

Chapter 7: Impulse and Momentum

Newton's Second Law in Another Guise

- Impulse-Momentum Theorem
- Principle of Conservation of Linear Momentum
- Collisions in One Dimension
- Collisions in Two Dimensions
- Centre of Mass

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Impulse and Momentum

Newton's second law: $\vec{F} = m\vec{a}$

$$\text{Or } \vec{F} = m \frac{\Delta \vec{v}}{\Delta t}$$

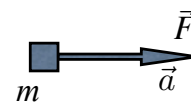
$$\text{So } \vec{F} \Delta t = m \Delta \vec{v}$$

$\vec{F} \Delta t$ is the *impulse* of the force \vec{F}

Define momentum $\vec{p} = m\vec{v}$

Then $\vec{F} \Delta t = m \Delta \vec{v} = \Delta p$ (Impulse-momentum theorem)

That is, impulse = change in momentum



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$$\vec{F} \Delta t = m \Delta \vec{v} = \Delta p$$

If $\vec{F} = 0$, then $\Delta \vec{p} = 0$

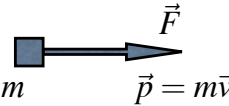
That is, momentum is conserved when the net force acting on an object is zero.

This applies also to an isolated system of two or more objects (no external forces) that may be in contact - the total momentum is conserved.

Compare Newton's first law: velocity is constant when the net force is zero.

Alternative formulation of Newton's second law

$\vec{F} \Delta t = m \Delta \vec{v} = \Delta p$



m \vec{F}
 $\vec{p} = m\vec{v}$

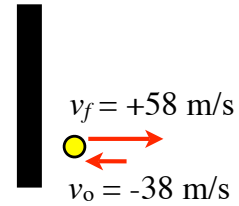
OR:

$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$

The net force acting on an object is equal to the rate of change of momentum of the object.

A 0.14 kg baseball has an initial velocity $v_0 = -38 \text{ m/s}$ as it approaches a bat.

The bat applies an average force F that is much larger than the weight of the ball.



After being hit by the bat, the ball travels at speed $v_f = +58 \text{ m/s}$.

a) The impulse applied to the ball is $mv_f - mv_0 = m(v_f - v_0)$

$$\text{Impulse} = (0.14 \text{ kg}) \times (58 - (-38)) = 13.44 \text{ N.s} \quad (\text{or kg.m/s})$$

b) The bat is in contact with the ball for 1.6 ms.

The average force of the bat on the ball is

$$F = \text{Impulse}/\text{time} = (13.44 \text{ N.s})/(0.0016 \text{ s}) = 8,400 \text{ N}$$

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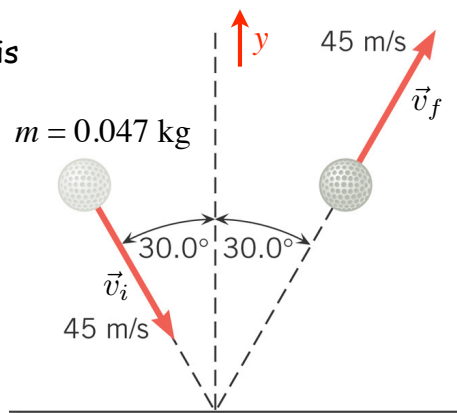
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7.13/9: A golf ball strikes a hard, smooth floor at an angle of 30° , and rebounds at the same angle. What is the impulse applied to the golf ball by the floor?

NB: velocity in sideways direction is unchanged

$$\begin{aligned} v_i &= -45 \cos 30^\circ \quad \leftarrow \text{In y-direction} \\ v_f &= +45 \cos 30^\circ \\ m &= 0.047 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Impulse} &= p_f - p_i \\ &= m(v_f - v_i) \\ &= 0.047(45 + 45)\cos 30^\circ \\ &= 3.7 \text{ N.s} \end{aligned}$$

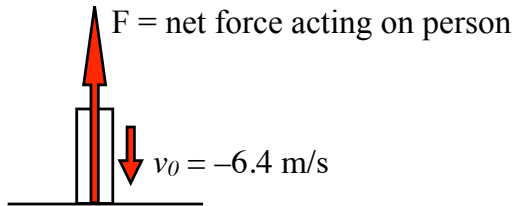


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7.50/8: Absorbing the shock when jumping straight down.

a) A 75 kg man jumps down and makes a stiff-legged impact with the ground at 6.4 m/s (eg, a jump from 2.1 m) lasting 2 ms. Find the average force acting on him in this time.



Change in momentum = impulse = force \times time

$$F \Delta t = \Delta p = 0 - mv_0$$

$$\text{So } F = -mv_0 / \Delta t = (75 \text{ kg} \times 6.4 \text{ m/s}) / (0.002 \text{ s}) = 240,000 \text{ N} \\ = 327mg !!$$

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b) After extensive reconstructive surgery, he tries again, this time bending his knees on impact to stretch out the deceleration time to 0.1 s.

