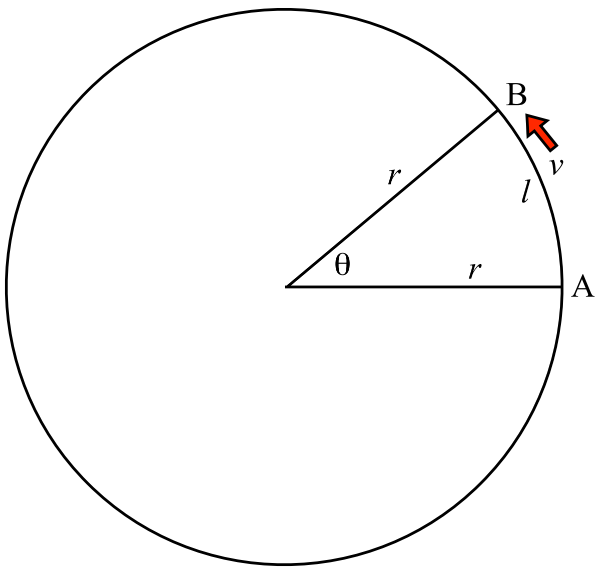
Chapter 9: Rotational Dynamics

Sections 1, 2, 3, 6 only

- Action of torques
- The two conditions of equilibrium
- Centre of gravity
- Conservation of angular momentum

Circular Motion



$l = r\theta$
$v = r\omega$

$\omega = \Delta\theta/\Delta t$
$\alpha = \Delta\omega/\Delta t$

θ in radians, $180^0 = \pi$ rad.

ω in rad/s
α in rad/s^2

Correspondence between Linear and Angular Motion

As seen in chapter 8:

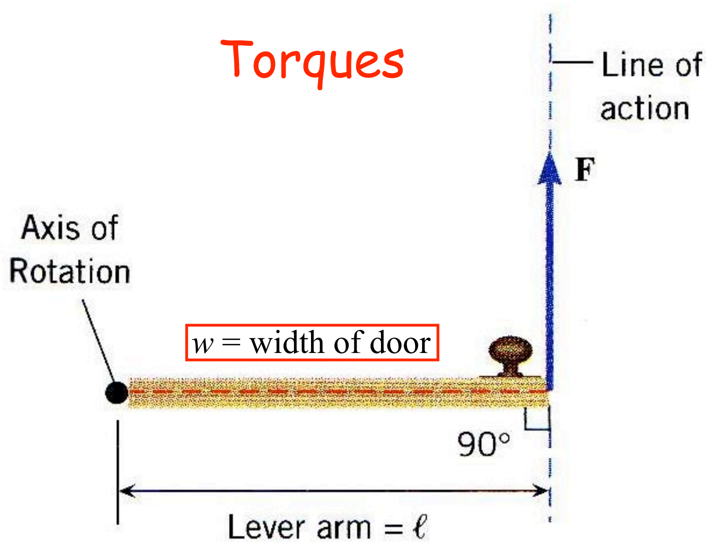
Linear		**Angular**
x	⟷	θ
v	⟷	ω
a	⟷	α

The "famous four equations" are the same once these substitutions have been made

Additionally:

$l = r\theta$, defines the radian, $v = r\omega$, relates speed to angular velocity

Torques

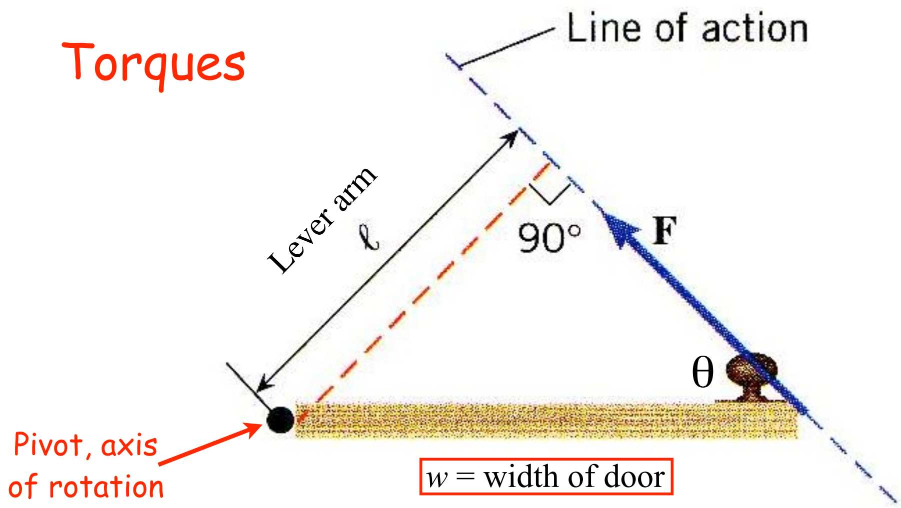


Torque = (magnitude of force) × (lever arm)

