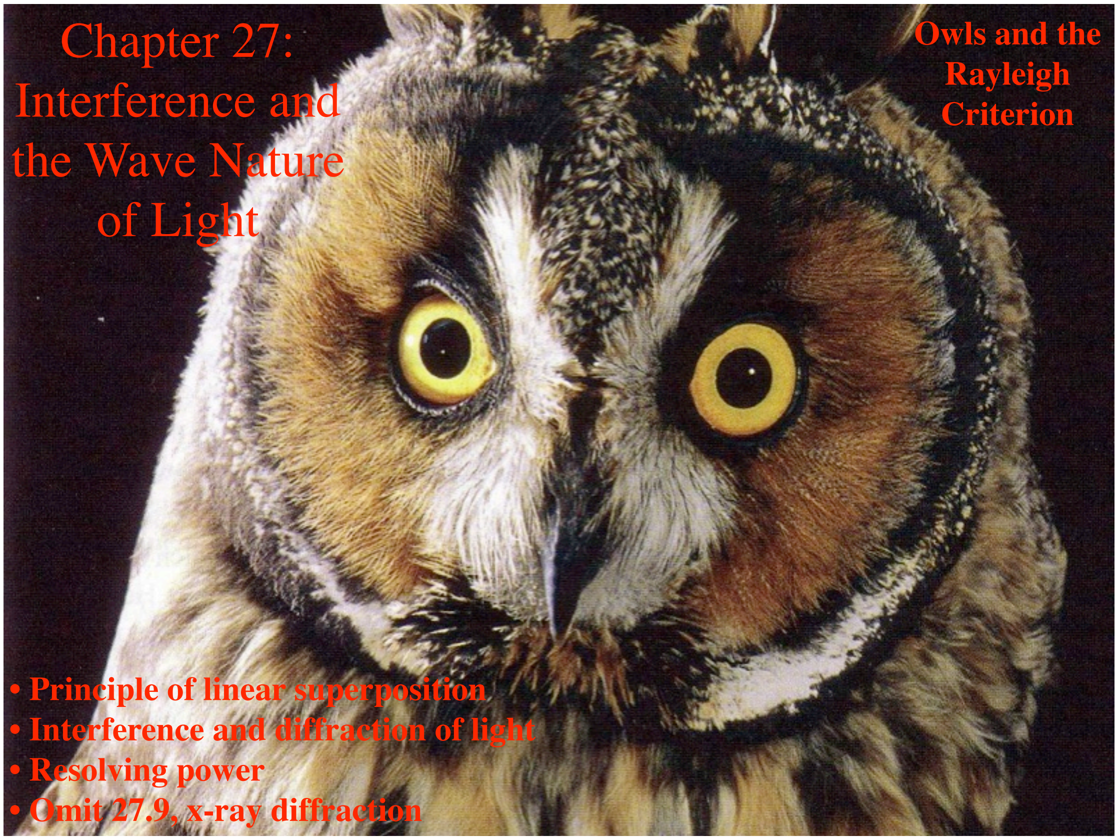
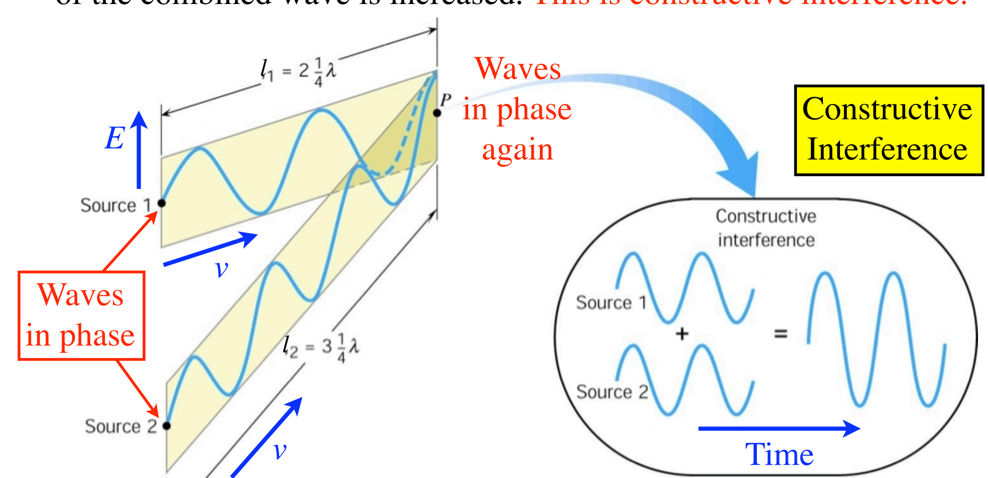


Principle of Linear Superposition

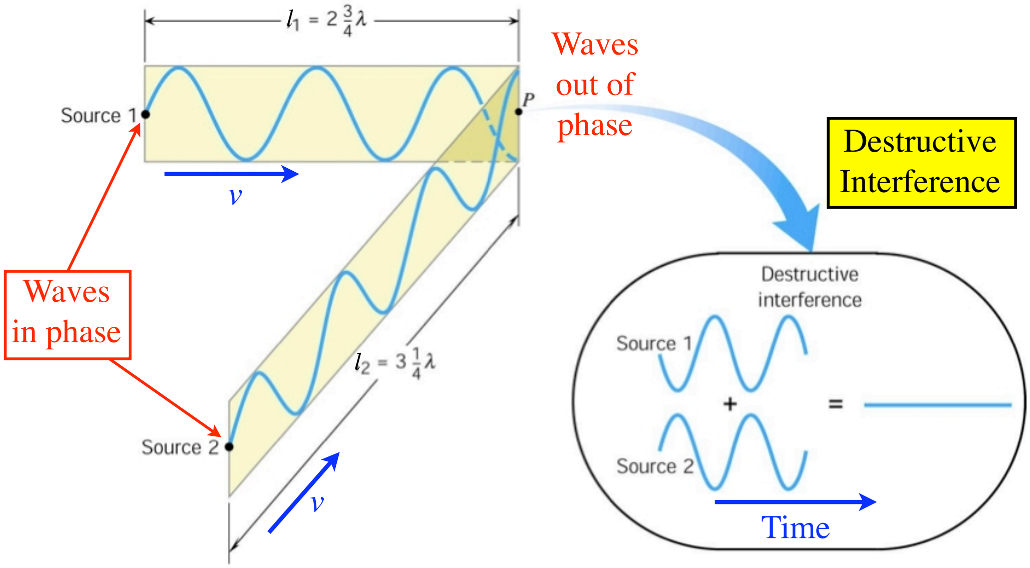
When two waves overlap, the resultant is the sum of the two.
Light waves: add the electric fields.

If the waves are of the same wavelength and are in phase, the amplitude of the combined wave is increased. This is constructive interference.



$$l_2 - l_1 = (3.25 - 2.25)\lambda = \lambda \rightarrow \text{waves in phase}$$

If the waves are of the same wavelength and out of phase, the amplitude of the combined wave is decreased, even zero if the two waves have the same amplitude. This is destructive interference.



$$l_2 - l_1 = (3.25 - 2.75)\lambda = 0.5\lambda \rightarrow \text{waves out of phase}$$

Coherent and incoherent light sources

Coherence: the waves maintain a constant phase relative to one another. Constructive and destructive interference can then occur, depending on the difference in path length. Example, light from a laser.

Constructive interference: the difference in path length is an integer number of wavelengths:

$$l_2 - l_1 = m\lambda, \;\; m = 0, 1, 2, 3... \;\; (m = \text{“order”})$$

Destructive interference occurs if:

$$l_2 - l_1 = (m + 1/2)\lambda$$

If the waves emitted by the sources do not maintain a constant phase relationship, the sources are “**incoherent**.” Example, light from a lamp.

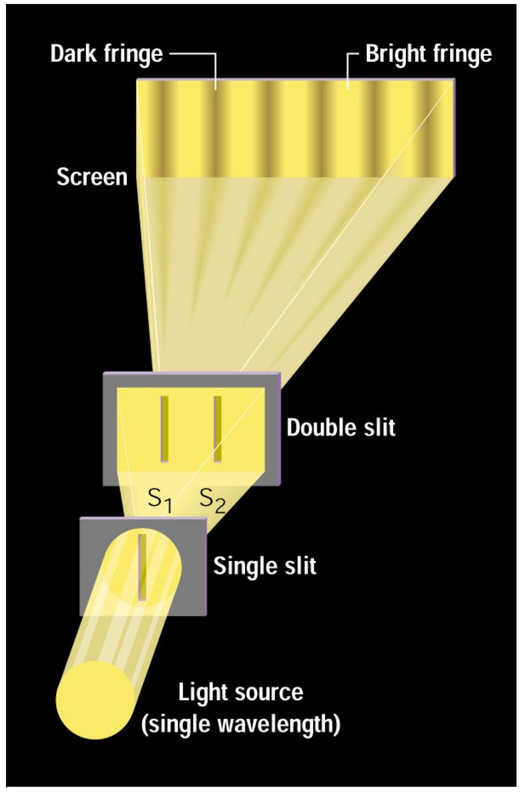
Young's double slit experiment

Light passes through a single slit.