The average force is now: $F = -mv_0 / \Delta t$

$$F = 75 \times 6.4 / 0.1 = 4,800 \text{ N} = 6.5mg$$

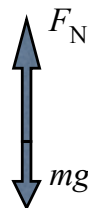
c) The net force acting on the person is:

$$F = F_N - mg$$

So the force of the ground on the person is:

$$F_N = F + mg = F + 75g$$

$$= 5535 \text{ N} = \text{momentary reading on bathroom scales,} \\ \text{equivalent to weight of a 565 kg mass.}$$



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Conservation of Momentum

Two isolated masses collide. The initial total momentum is:

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

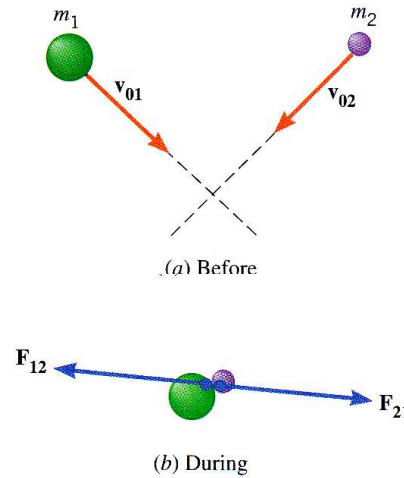
$$\text{with } \vec{p}_1 = m_1 \vec{v}_{01}$$

$$\vec{p}_2 = m_2 \vec{v}_{02}$$

While the masses are in contact, they exert equal and opposite forces on each other (Newton's third law).

$$\vec{F}_{12} = -\vec{F}_{21}$$

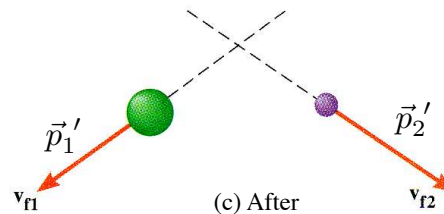
So the impulse acting on m_1 is equal in magnitude and opposite in direction to the impulse acting on m_2



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So the *impulse* acting on m_1 is equal in magnitude and opposite in direction to the *impulse* acting on m_2



Therefore, $\Delta \vec{p}_1 = -\Delta \vec{p}_2$ (change in momentum = impulse)

After the collision: $\vec{p}'_1 = \vec{p}_1 + \Delta \vec{p}_1$

$$\vec{p}'_2 = \vec{p}_2 + \Delta \vec{p}_2 = \vec{p}_2 - \Delta \vec{p}_1$$

So, the total momentum after the collision is:

$$\begin{aligned} \vec{p}' &= \vec{p}'_1 + \vec{p}'_2 = (\vec{p}_1 + \cancel{\Delta \vec{p}_1}) + (\vec{p}_2 - \cancel{\Delta \vec{p}_1}) \\ &= \vec{p}_1 + \vec{p}_2 \\ &= \vec{p} \end{aligned}$$

That is, $\vec{p}' = \vec{p}$ and the total momentum is conserved.

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GENERAL PHYSICS I: PHYS 101

Schedule - Fall 2007 (lecture schedule is approximate)

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	F	9	27			

Week of October 29 Experiment 3: Forces in Equilibrium

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Mastering Physics Assignment 3

Is due Friday, October 26 at 11 pm

Covers material from chapters 4 and 5

There are 8 questions for practice and 6 for credit

Assignment 4 arrives on Monday

Chapters 6, 7

Due November 12 at 11 pm

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Newton's Second Law in Various Forms

$$\vec{F} = m\vec{a}, \text{ or, } \vec{F} = m \frac{\Delta \vec{v}}{\Delta t}$$

Define momentum $\vec{p} = m\vec{v}$, so,

$$\text{impulse } \vec{F}\Delta t = m\Delta \vec{v} = \Delta \vec{p}$$

(impulse-momentum theorem,
the change in momentum is equal to the impulse imparted)

$$\text{and, } \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

If $\vec{F} = 0$, then momentum is constant

Clickers!

You are standing on the edge of a dock and jump straight down. If you land on sand your stopping time is much shorter than if you land on water. Using the impulse-momentum theorem as a guide, $F\Delta t = \Delta p$, determine which one of the following statements is correct.

A) In bringing you to a halt, the sand exerts a greater impulse on you than does the water.

B) In bringing you to a halt, the sand and the water exert the same impulse on you, but the sand exerts a greater average force.