Lever arm, $l = w\sin 90^\circ$

Torque, $\tau = Fl = Fw$

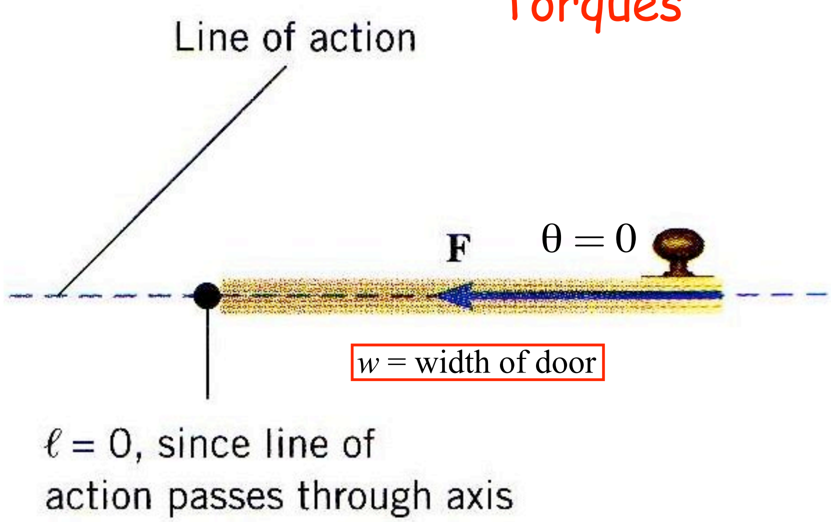
Torques



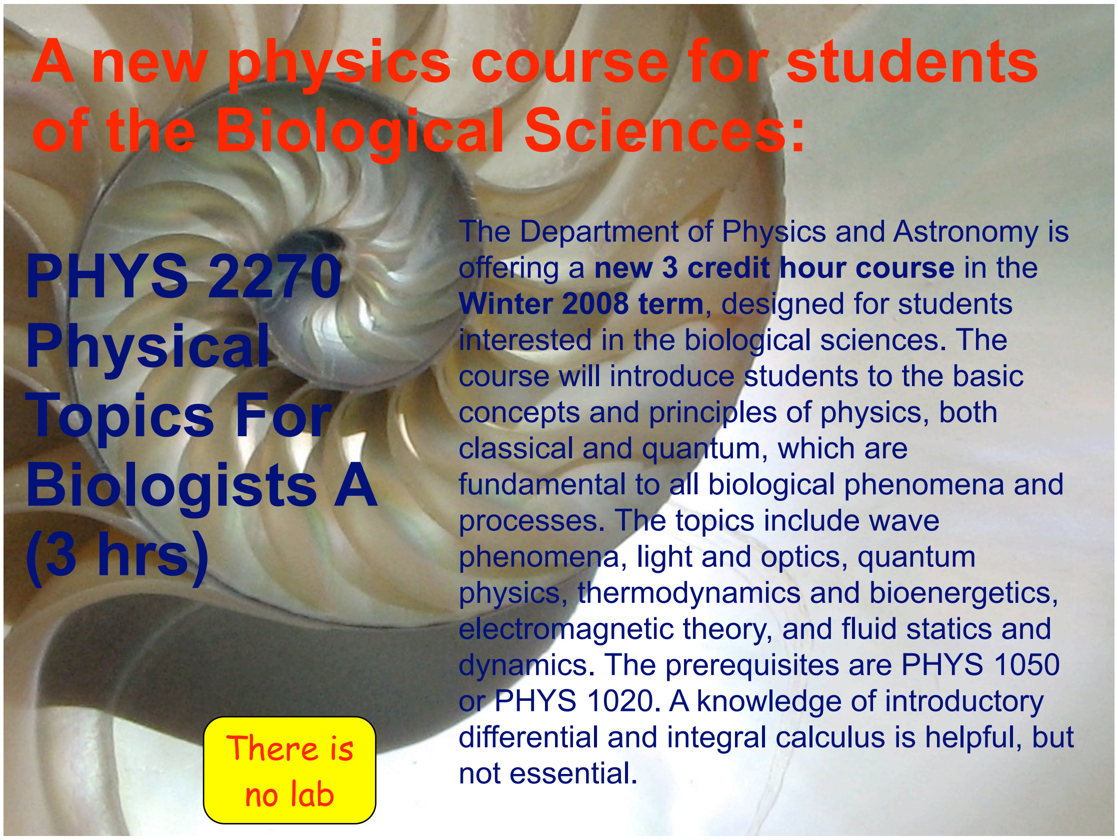
Lever arm: $l = w\sin\theta$

Torque, $\tau = Fl = Fw\sin\theta$

Torques



Torque, $\tau = Fl = Fw\sin\theta = 0$



A new physics course for students of the Biological Sciences:

PHYS 2270 Physical Topics For Biologists A (3 hrs)

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There is no lab

GENERAL PHYSICS I: PHYS 1020

Schedule - Fall 2007
(lecture schedule is approximate)

8	M	22	19	Chapter 7	Impulse and momentum	No lab or tutorial
	Tue	**23**	**MID-TERM TEST, Ch 1-5, Tuesday, October 23, 7-9 pm**			
	W	24	20	Chapter 7	Impulse and momentum	
	F	26	21	Chapter 8, sections 1-3 only	Rotational kinematics	
9	M	29	22			Experiment 3: Forces in Equilibrium
	W	31	23	Chapter 9 sections 1 - 3, 6	Rotational dynamics	
	F	**Nov 2**	24			
10	M	5	25	Chapter 10 exclude 10.7 and 10.8	Simple harmonic motion, sections 10.5 and 10.6, for self study only	Tutorial and Test 3 ~~(chapters 7, 8)~~ Chapters 6, 7
	W	7	26			
	F	9	27	Chapter 11 exclude 11.11	Fluids	