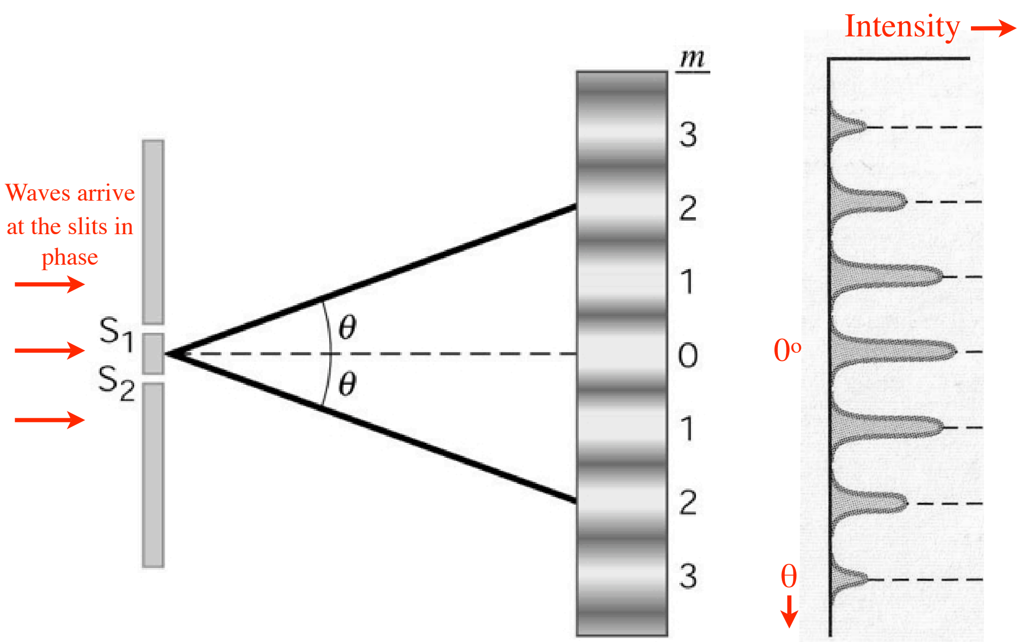
The light wave from the slit spreads out to pass through two slits which act as coherent sources of light.

The light waves from the two slits overlap on a screen.

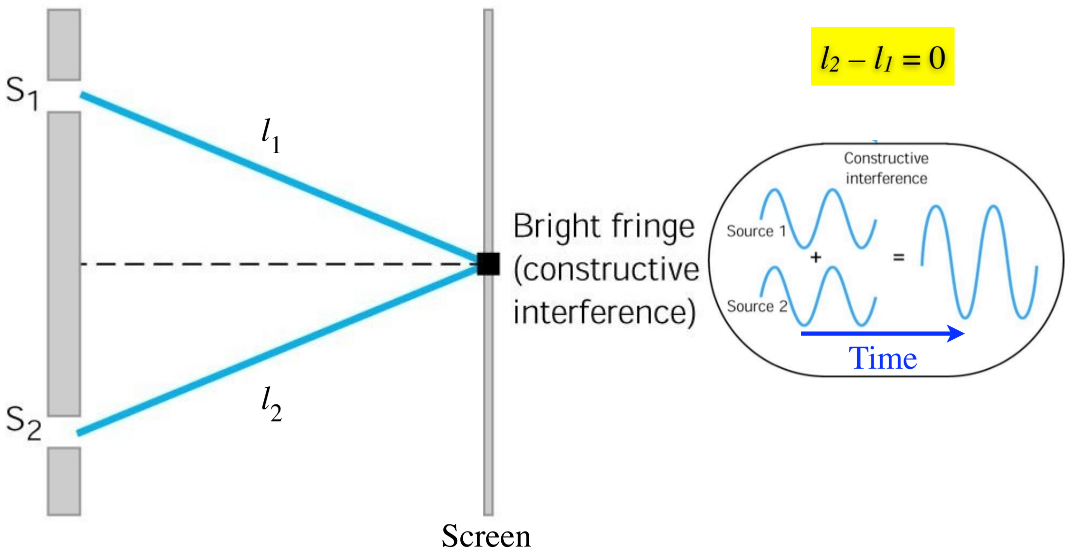
Constructive and destructive interference is seen as a series of bright and dark bands – not as images of the two slits.



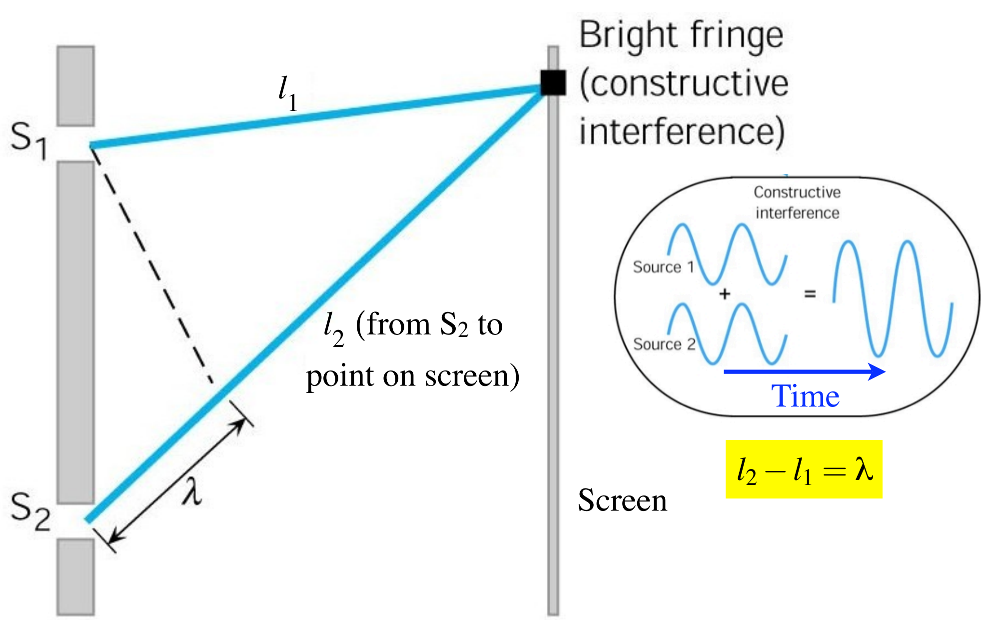
Young's double slit experiment



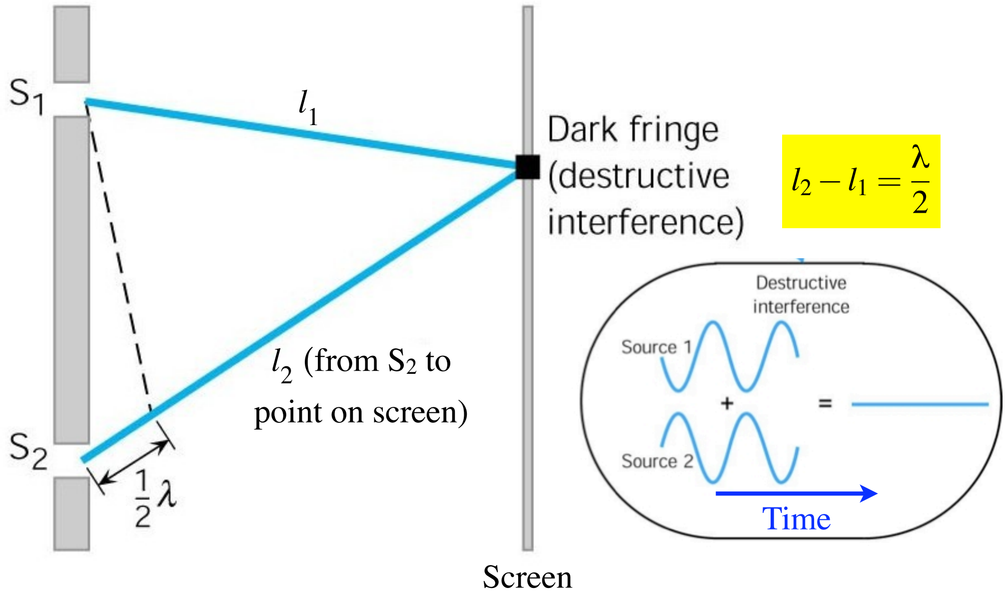
Constructive and destructive interference



Constructive and destructive interference



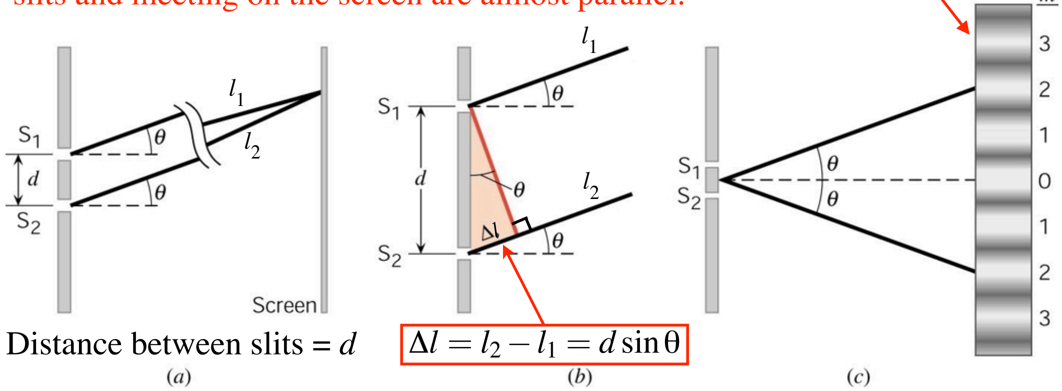
Constructive and destructive interference



$$l_2 - l_1 = \frac{\lambda}{2}$$

Young's double slits

The screen is far away, so the waves leaving the slits and meeting on the screen are almost parallel.