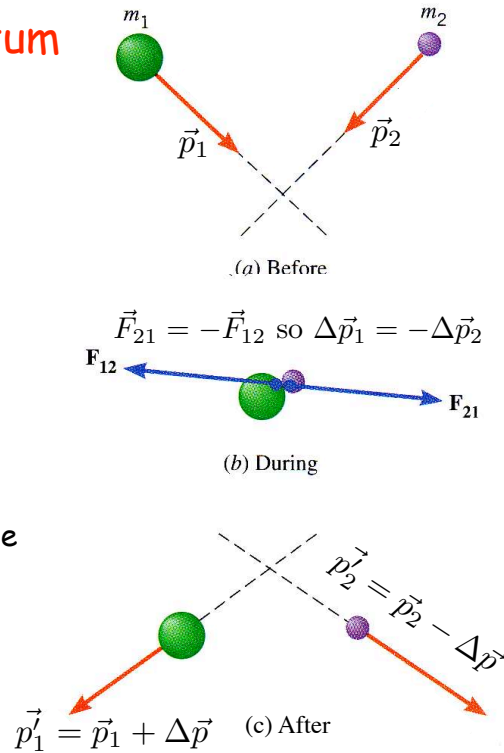
C) In bringing you to a halt, the sand and the water exert the same impulse on you, but the sand exerts a smaller average force.

Conservation of Momentum

In a collision between two isolated masses (no applied or friction forces) the total momentum is conserved:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

This is because the action and reaction forces that act between the masses while they're in contact are equal in magnitude, but opposite in direction, so what one mass gains in momentum the other loses...

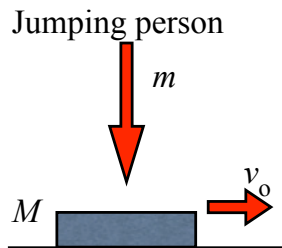


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7.C13: An ice boat slides without friction horizontally across the ice. Someone jumps vertically down from a bridge onto the boat.

Does the momentum of the boat change?



As the momentum of the person is downward, not sideways, the horizontal momentum of the boat is unchanged.

As the mass of the boat is increased by the mass of the person, the boat moves more slowly, so that the momentum is unchanged -

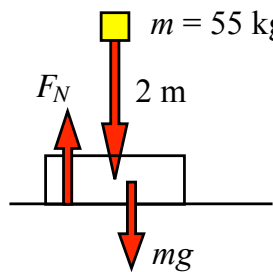
before $\rightarrow Mv_0 = (M+m)v \leftarrow$ after

$$v = \frac{Mv_0}{M+m}$$

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7.14: A dump truck is being filled with sand at a rate of 55 kg/s. The sand falls straight down from rest from a height of 2 m above the truck bed.



The truck stands on a weigh scale.

By how much does the reading of the scale exceed the weight of the truck?

The truck and scale absorb the momentum of the falling sand.

The force exerted by the truck and scale to bring the sand to rest is equal to the rate at which they absorb momentum.

That is, $F = \frac{\Delta p}{\Delta t}$

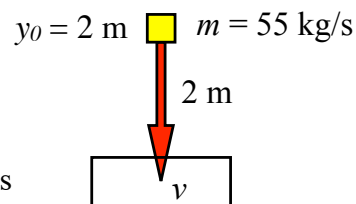
Work out speed and momentum of the sand at impact.

Conservation of mechanical energy:

$$PE_0 + KE_0 = PE + KE$$

$$mgy_0 + 0 = 0 + mv^2/2$$

$$\text{So } v = \sqrt{2gy_0} = \sqrt{2g \times (2 \text{ m})} = 6.261 \text{ m/s}$$



The momentum carried by the sand is then,

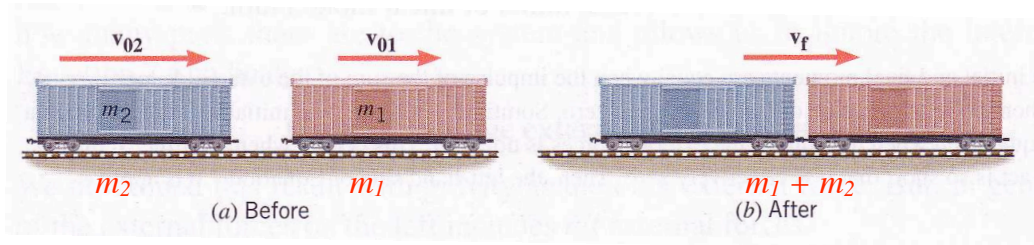
$$mv = (55 \text{ kg}) \times (6.261 \text{ m/s}) = 344.4 \text{ kg.m/s, each second}$$

This is the rate at which the momentum of the sand is changed by impact with the truck and scale, i.e. $\Delta p / \Delta t$.

Then, $F = \frac{\Delta p}{\Delta t}$

So the force exerted by the truck and scale is 344.4 N.

Conservation of Momentum



The freight car on the left catches up with the one on the right and connects up with it. They travel on with the same speed v_f .