Week of November 5

Tutorial and Test 3: Chapters 6 & 7

Mastering Physics Assignment 4

Is due Monday, November 12 at 11 pm

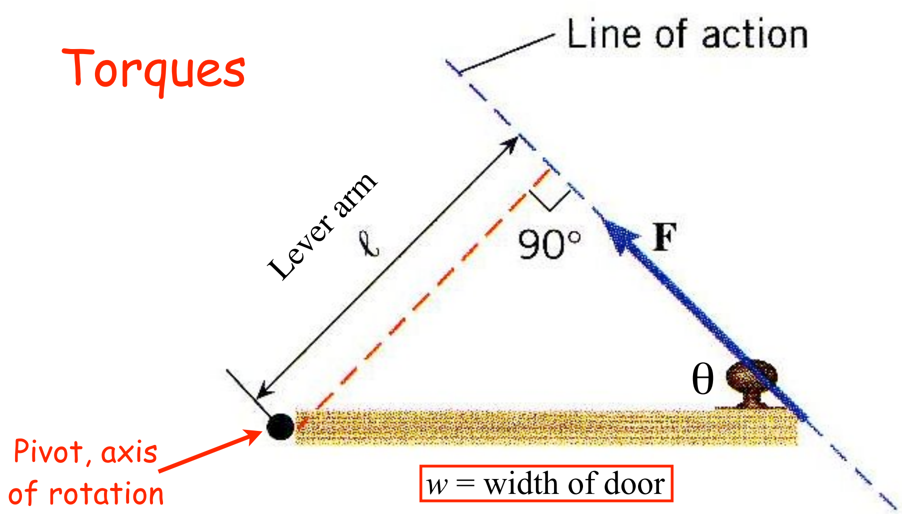
Covers material from chapters 6 and 7

There are 8 questions for practice and 6 for credit

The Final Exam Schedule is Now Final!

PHYS 1020: Monday, December 17, 6 - 9 pm
Frank Kennedy Brown & Gold Gyms
The whole course
30 multiple choice questions
Formula sheet provided

Torques



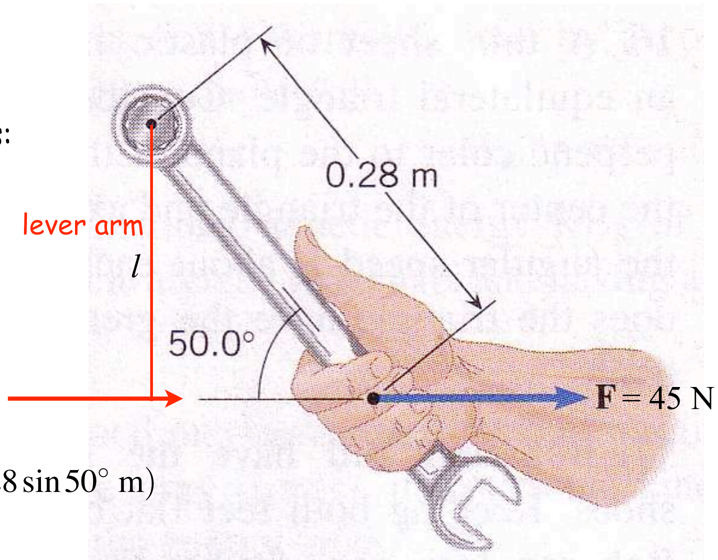
Lever arm: $l = w \sin\theta$

Torque, $\tau = Fl = Fw\sin\theta$

9.1/2: The torque applied to the bolt is:

$$\tau = Fl$$

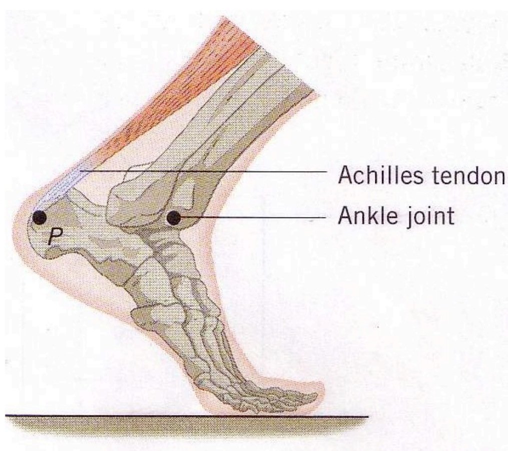
$$l = 0.28 \sin 50^\circ \text{ m}$$

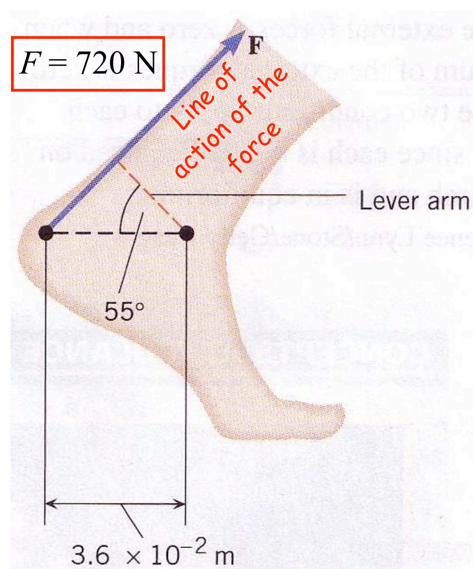


Line of action of the force

$$\tau = (45 \text{ N}) \times (0.28 \sin 50^\circ \text{ m})$$

$$= 9.65 \text{ N.m}$$





Torque generated by Achilles tendon about ankle joint:

$$l = (0.036 \text{ m}) \times \cos 55^\circ = 0.0207 \text{ m}$$

$$\tau = Fl = (720 \text{ N})(0.0207 \text{ m}) = 14.9 \text{ N.m}$$

Sign convention: torque is positive when tending to rotate counterclockwise about reference point. So, torque = –14.9 N.m.

9.8: One end of a metre stick is pinned to a table, so that the stick can rotate freely around the surface of the tabletop. Two forces, both parallel to the tabletop, are applied in such a way that the net torque is zero. Where along the stick must the 6 N force be applied?

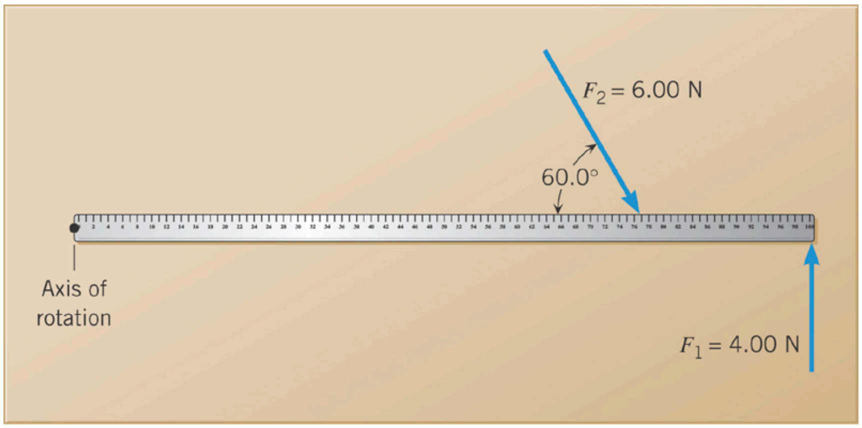
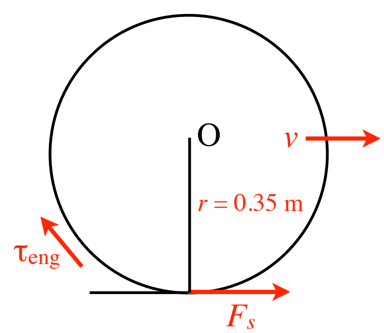


Table (top view)

9.3: The engine applies a torque of τ_{eng} = 295 N.m to the wheel of a car, which does not slip against the road surface because the static friction force applies a countertorque. The car is travelling at constant velocity.

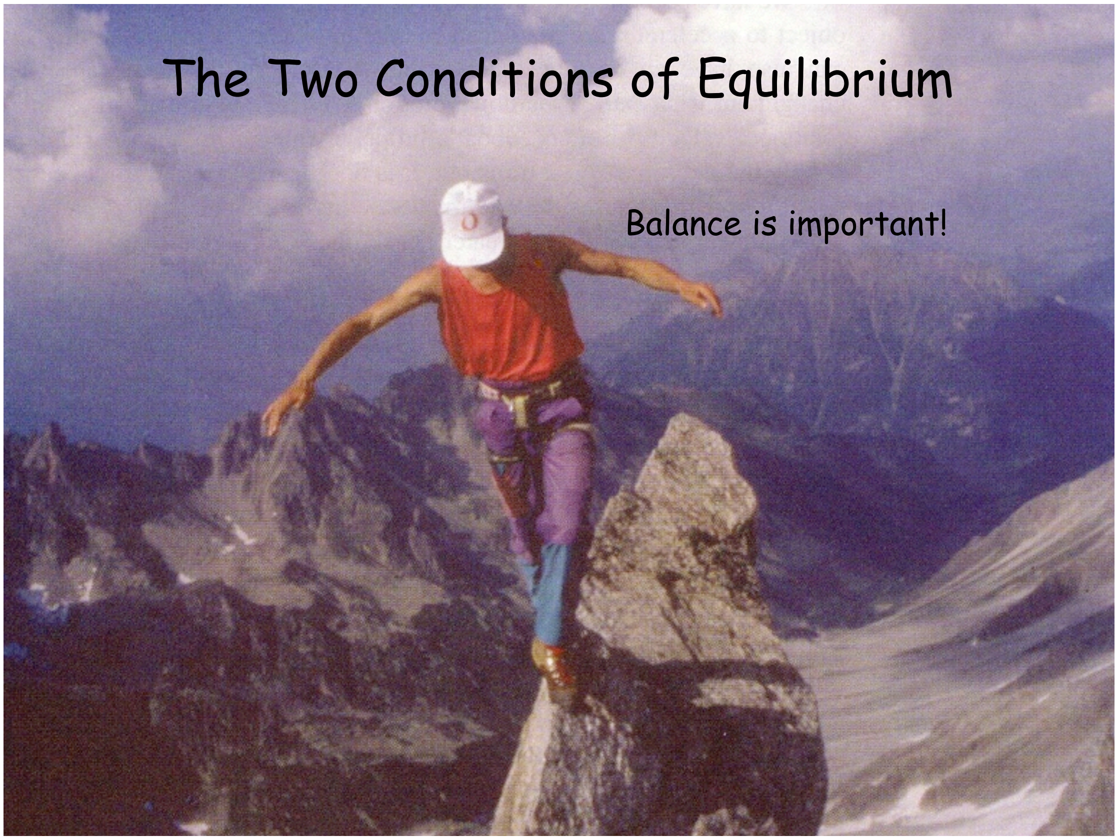
What is the static friction force?