Distance between slits = d

$$\Delta l = l_2 - l_1 = d\sin\theta$$

Bright and dark fringes on the screen

For constructive interference: $d\sin\theta = m\lambda, \quad m = 0, 1, 2\ldots$

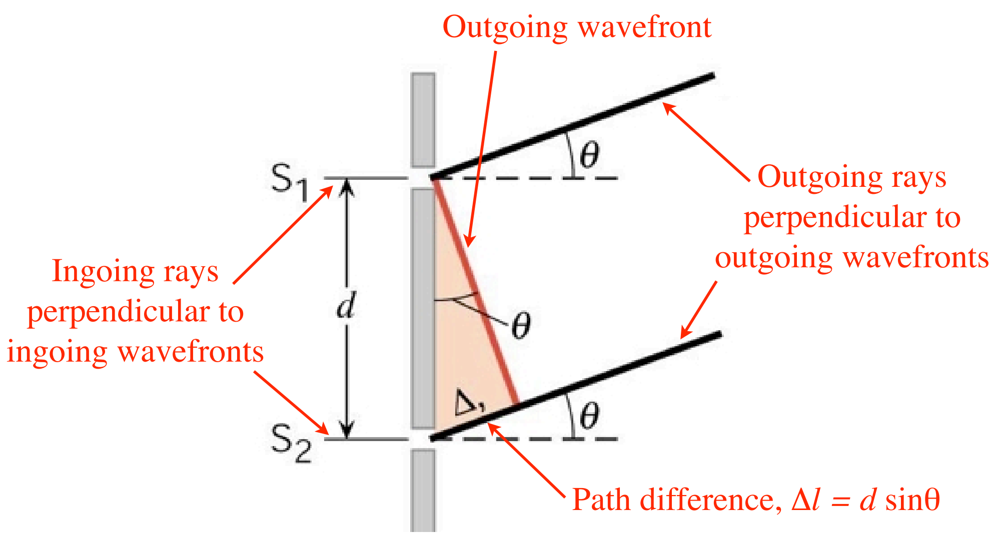
For destructive interference: $d\sin\theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2\ldots$

m = "order" of the fringe

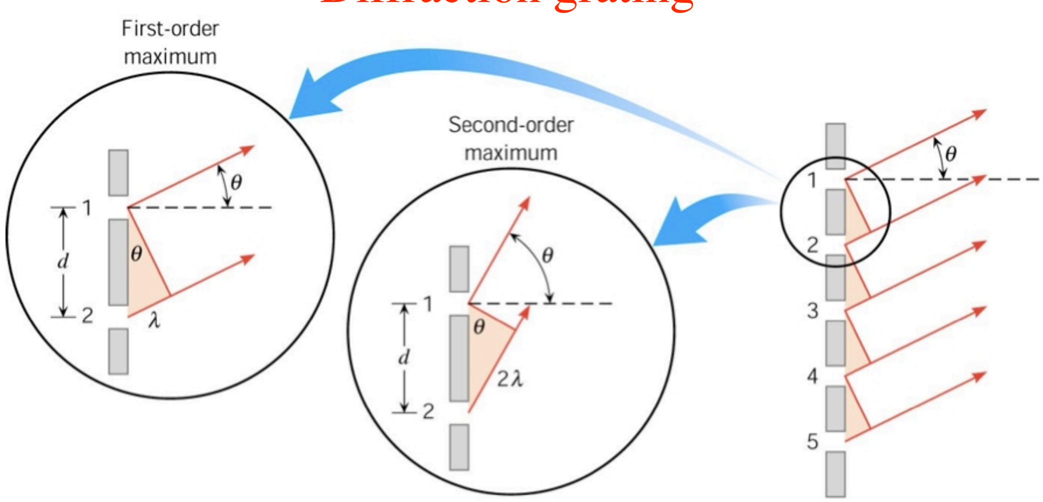
Principle of Linear Superposition

When two waves overlap, the resultant is the sum of the two.
Light waves: add the electric fields.

Young's Double Slits



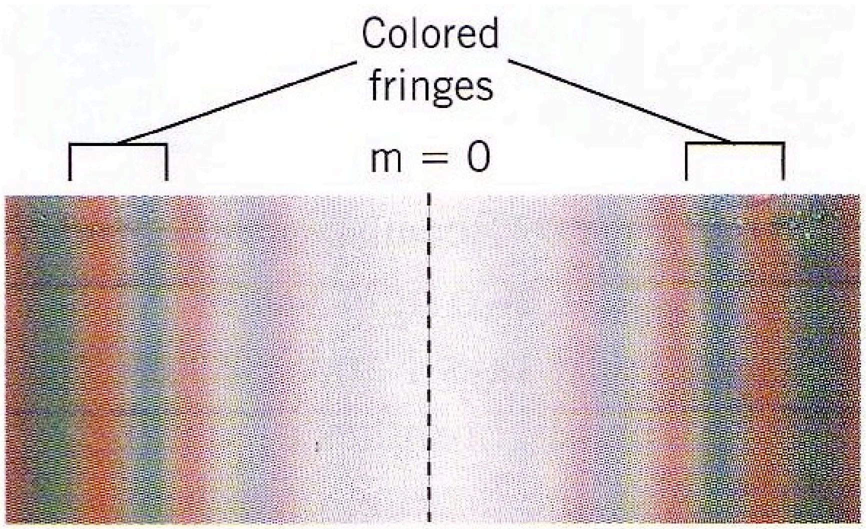
Diffraction grating



Pairs of slits act as two Young's slits. The bright fringes ("principal maxima") are at the same angles as for Young's double slits.
Interference also occurs between more distant slits → sharper peaks

Bright fringes (principal maxima): $d \sin\theta = m \lambda$

Young's double slits with white light



d sinθ = $m\lambda$ for bright fringes of wavelength λ. The different wavelengths have their bright fringes at different angles. If d is large, the angle scale is too small for the fringes to be visible.

27.6

Prob. 27.52/6: Two slits are 0.158 mm apart. A mixture of red light (λ = 665 nm) and yellow-green light (λ = 565 nm) falls on the slits. A screen is 2.24 m away.

What is the distance on the screen between the third-order red fringe and the third-order yellow-green fringe?

In a Young's double slit experiment, blue light of wavelength 440 nm produces a second order bright fringe on a screen.

What wavelength of visible light would produce a dark fringe at the same place?

Visible light: λ = 380 – 750 nm.

Prob. 27.9: A sheet of plastic (n = 1.6) covers one slit of a double slit. When the double slit is illuminated by monochromatic light (λ_0 = 586 nm in vacuum), the centre of the screen appears dark rather than bright.

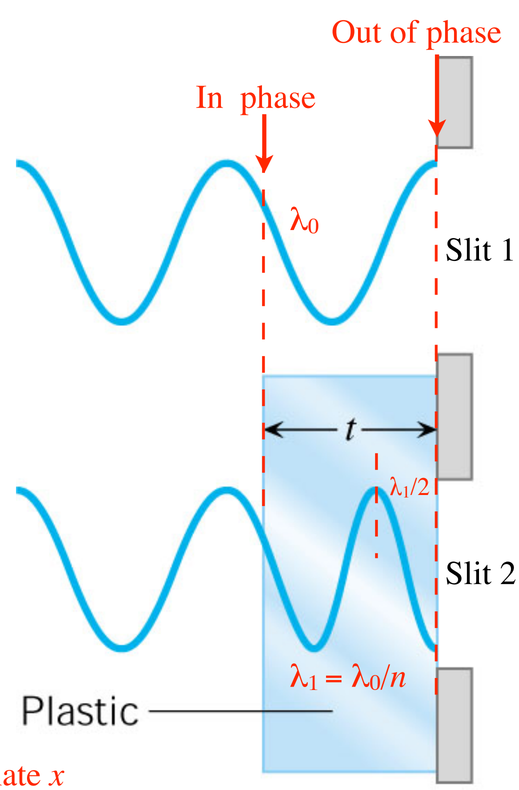
What is the minimum thickness of the plastic?

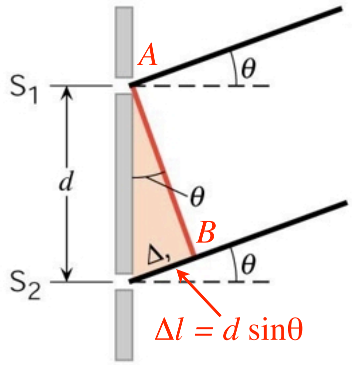
There are x wavelengths in thickness t of air (with wavelength λ_0 = 586 nm)

$t = x\,\lambda_0$

There are $(x + 1/2)$ wavelengths in thickness t of plastic (with wavelength $\lambda_1 = \lambda_0/n$).