Conservation of momentum:

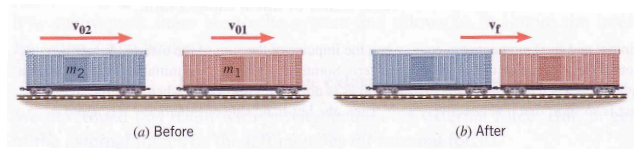
$$m_2 v_{02} + m_1 v_{01} = (m_1 + m_2) v_f$$

$$\text{So } v_f = \frac{m_2 v_{02} + m_1 v_{01}}{m_1 + m_2}$$

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$$v_f = \frac{m_2 v_{02} + m_1 v_{01}}{m_1 + m_2}$$



Example: $m_1 = 65,000 \text{ kg}$, $m_2 = 92,000 \text{ kg}$

$v_{01} = 0.8 \text{ m/s}$, $v_{02} = 1.3 \text{ m/s}$

$$v_f = \frac{92,000 \times 1.3 + 65,000 \times 0.8}{92,000 + 65,000} = 1.09 \text{ m/s}$$

Kinetic energy:

$$KE_i = \frac{m_1 v_{01}^2}{2} + \frac{m_2 v_{02}^2}{2} = 98,540 \text{ J}$$

$$KE_f = \frac{(m_1 + m_2) v_f^2}{2} = 93,266 \text{ J}$$

Missing
5,274 J

What happened to the missing kinetic energy?

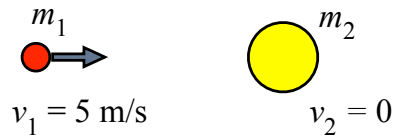
The collision was **inelastic**, some KE got turned into heat.

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- **Elastic collision:** the total kinetic energy after collision is equal to the total before collision.
- **Inelastic collision:** the total kinetic energy is not conserved. If objects stick together after collision, the collision is “**perfectly inelastic**” – there is no bounce.

Example: A ball of mass $m_1 = 0.25$ kg makes a perfectly elastic collision with a ball of mass $m_2 = 0.8$ kg.



$$\text{Initial momentum} = m_1 v_1 + 0$$

$$\text{Momentum after impact} = m_1 v_{f1} + m_2 v_{f2}$$

$$\text{Momentum conserved: } m_1 v_1 = m_1 v_{f1} + m_2 v_{f2}$$

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Momentum conserved: $m_1 v_1 = m_1 v_{f1} + m_2 v_{f2}$

m_1 m_2
 $v_1 = 5 \text{ m/s}$ $v_2 = 0$

$$\text{So, } v_{f2} = \frac{m_1 v_1 - m_1 v_{f1}}{m_2}$$

The collision is elastic, so:

$$\text{KE: } \frac{m_1 v_1^2}{2} + 0 = \frac{m_1 v_{f1}^2}{2} + \frac{m_2 v_{f2}^2}{2}$$

Solution, after some algebra, is:

$$v_{f1} = v_1 \left[\frac{m_1 - m_2}{m_1 + m_2} \right]$$

$$v_{f2} = v_1 \left[\frac{2m_1}{m_1 + m_2} \right]$$

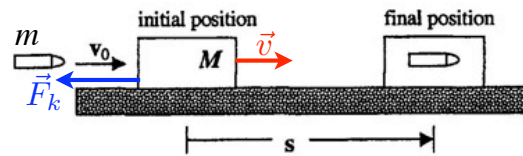
If $m_1 = m_2$, then $v_{f1} = 0$, $v_{f2} = v_1$

If $m_1 = 0.25$ kg, $m_2 = 0.8$ kg, then $v_{f1} = -2.62$ m/s, $v_{f2} = 2.38$ m/s

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Dec 2003 Final, Q26: A bullet of mass m is fired with speed v_0 into a block of wood of mass M . The bullet comes to rest in the block. The block with the bullet inside slides along a horizontal surface with coefficient of kinetic friction μ_k . How far does the block slide before it comes to rest?



For the impact -

Conservation of momentum: $mv_0 = (m + M)v$ right after impact

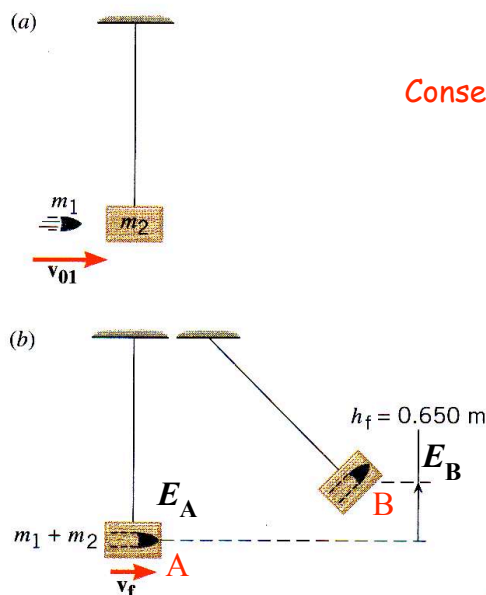
So, speed of block + bullet immediately after impact is: $v = \frac{mv_0}{m + M}$

Work-energy theorem, block coming to rest: $W_{nc} = F_k s = \Delta KE + \Delta PE$

That is: $-\mu_k(m + M)g \times s = -(m + M)v^2/2 + 0$

$$\text{So, } s = \frac{v^2/2}{\mu_k g} = \frac{1}{2\mu_k g} \left[\frac{mv_0}{m + M} \right]^2$$

Ballistic pendulum Measure the speed of a bullet



Conservation of momentum for initial impact:

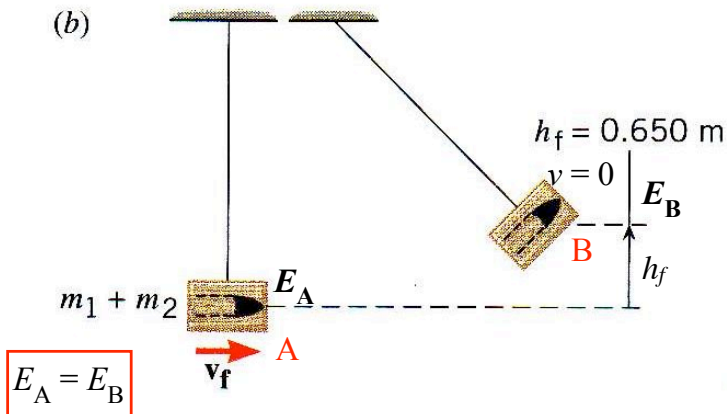
$$m_1 v_{01} = (m_1 + m_2) v_f$$

The collision is inelastic - the bullet drills into the block, generating a lot of heat.

BUT, after the collision, the block + bullet swing up to a highest point and conserve mechanical energy.

$$E_A(\text{after impact}) = E_B$$

$$m_1 v_{01} = (m_1 + m_2) v_f \rightarrow v_f = \frac{m_1 v_{01}}{(m_1 + m_2)}$$



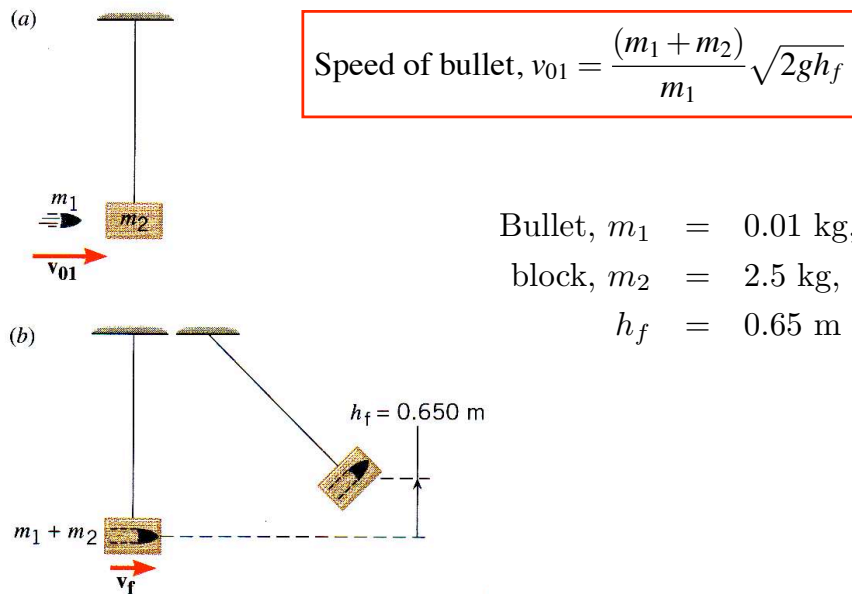
That is, $\frac{1}{2}(m_1 + m_2)v_f^2 = 0 + (m_1 + m_2)gh_f$

$$v_f = \sqrt{2gh_f} = \frac{m_1 v_{01}}{m_1 + m_2}$$

Speed of bullet, $v_{01} = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh_f}$

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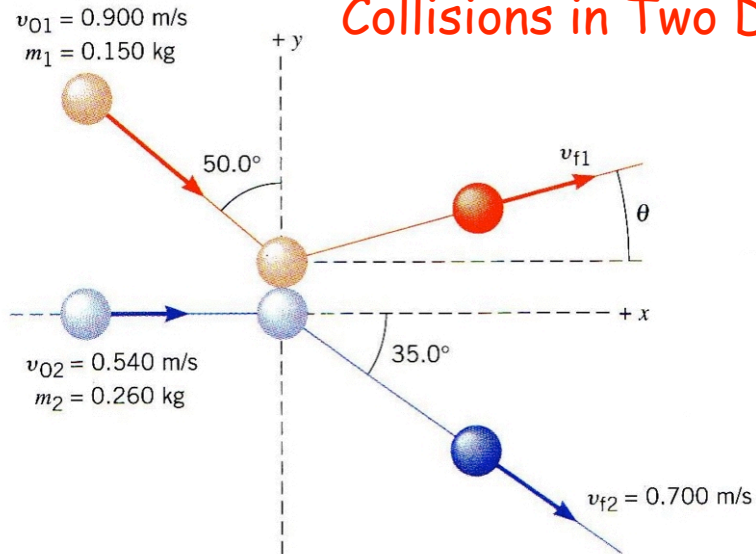


Speed of bullet, $v_{01} = \frac{(0.01 + 2.5)}{0.01} \sqrt{2g \times 0.65} = 896$ m/s

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Collisions in Two Dimensions