As the wheel is turning at constant rate, the net torque applied to it must be zero.

Therefore, $\tau_{eng} = F_s \times r$

$F_s = \tau_{eng}/r = 295/0.35 = 843$ N



The Two Conditions of Equilibrium

For an object to be in equilibrium:

1) The sum of forces acting on the object must be zero:

$$\Sigma\vec{F}_i = \vec{F}_1 + \vec{F}_2 + \ldots = 0$$

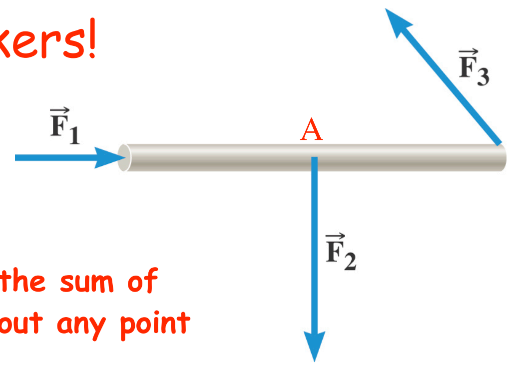
Then the acceleration is zero by Newton's first law.

2) Balance: the sum of torques **about any point** must be zero:

$$\Sigma\tau_i = \tau_1 + \tau_2 + \ldots = 0$$

Then the angular acceleration is zero.

Clickers!



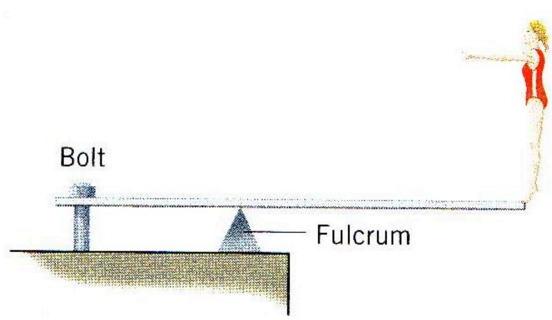
9.C8: Are the three forces shown sufficient to keep the rod in equilibrium?

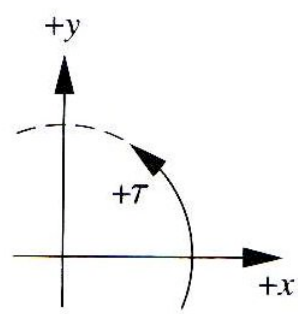
For the rod to be in equilibrium, the sum of the forces and of the torques about any point must be zero.

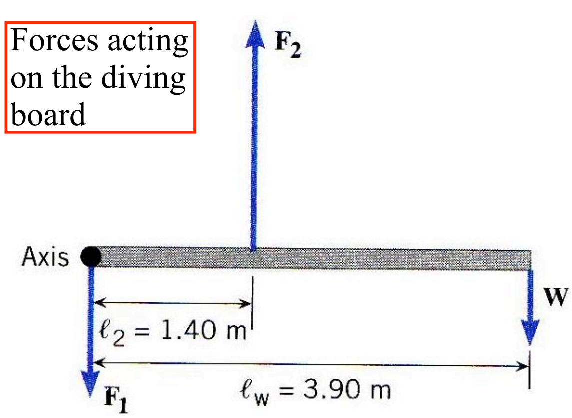
A) It is not possible to adjust the magnitudes of the forces so that the net force is zero.

B) It is possible to adjust the magnitudes of the forces so that the net force is zero, but the net torque cannot be zero.

C) It is possible to adjust the magnitudes of the forces so that the net force and the net torque are both zero.