$t = (x + 1/2)\,\lambda_1$ → eliminate x



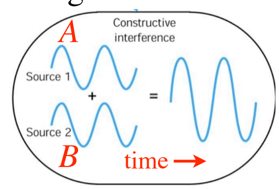


Young's Double Slits

(Outgoing rays parallel – "Fraunhofer diffraction")

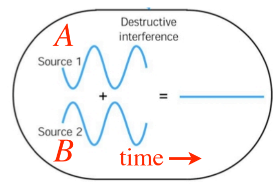
The light waves from the two slits produce bright bands of light when:

$$d \sin\theta = m\,\lambda, \quad m = 0, 1, 2\ldots$$



They produce dark bands when:

$$d \sin\theta = (m + 1/2)\,\lambda, \quad m = 0, 1, 2\ldots$$



Thin film interference in a soap bubble

Thin film interference

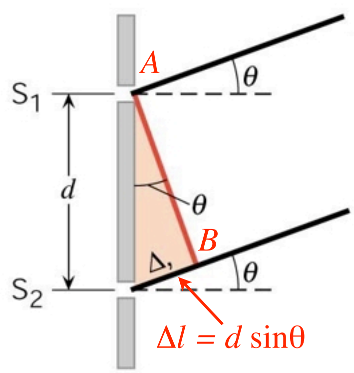
- Anti-reflective coatings on lenses (cameras, eye glasses...)
- Reflective coatings (mirrors, "aviator" sun glasses)

Optical interference devices

- "Interferometers" – check smoothness of a surface at the level of the wavelength of light

Will look at:

– thin film interference
– air wedge
– Newton's rings
– Michelson interferometer



Young's Double Slits

(Outgoing rays parallel – "Fraunhofer diffraction")

The light waves from the two slits produce bright bands of light when:

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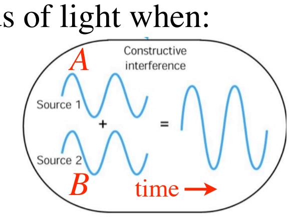
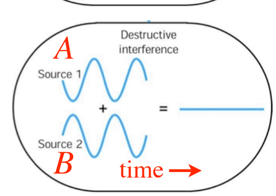


They produce dark bands when:

$$d \sin\theta = (m + 1/2)\lambda, \quad m = 0, 1, 2...$$



Thin film interference

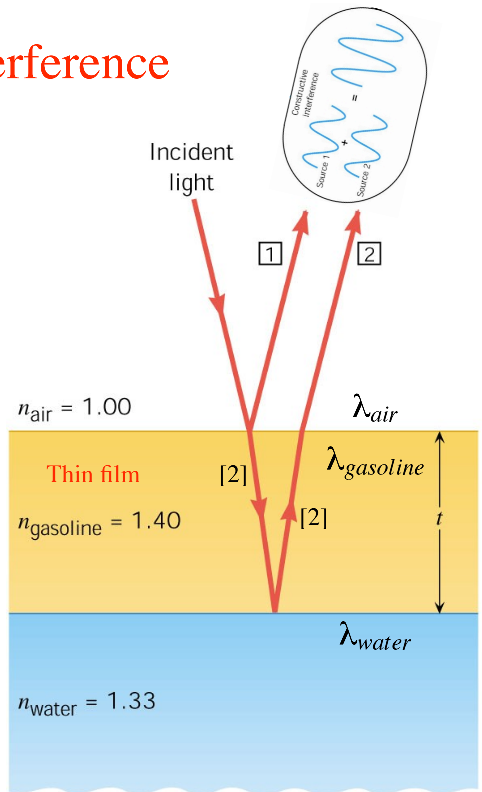
A light wave is split into reflected [1] and refracted [2] parts at the upper surface.

Interference occurs between them when they recombine.

Ray [2] travels an extra distance ≈ $2t$ compared with ray [1].
(small angle of incidence assumed)

$2t = m\lambda_{gasoline}$ for bright fringe?

No! There is a wrinkle due to reflection

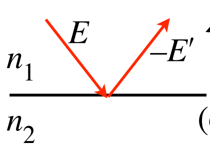


Phase change on reflection

For an **external** reflection (reflection at a surface of higher refractive index) there is a phase change of the reflected ray equivalent to half a wavelength.

Sign is reversed due to phase change

$n_2 > n_1$



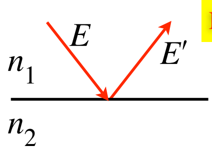
(eg, air, $n_1 = 1$)

(eg, water, $n_2 = 1.333$)

For an **internal** reflection (reflection at a surface of lower refractive index) there is no phase change.

No reversal of sign

$n_2 < n_1$

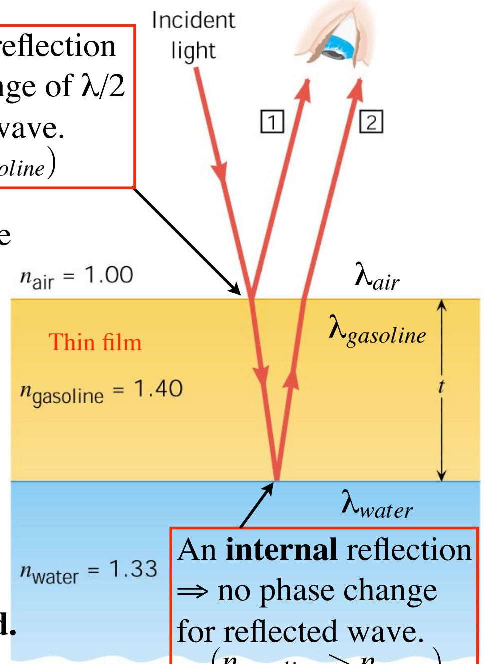


(eg, glass, $n_1 = 1.5$)

(eg, air, $n_2 = 1$)

Thin film interference – the wrinkle

An **external** reflection
$\Rightarrow$ phase change of $\lambda/2$
for reflected wave.
$(n_{air} < n_{gasoline})$



An **internal** reflection
$\Rightarrow$ no phase change
for reflected wave.
$(n_{gasoline} > n_{water})$

Then, $2t = (m + \frac{1}{2})\lambda_{gasoline}$ for bright fringe

$$\lambda_{gasoline} = \frac{\lambda_{air}}{n_{gasoline}}$$

So, $2t = (m + \frac{1}{2})\frac{\lambda_{air}}{n_{gasoline}}$ bright fringe

and $2t = \frac{m\lambda_{air}}{n_{gasoline}}$ for dark fringe

If both reflections are internal or both are external, these results are exchanged.

Summary of thin film interference results

Reflective coating (mirror, aviator sun glasses...) Constructive

2 internal or 2 external reflections:	$2t = m\lambda_{film}, \quad m = 1,2,3\ldots$
1 internal and 1 external reflection:	$2t = (m+\frac{1}{2})\lambda_{film}, \quad m = 0,1,2\ldots$

Anti-reflective coating (camera lens, eye glasses...) Destructive

2 internal or 2 external reflections:	$2t = (m+\frac{1}{2})\lambda_{film}, \quad m = 0,1,2\ldots$
1 internal and 1 external reflection:	$2t = m\lambda_{film}, \quad m = 1,2,3\ldots$

t = thickness of film, $\lambda_{film} = \lambda_{vacuum}/n_{film}$

Prob. 27.48/11: A non-reflective coating coating of magnesium fluoride (n = 1.38) covers the glass (n = 1.52) of a camera lens.

Assuming the coating prevents reflection of yellow-green light (wavelength in vacuum = 565 nm), determine the minimum non-zero thickness that the coating can have.

- Are the reflections at the top and bottom layers internal (no phase change) or external (phase change)?
- Choose the appropriate formula for destructive interference.

Prob. 27.11/10: Light of wavelength 691 nm (in vacuum) is incident on a soap film (n = 1.33) suspended in air. What are the two smallest non-zero film thicknesses for which the reflected light undergoes constructive interference?

- Are the reflections at the top and bottom layers internal (no phase change) or external (phase change)?
- Choose the appropriate formula for constructive interference.

Fringes produced by an air wedge

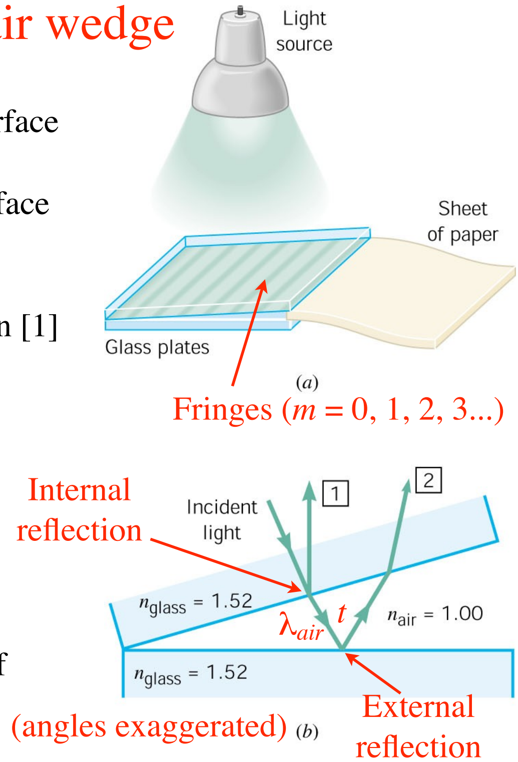
External reflection for [1] at upper surface
$\Rightarrow$ phase change of $\lambda/2$
Internal reflection for [2] at lower surface
$\Rightarrow$ no phase change