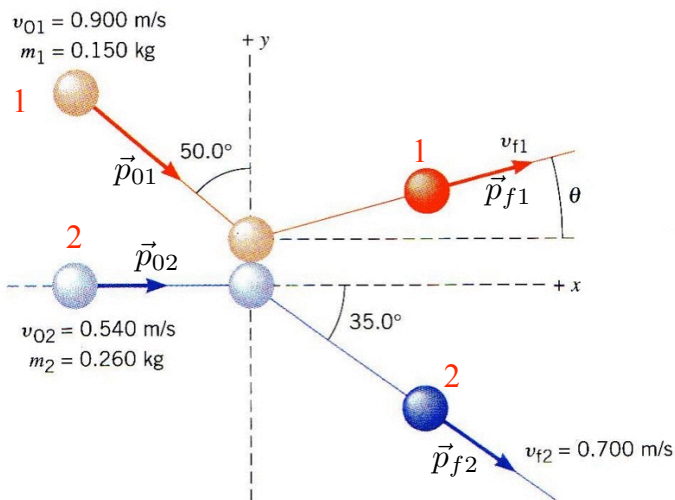
Conservation of momentum: $\vec{p}_0 = \vec{p}_f$

x and y components: $p_{0x} = p_{fx}$

$$p_{0y} = p_{fy}$$

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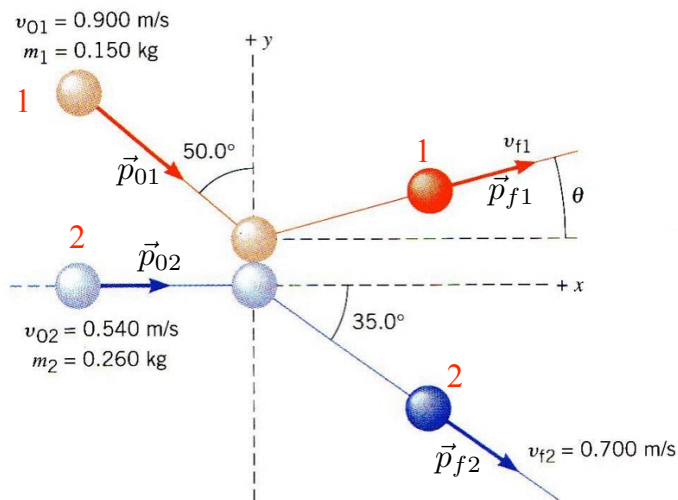
x: $p_{01} \sin 50^\circ + p_{02} = p_{f1x} + p_{2f} \cos 35^\circ$

That is, $p_{f1x} = 0.9 \times 0.15 \sin 50^\circ + 0.26 \times 0.54 - 0.26 \times 0.7 \cos 35^\circ$

$$p_{f1x} = 0.0947 \text{ kg.m/s}$$

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$$y: -p_{01} \cos 50^\circ + 0 = p_{f1y} - p_{f2} \sin 35^\circ$$

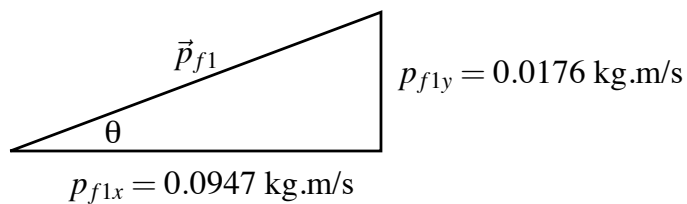
$$\text{So, } p_{f1y} = p_{f2} \sin 35^\circ - p_{01} \cos 50^\circ$$

$$= 0.26 \times 0.7 \sin 35 - 0.15 \times 0.9 \cos 50^\circ$$

$$p_{f1y} = 0.0176 \text{ kg.m/s}$$

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$$p_{f1} = \sqrt{0.0947^2 + 0.0176^2} = 0.0963 \text{ kg.m/s}$$

$$v_{f1} = p_{f1}/m_1 = 0.64 \text{ m/s}$$

$$\tan \theta = \frac{0.0176}{0.0947} = 0.1859$$

$$\theta = 10.5^\circ$$

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GENERAL PHYSICS I: PHYS 101

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Week of October 29 Experiment 3: Forces in Equilibrium

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PHYS 1020 Web Page

Instructors	Required Materials	Schedule	Policies/Evaluation	Suggested Problems	Formula Sheet
Answers to Even-Numbered Problems					
Answers for tutorial test problems					
Answers for midterm test					
<p>Mastering Physics Assignment #3 Due Friday, October 26 at 11 pm</p> <p>Information on "Mastering Physics"</p>					
 Marks files					