Conditions of equilibrium:

Forces:

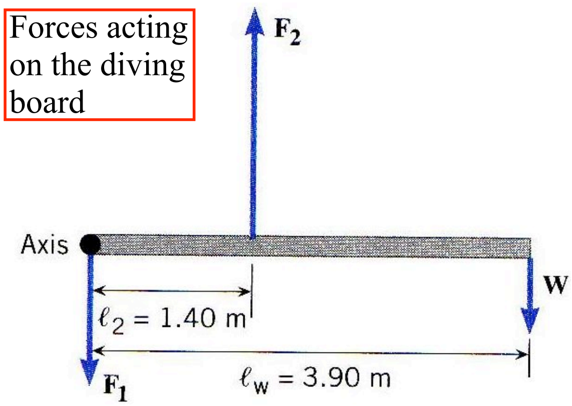
$$-F_1 + F_2 - W = 0$$

Torques about axis:

$$0 \times F_1 + 1.4F_2 - 3.9W = 0$$

That is, $F_2 = \dfrac{3.9W}{1.4} = 2.786W$

Forces acting on the diving board



$$-F_1 + F_2 - W = 0$$

$$F_2 = 2.786W$$

$$F_1 = F_2 - W = 1.786W$$

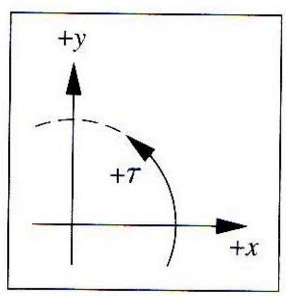
If the weight of the diver is 530 N, then

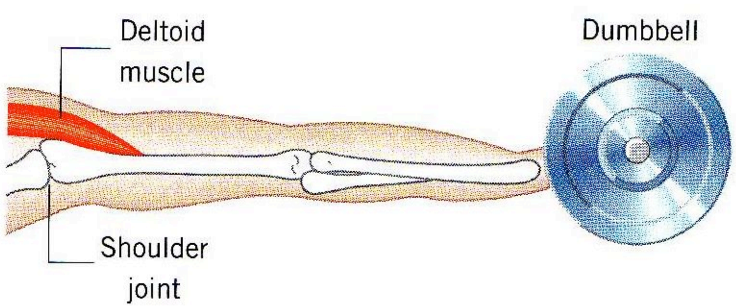
$F_1 = 947$ N,

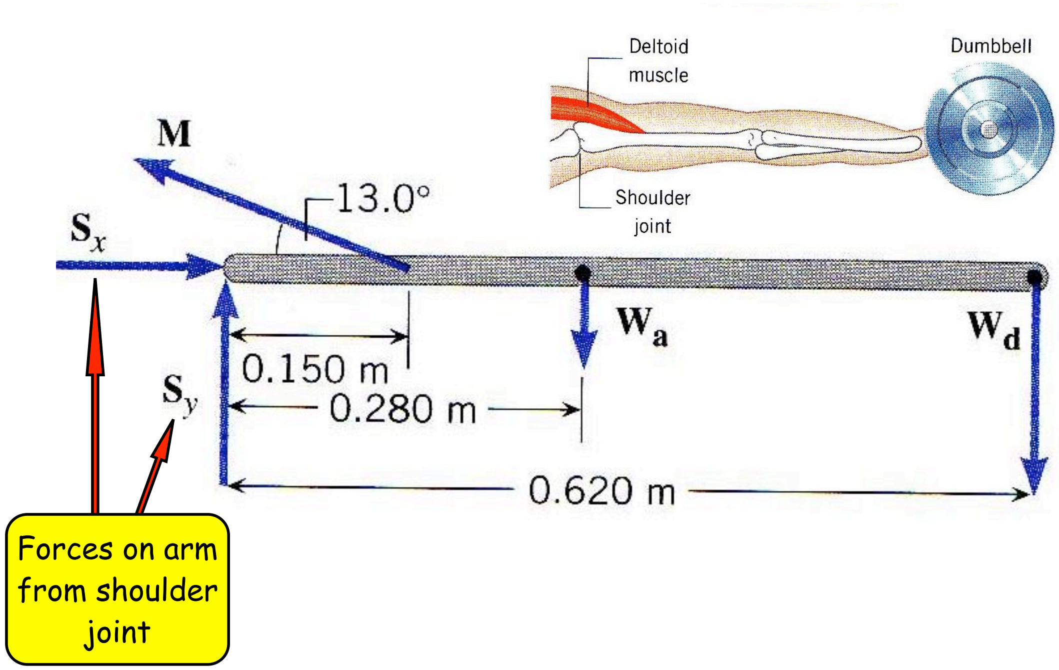
$F_2 = 1480$ N

What weight can the muscle lift?

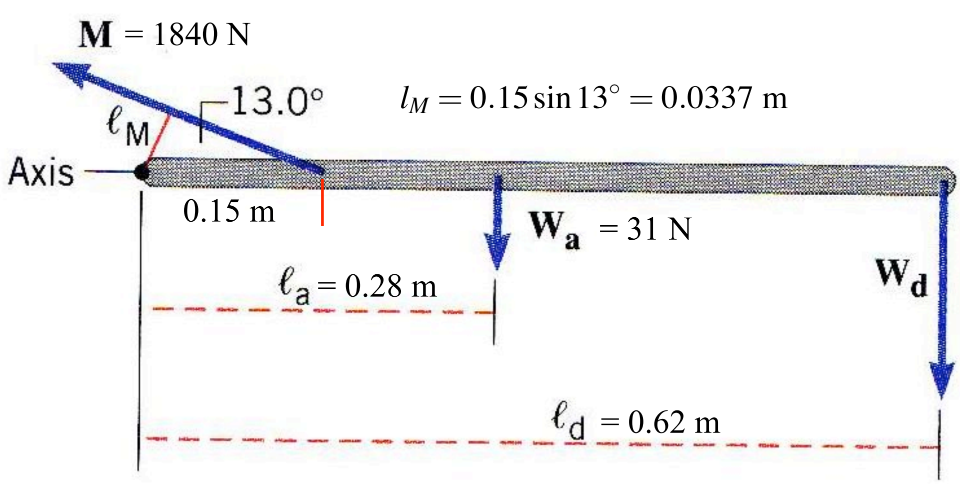
Maximum tension in muscle = 1840 N







Calculate torques about the shoulder joint to eliminate S_x, S_y from the torque equations.