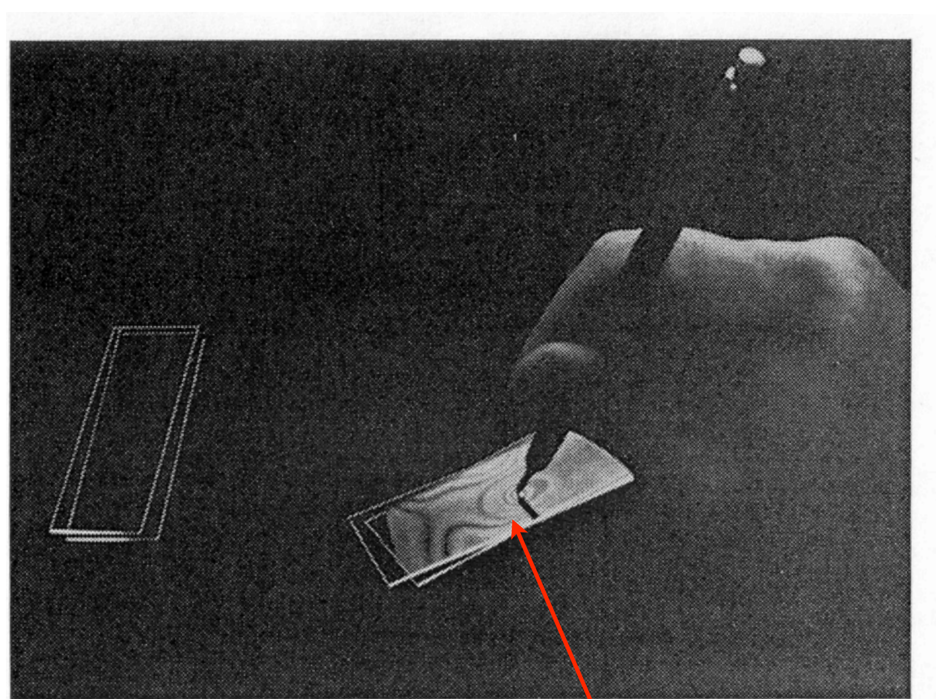
For **constructive** interference between [1] and [2]:

$$2t = (m + \frac{1}{2})\lambda_{air}, \quad m = 0, 1, 2 \ldots$$

$2t$ increases by λ_{air} between bright fringes (m increasing by 1)

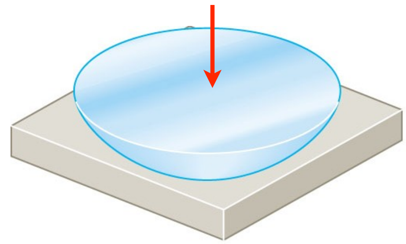
Uneven surfaces $\Rightarrow$ jogs or curving of the fringes





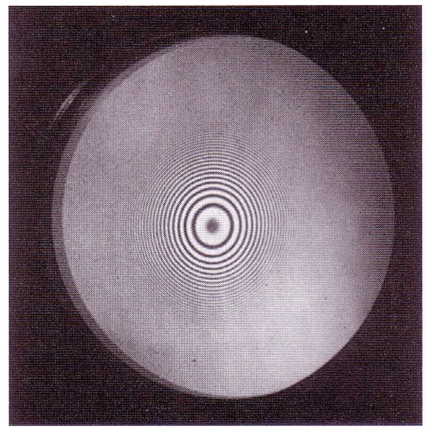
Fringes follow contours of constant air gap
$2t$ changes by λ between fringes

Newton's Rings



The air gap is between the lower surface of the lens and a flat glass plate.

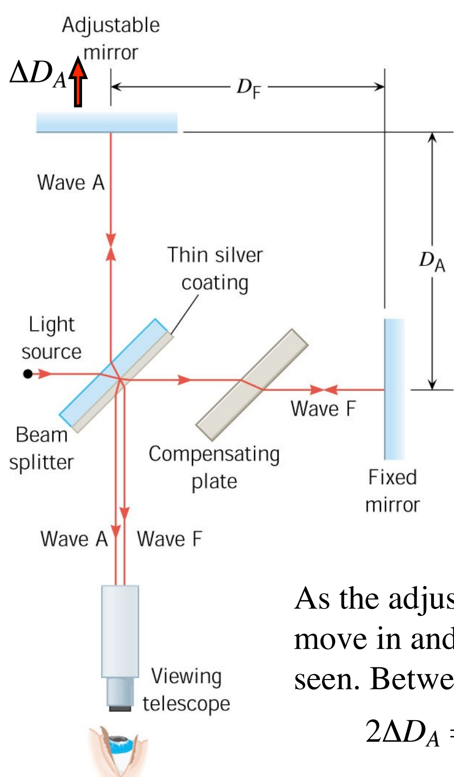
Interference of the light reflected from the flat glass plate and the lower surface of the lens is viewed from above.



$\Rightarrow$ concentric circular fringes.

Nonuniformity of surfaces distorts the rings.

Michelson interferometer – measurement of wavelength of light

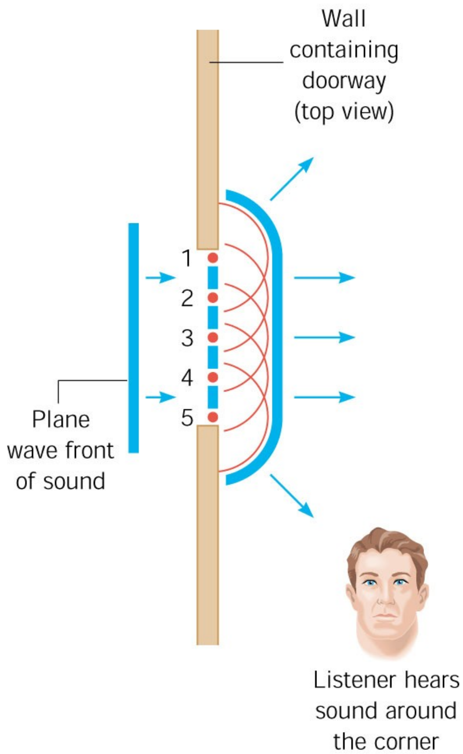


The light from the source is divided into reflected and refracted beams, wave A and wave F, by a beam splitter (partially silvered mirror).

The two beams are reflected back and recombine, interfere, and are observed through a telescope.

As the adjustable mirror is moved, the waves A and F move in and out of phase and bright and dark fringes are seen. Between bright fringes:

$$2\Delta D_A = \lambda \quad \Rightarrow \text{wavelength of the light}$$



Diffraction – Huygens construction

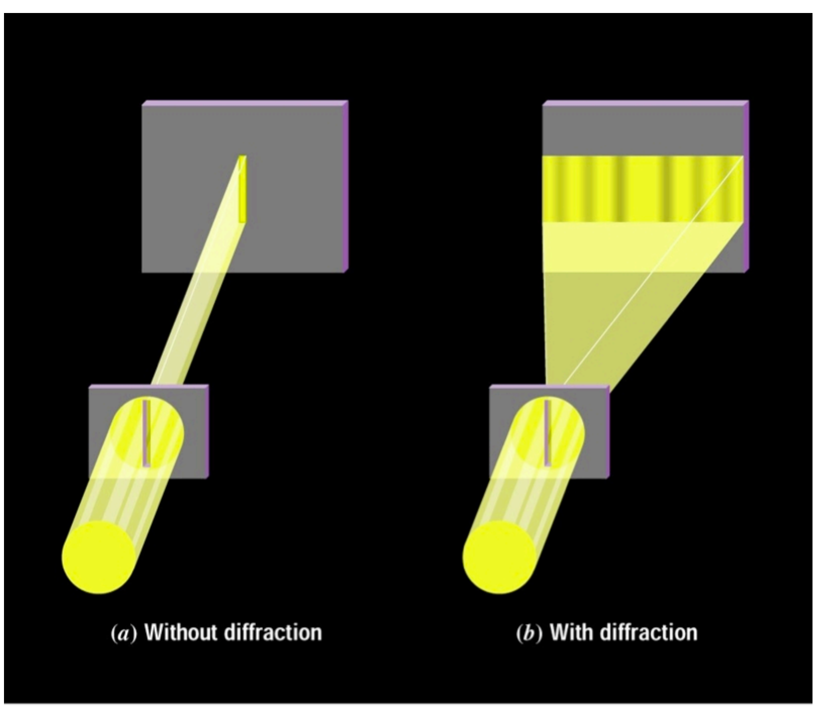
Each point on a wavefront acts as a source of secondary waves.

The new wavefront is tangent to the secondary waves.

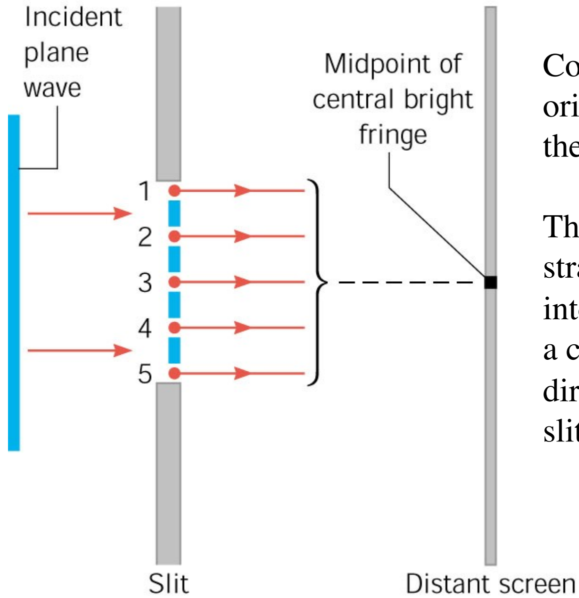
⇒ waves spread out – are diffracted – around a barrier or the edges of an opening.