Midterm Marks

Average: 13.6/20

Papers without responses were marked by hand

ID	Mark	3	5	2	3	2	4	2	4	3	1	1	4	2	2	3	5	4	1	5	4	1	1	1	2	4	2	4	1	1	5	1	4	2	3	1	4	3	5	1				
	16																																											
	13																								1	1	1	2	4	2	4	2	1		1	2	4	2	5	1	4	3	2	3
	13																								1	1	1	2	4	3	4	1	1	1	5	5	4	3	5	2	4	3	3	
	11																								3	1	3	2	4	4	4	1	1	1	1	1	1	3	5	1	4	3	4	5
	14	5	5	2	3	2	4	2	4	3	2	1	4	2	1	2	5	4	1	3	5																							
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	10	4	5	4	3	2	2	2	4	3	4	1	4	3	3	2	3	4	1	4	3				5	1	1	2	4	4	1	2	1	3	5	3	3	1	1	1	5	5	4	5

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Mastering Physics Assignment 4

Is due Monday, November 12 at 11 pm

Covers material from chapters 6 and 7

There are 8 questions for practice and 6 for credit

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Impulse and Momentum

Impulse = $F \Delta t = \Delta p$ = change in momentum

Momentum is constant if there is zero net applied force

Alternative statement of Newton's second law: $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$

For a system of 2 or more colliding objects, momentum is conserved if there is zero net applied force:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

Total momentum before = total momentum after

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Clickers!

A canoe with two people aboard is coasting with an initial momentum of +110 kg.m/s. Then, one of the people (person 1) dives off the back of the canoe. During this time, the net average external force acting on the system of canoe and two people is zero.

The table lists four possibilities for the final momentum of person 1 and the final momentum of the canoe with person 2, immediately after person 1 leaves the canoe. Only one possibility could be correct. Which is it?

Person 1	Person 2 and canoe
A) -60 kg.m/s	+170 kg.m/s
B) -30 kg.m/s	+110 kg.m/s
C) -40 kg.m/s	-70 kg.m/s
D) +80 kg.m/s	-30 kg.m/s

Momentum is conserved!

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Dec 2003 Final, Q25. A boy who weighs 50 kg runs at a speed of 10 m/s and jumps onto a cart, as shown. What is the mass of the cart?

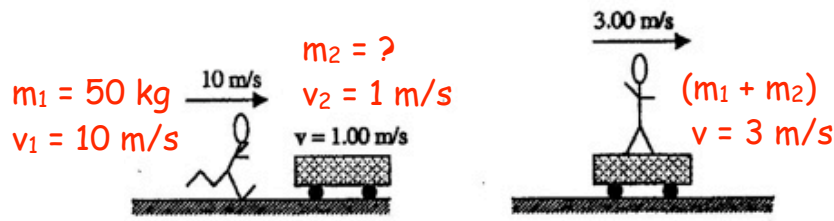
(a) 175 kg

(b) 350 kg

(c) 117 kg

(d) 88 kg

(e) 163 kg



Conservation of momentum: $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

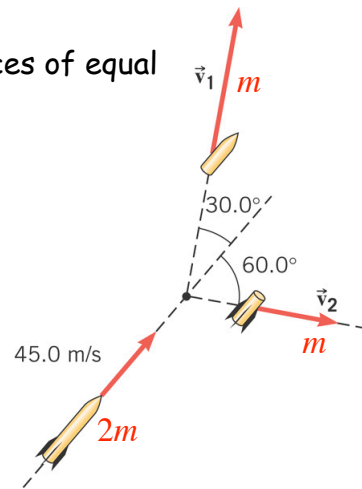
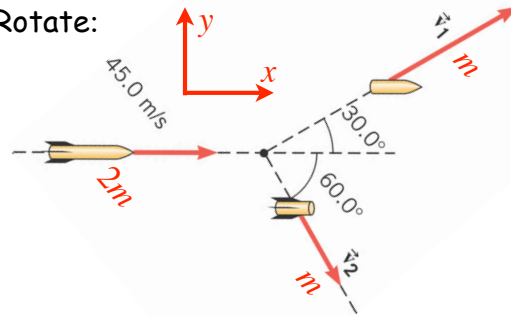
$$\text{Solve for } m_2: m_2 = \frac{m_1(v_1 - v)}{v - v_2} = \frac{50(10 - 3)}{3 - 1} = 175 \text{ kg}$$

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7.19: A fireworks rocket breaks into two pieces of equal mass. Find the velocities of the pieces.

Rotate:



Momentum in x -direction: $2m \times 45 = mv_1 \cos 30^\circ + mv_2 \cos 60^\circ$ (1)

Momentum in y -direction: $0 = mv_1 \sin 30^\circ - mv_2 \sin 60^\circ$

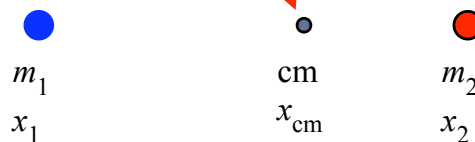
$$\rightarrow v_2 = \frac{v_1 \sin 30^\circ}{\sin 60^\circ} = 0.5774v_1 \quad (2) \quad \text{Substitute into (1)}$$

$$v_1 = \frac{90}{\cos 30^\circ + 0.5774 \cos 60^\circ} = 77.94 \text{ m/s} \quad v_2 = 45.0 \text{ m/s}$$

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Centre of Mass (or, centre of gravity)