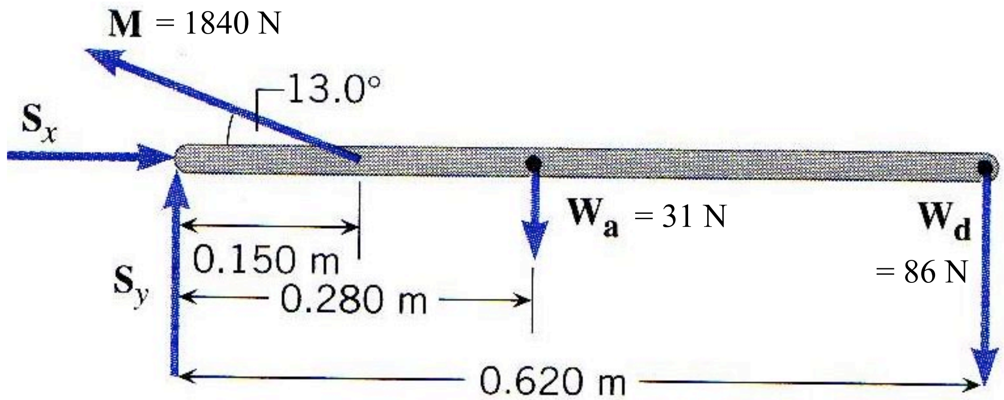


$l_M = 0.15 \sin 13^\circ = 0.0337$ m

Torques about shoulder joint (axis):

$$1840 \times 0.0337 - 31 \times 0.28 - W_d \times 0.62 = 0$$

$$W_d = 86 \text{ N}$$



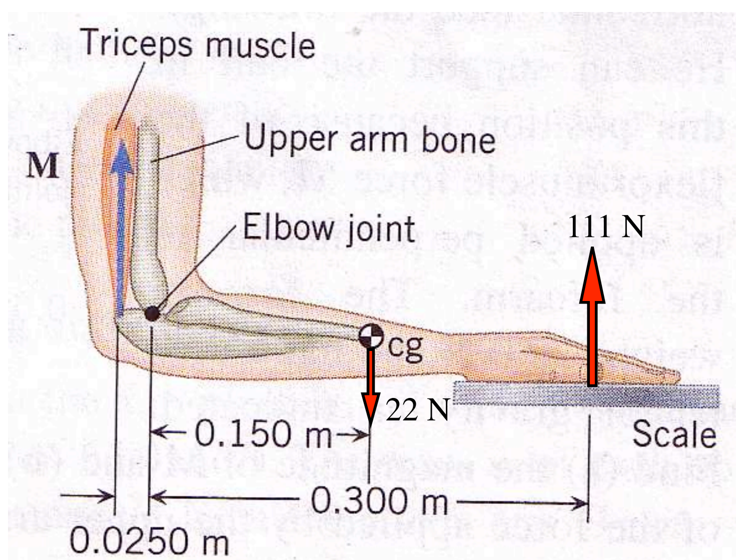
Forces:

$$x:\ S_x - M\cos 13^\circ = 0 \qquad S_x = 1840\cos 13^\circ = 1793\ \text{N}$$

$$y:\ M\sin 13^\circ + S_y - W_a - W_d = 0$$

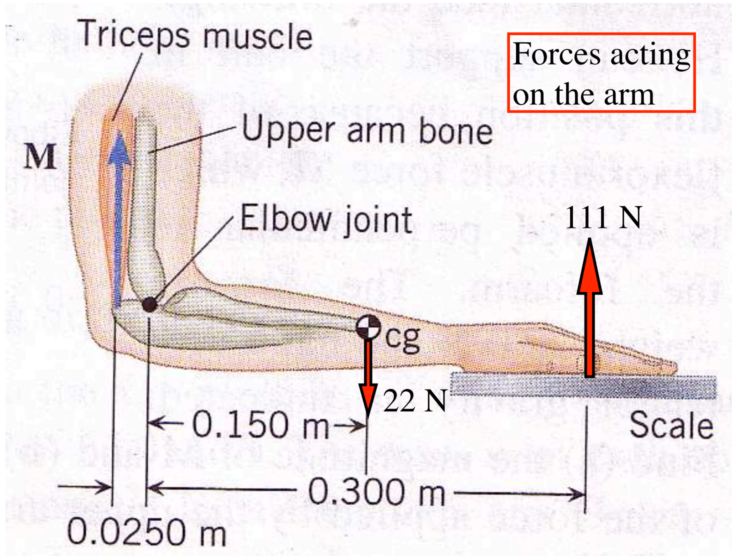
$$S_y = -1840\sin 13^\circ + 31 + 86 = -297\ \text{N}$$

9.18



The hand presses down on the scale.
The scale reads 111 N.
The weight of the forearm is 22 N.