The same phenomenon occurs with water and light waves ⇒ light does not always travel in perfect straight lines.

Diffraction by a single vertical slit



Diffraction by a single vertical slit

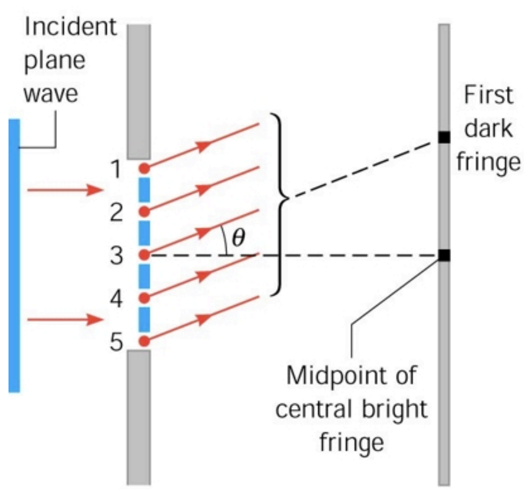


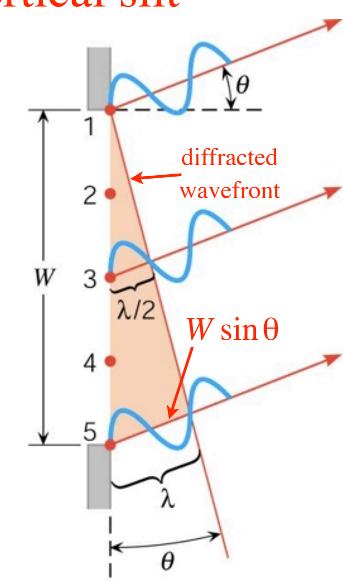
Much magnified view of the slit

Consider secondary waves originating from points all across the slit.

The secondary waves travelling straight ahead are all in phase and interfere constructively to produce a central bright fringe at $\theta = 0$, directly opposite the centre of the slit.

Diffraction by a single vertical slit

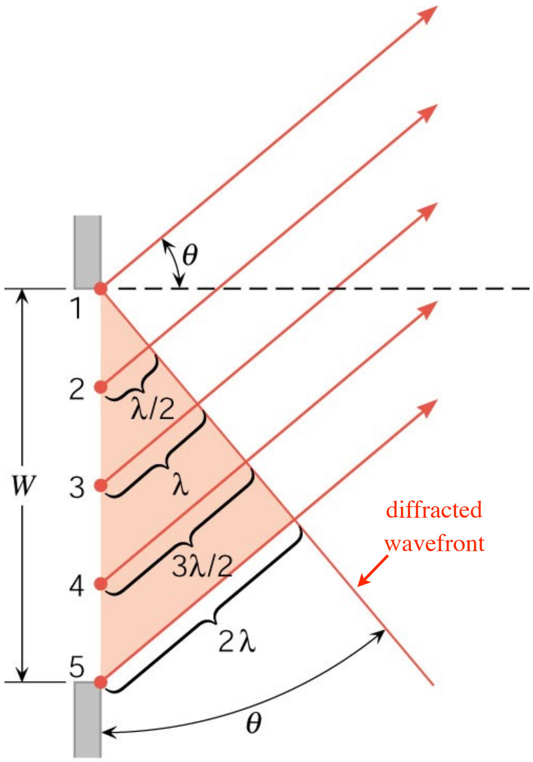




Much magnified view of the slit

At angle θ secondary waves from 1 and 3 are out of phase and cancel. So are waves from 2 and 4, and all other waves that are half a slit apart.

$\rightarrow$ first dark fringe when $W \sin\theta = \lambda$



Diffraction by a single vertical slit

At a larger angle, all points across the slit separated by a distance $W/4$ are also out of phase, can be paired off to interfere destructively and produce a dark fringe when:

$$W \sin\theta = 2\lambda$$

In general, the dark fringes are seen when:

$$W \sin\theta = m\lambda, \quad m = 1, 2, 3\ldots$$

Prob. 27.-/20: A slit of width $W = 4.3\times10^{-5}$ m is located 1.32 m from a flat screen. Light shines through the slit and falls on the screen.

Find the width of the central fringe of the diffraction pattern when the wavelength of the light is 635 nm.

- At what angle is the first minimum found?

Thin film interference

2 internal or 2 external reflections:
(refractive indices in increasing or decreasing order)

$$2t = m\lambda_{film} \quad \rightarrow \quad \text{constructive interference}$$
$$2t = (m + \frac{1}{2})\lambda_{film} \quad \rightarrow \quad \text{destructive interference}$$

1 internal and 1 external reflection:
(refractive indices not in increasing or decreasing order)

$$2t = (m + \frac{1}{2})\lambda_{film} \quad \rightarrow \quad \text{constructive interference}$$
$$2t = m\lambda_{film} \quad \rightarrow \quad \text{destructive interference}$$

$$\lambda_{film} = \frac{\lambda_{air}}{n_{film}}$$

Young's double slits and diffraction grating

Bright fringes when: $d \sin\theta = m\lambda, \;\; m = 0, 1, 2 \ldots$

Dark fringes when: $d \sin\theta = (m + \frac{1}{2})\lambda, \;\; m = 0, 1, 2 \ldots$

d = distance between slits

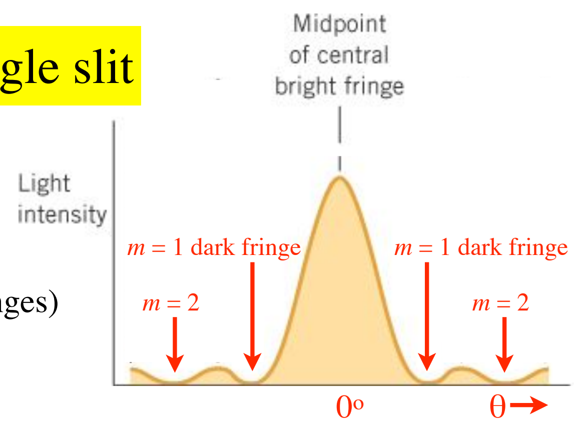
Diffraction from a single slit

Dark fringes when:

$$W \sin\theta = m\lambda, \;\; m = 1, 2, 3 \ldots$$

(no simple formula for bright fringes)

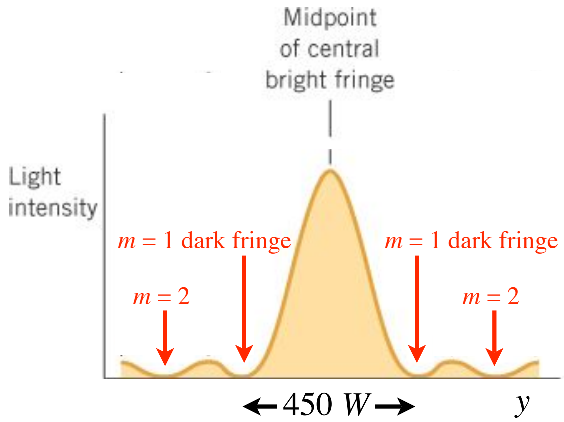
W = width of slit



Prob. 27.27: In a single slit diffraction pattern, the central fringe is 450 times as wide as the slit. The screen is 18,000 times farther from the slit than the slit is wide.

What is the ratio λ/W, where W is the slit width?

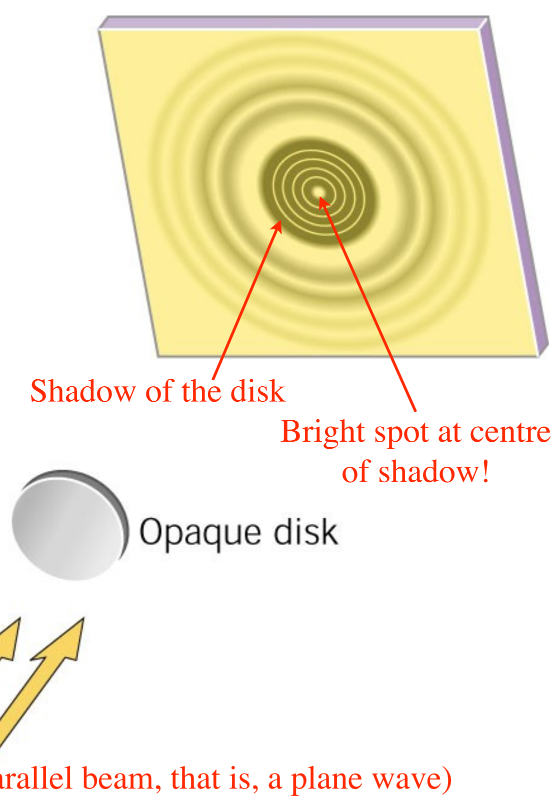
Assume that angles are small, so $\sin\theta \approx \tan\theta \approx \theta$ radians.