The centre of mass of the two objects is defined as:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(The mean position
weighted by the masses)

If the masses are moving, the centre of mass moves too:

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

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Motion of cm:
$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

The speed of the centre of mass is:

$$v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{p_{tot}}{m_1 + m_2} \quad (p_{tot} = p_1 + p_2)$$

Or, $p_{tot} = (m_1 + m_2)v_{cm}$

If zero net force acts on the masses, the total momentum is constant and the speed of the centre of mass is constant also, **even after a collision.**

In two or three dimensions:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \text{velocity of centre of mass}$$

$$\vec{p}_{tot} = (m_1 + m_2)\vec{v}_{cm}$$

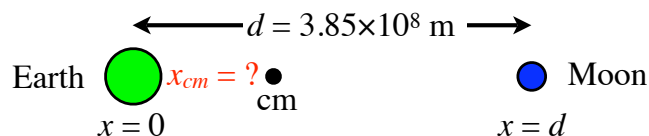
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7.41: The earth and the moon are separated by a centre-to-centre distance of 3.85×10^8 m. If:

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}, \quad M_{\text{moon}} = 7.35 \times 10^{22} \text{ kg},$$

how far does the centre of mass lie from the centre of the earth?



Measuring distances from the centre of the earth:

$$\begin{aligned}
 x_{cm} &= \frac{M_{\text{earth}} x_{\text{earth}} + M_{\text{moon}} x_{\text{moon}}}{M_{\text{earth}} + M_{\text{moon}}} = \frac{M_{\text{earth}} \times 0 + M_{\text{moon}} \times d}{M_{\text{earth}} + M_{\text{moon}}} \\
 &= \frac{7.35 \times 10^{22} \times 3.85 \times 10^8}{5.98 \times 10^{24} + 7.35 \times 10^{22}} = 4,675,000 \text{ m}
 \end{aligned}$$

Note, the radius of the earth is 6,380 km, so the cm is inside the earth.

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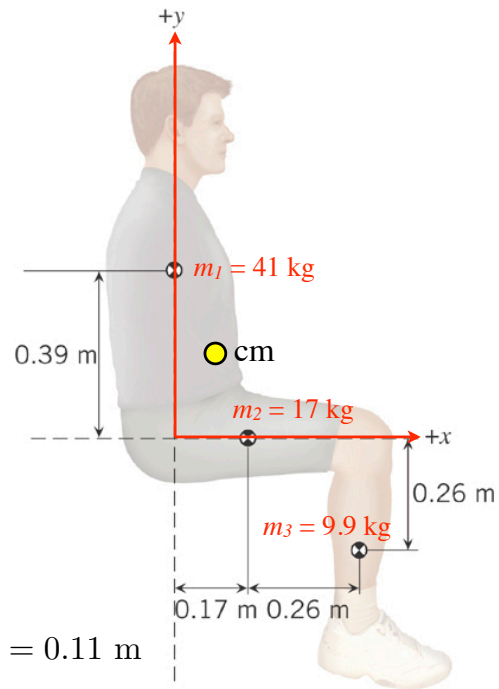
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7.44/-: The figure shows the centres of mass of:

- 1) torso, neck and head, mass $m_1 = 41$ kg,
- 2) upper legs, mass $m_2 = 17$ kg,
- 3) lower legs and feet, mass $m_3 = 9.9$ kg.

Find the position of the centre of mass of the seated person.

(NB minor appendages such as arms and hands have been left out for simplicity).



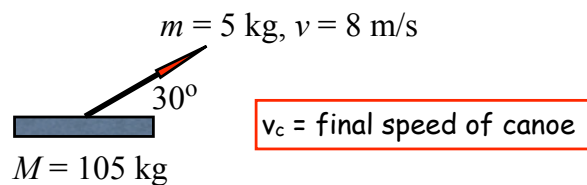
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{41 \times 0 + 17 \times 0.17 + 9.9 \times 0.43}{41 + 17 + 9.9} = 0.11 \text{ m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{41 \times 0.39 + 17 \times 0 - 9.9 \times 0.26}{41 + 17 + 9.9} = 0.20 \text{ m}$$

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7.54: A person stands in a stationary canoe and throws a 5 kg stone at 8 m/s at 30° above the horizontal. What is the recoil velocity of the canoe?



Conservation of momentum: canoe initially at rest, so momentum = 0
Or, centre of mass is at rest and remains so.

$$\begin{aligned} x: \quad 0 &= mv \cos 30^\circ + Mv_c \\ v_c &= \frac{-mv \cos 30^\circ}{M} \\ &= \frac{-5 \times 8 \cos 30^\circ}{105} = -0.33 \text{ m/s} \end{aligned}$$

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Summary

Momentum

$$\vec{p} = m\vec{v}$$

Impulse and Momentum - Newton's 2nd law in another form

$$\text{Impulse: } \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$$

Conservation of Momentum

$$\text{In the absence of applied forces: } \vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

Centre of Mass

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

continues to move at constant velocity if no applied forces