases} 1 & \text{hollow disc} \\ \frac{1}{2} & \text{solid disc} \\ \frac{2}{5} & \text{solid sphere} \end{cases}$$

Special Relativity

$$L = \frac{L_0}{\gamma} \qquad \Delta t = \gamma\Delta t_0 \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Velocity addition:

$$u = \frac{u' + v}{\left(1 + vu'/c^2\right)}$$