Diffraction by a small opaque disk

Because of diffraction of light from the edge of the circular disk, the shadow cast by the disk has:

- a central bright spot (constructive interference of light waves from around edge of disk)
- circular bright fringes inside the shadow
- bright and dark fringes outside the shadow



Diffraction by a circular aperture

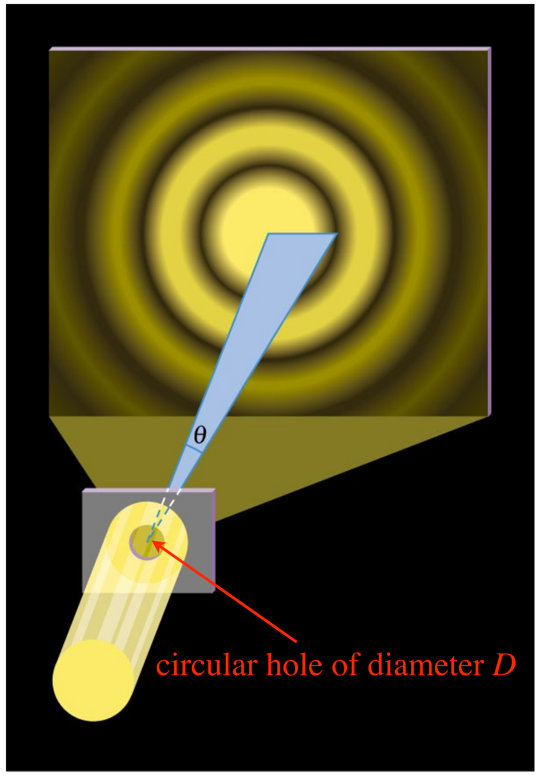
The bright and dark fringes are circular and concentric with the aperture.

The first dark fringe from the centre is at the angle shown in the diagram:

$$\theta = 1.22\frac{\lambda}{D} \text{ radians}$$

D = diameter of aperture

The smaller the aperture, the larger the angle of diffraction.

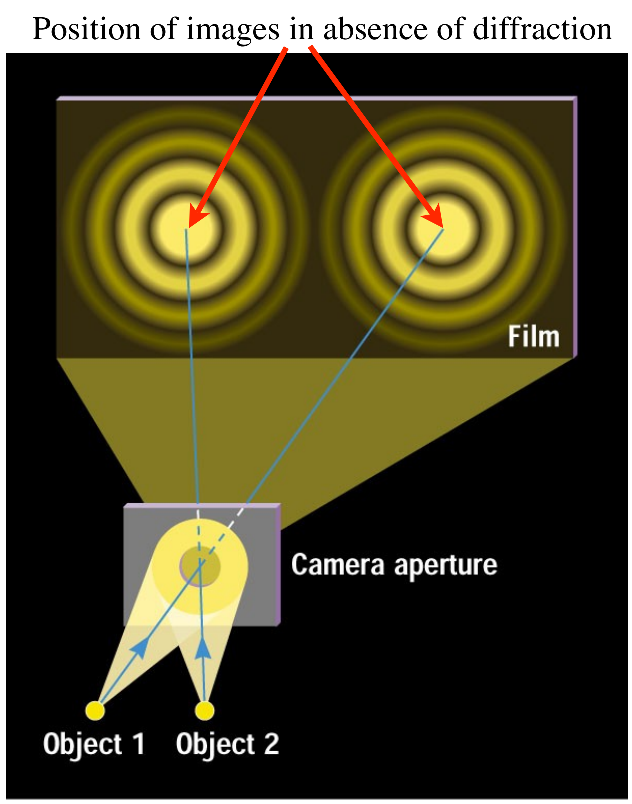


Resolving power

Light passing through any aperture, such as a camera lens, or an eye lens, is diffracted.

If two objects are close together, the diffraction patterns from them may overlap.

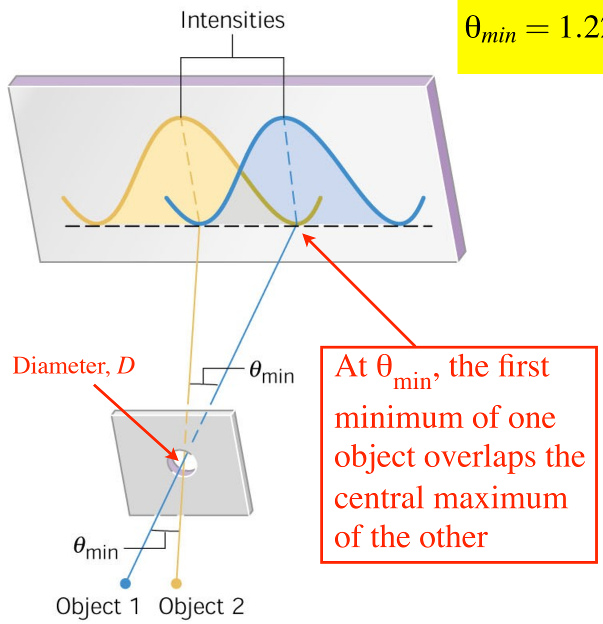
If the overlap of the patterns is large enough, it may not be possible to tell there are two objects – "resolution" is limited by diffraction.



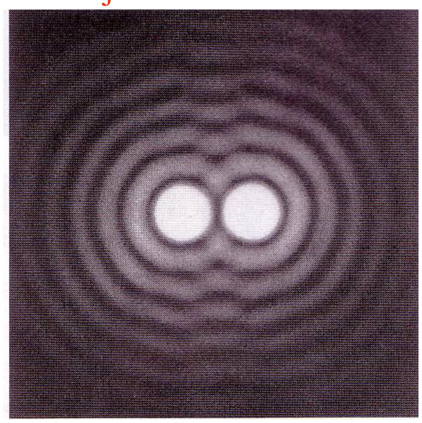
Resolving power – Rayleigh criterion

Two objects can be resolved when their angular separation is greater than:

$$\theta_{min} = 1.22\frac{\lambda}{D} \text{ radians} \quad \text{(Rayleigh criterion)}$$



Objects well-resolved



Resolving Power

For green light, λ = 555 nm, and the diameter of the pupil of the eye, D = 2.5 mm, the angular resolution is:

$$\theta_{min} = 1.22\frac{\lambda}{D} = 2.7 \times 10^{-4} \text{ rad}$$

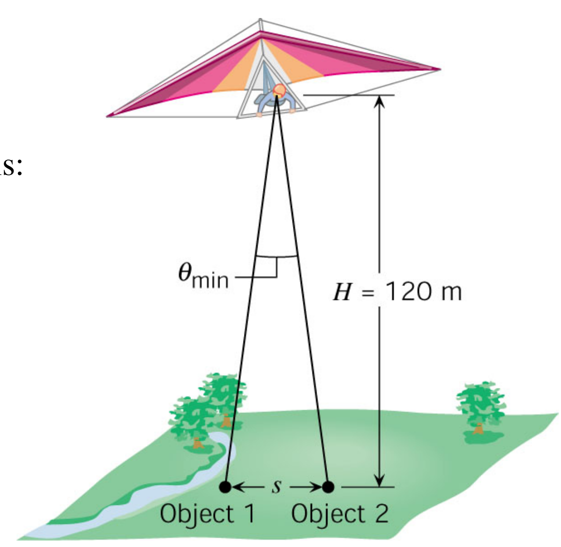
At 120 m altitude, the minimum resolvable distance on the ground is:

$$s = H\theta_{min} = 0.033 \text{ m}$$

Eagle/owl: D = 6.2 mm

$$\theta_{min} = 1.1 \times 10^{-4} \text{ rad},$$
$$s = 0.013 \text{ m}$$

→ better ability to find small furry animals on the ground

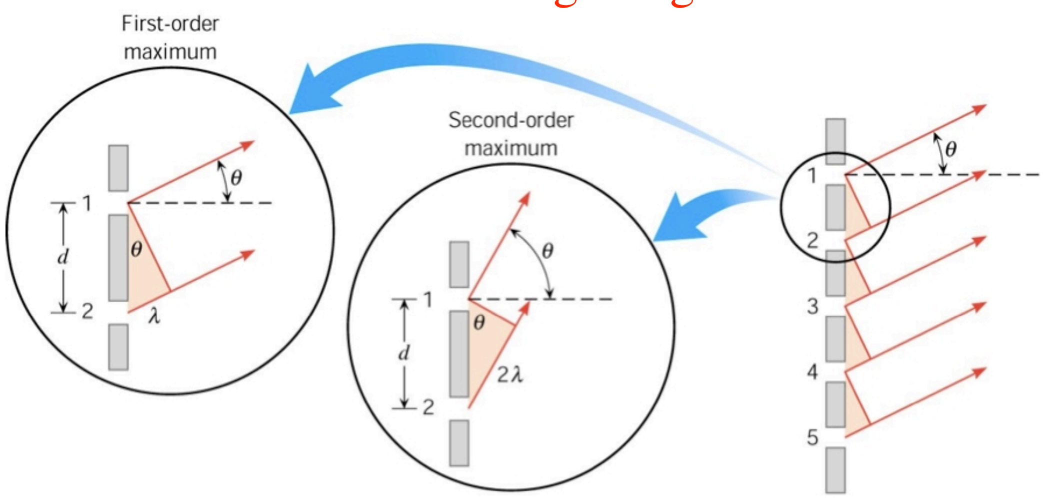


Prob. 27.30: Two stars are 3.7×10^{11} m apart and are equally distant from the earth. A telescope has an objective lens with a diameter of 1.02 m and just detects these stars as separate objects.

Assume that light of wavelength 550 nm is being observed and that diffraction effects rather than atmospheric turbulence limit the resolving power of the telescope.

Find the maximum distance that these stars could be from earth.

Diffraction grating



Pairs of slits act as two Young's slits. The bright fringes ("principal maxima") are at the same angles as for Young's double slits. Interference also occurs between more distant slits → sharper peaks

Bright fringes (principal maxima): $d \sin\theta = m \lambda$

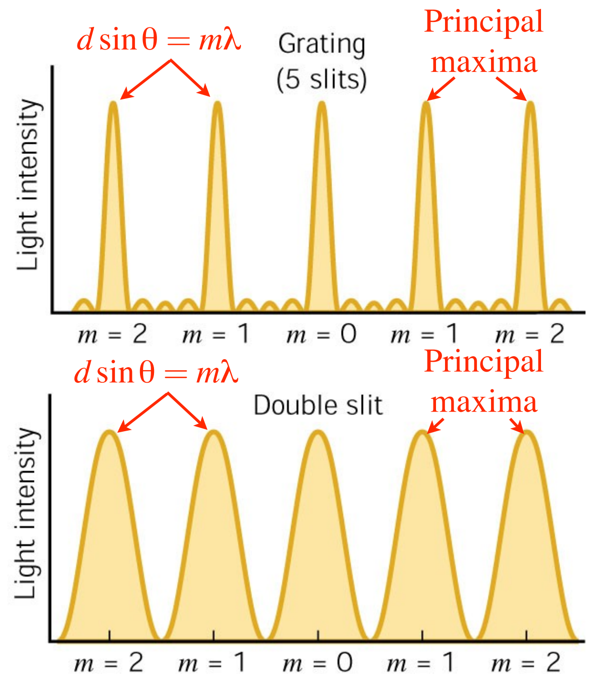
Diffraction grating

Much sharper fringes than Young's double slits

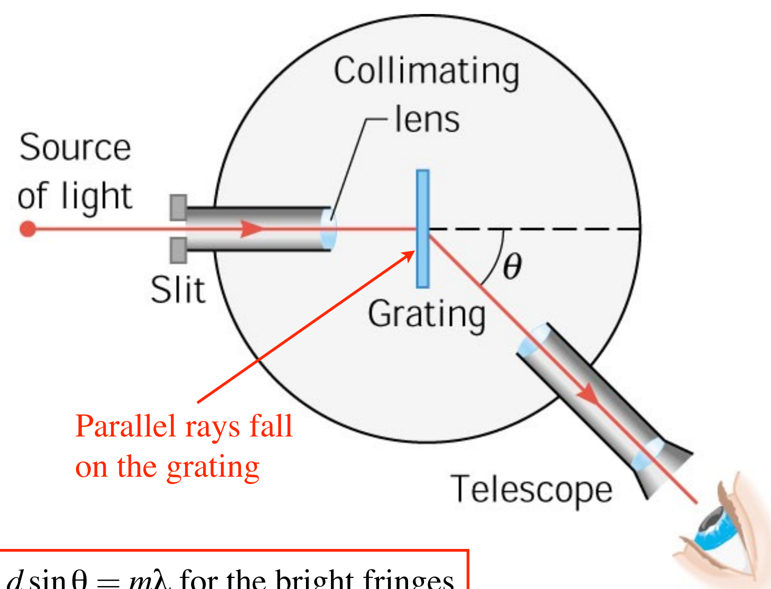
$\Rightarrow$ much more precise measurement of wavelength

The small peaks are "subsidiary maxima"

Young's double slits with same distance between slits, d



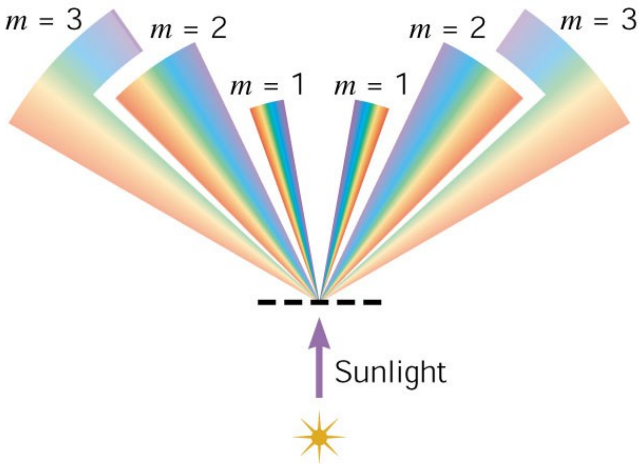
A grating spectrometer



Grating equation: $d\sin\theta = m\lambda$ for the bright fringes

$$\lambda = \frac{d\sin\theta}{m} \rightarrow \text{ wavelength, } \lambda$$

Spectrum of visible light seen by a grating



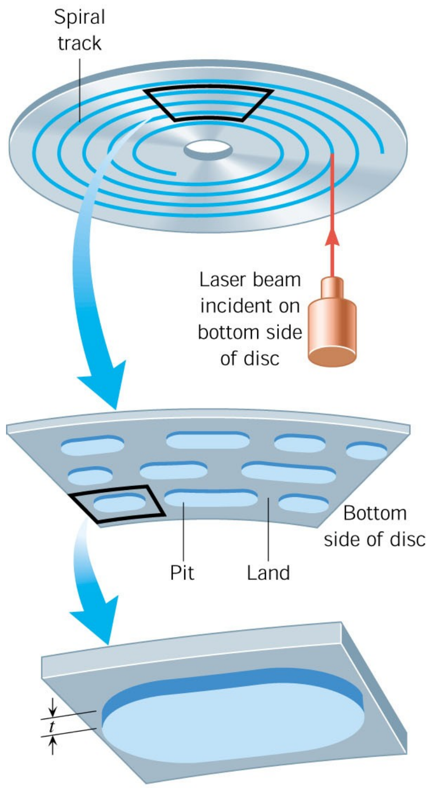
The second order red overlaps the third order blue.

The spectrum of light from a star allows elements in the star's "atmosphere" to be identified. Helium was observed in the sun's atmosphere before it had been discovered on earth. Named after the Greek word for the sun.

27.42

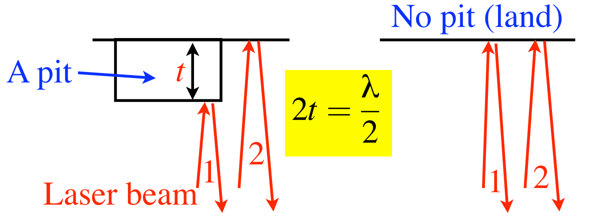
Prob. 27.42: A diffraction grating has 2604 lines per centimetre and produces a principal maximum at $\theta = 30^o$. The grating is used with light that contains all wavelengths between 410 and 660 nm.

What wavelengths of the incident light that could have produced this maximum?



CDs, DVDs

Information is stored along spiral tracks 1600 nm apart as binary 0's and 1's that are read by how much a laser beam is reflected from the surface.

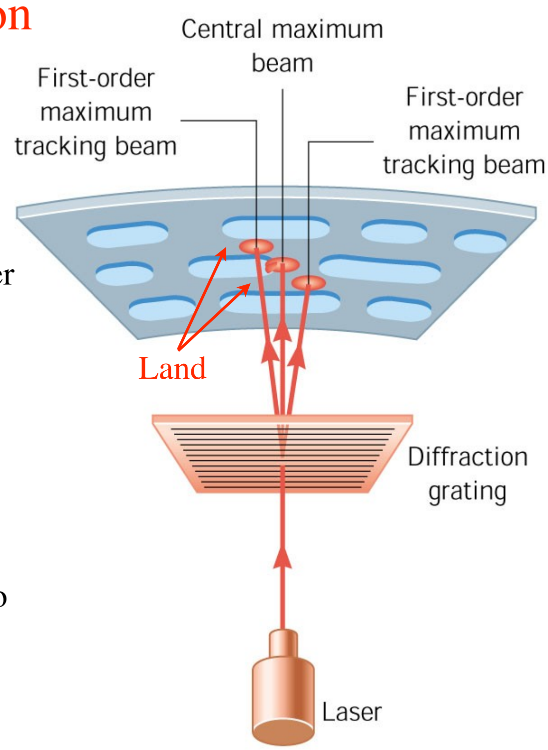


Destructive interference, weak reflection

Constructive interference, strong reflection

If λ = 520 nm, $t = \lambda/4 = 130$ nm for destructive interference.

Keeping the laser beam on track



Two tracking beams are the first order maxima produced by a diffraction grating.

If the laser drifts off track, one of the tracking beams will start to hit pits and will be reflected less strongly.

A feedback system steers the beam to centre it so that both tracking beams are once again strongly reflected.

Summary of Chapter 27

- Principle of Linear Superposition → interference and diffraction
- Diffraction by Young's double slits, by a single slit, and the diffraction grating
- Interference in thin films, phase change on reflection (for external reflections only)
- Michelson interferometer
- Interference by a circular aperture → Rayleigh criterion and resolving power