Find the force due to the muscle, M.



Torques about elbow joint:

$$-M \times 0.025 - 22 \times 0.15 + 111 \times 0.3 = 0$$

$$M = 1200 \text{ N}$$

The Two Conditions of Equilibrium

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1) The sum of forces acting on the object must be zero:

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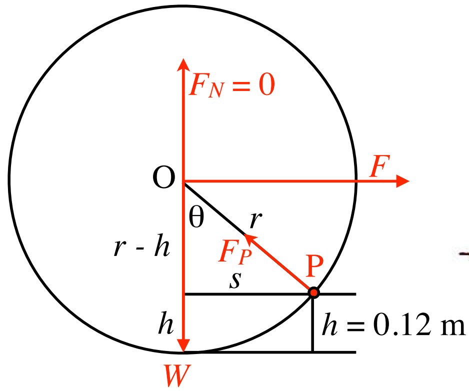
Then the acceleration is zero by Newton's first law.

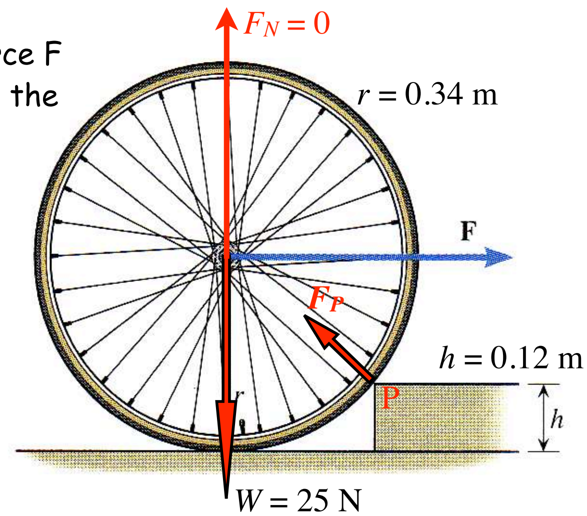
2) The sum of torques **about any point** must be zero:

$$\Sigma\tau_i = \tau_1 + \tau_2 + \ldots = 0$$

The angular acceleration is then zero.

9.66/20: What horizontal force F is needed to pull the wheel up the step?



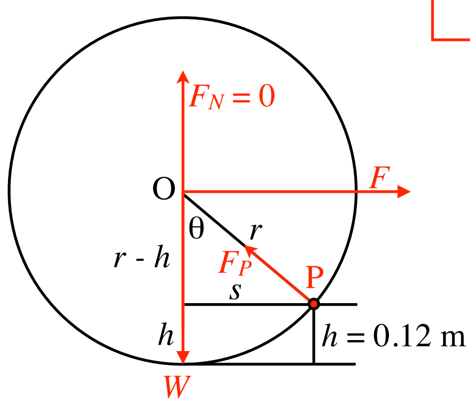


Torques about P: $W\,s - F(r-h) = 0$

$$F = \frac{W\,s}{r-h}$$

(normal force of ground on tire drops to zero as tire is about to leave ground)

$r = 0.34$ m



$$F = \frac{W\,s}{r-h}$$

What is s ?

$s = r\sin\theta$

$$\cos\theta = \frac{r-h}{r} = \frac{0.34-0.12}{0.34}$$

$\theta = 49.68°$

$s = r\sin\theta = 0.259$ m

$W = 25$ N

$$\text{So, } F = \frac{25 \times 0.259}{0.34-0.12} = 29.4 \text{ N}$$

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Week of November 5

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The whole course
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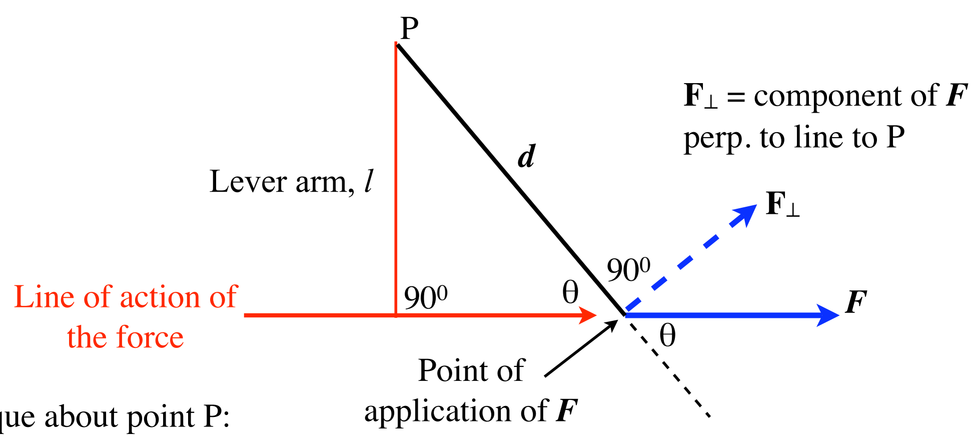
Two ways to calculate torques (moments)



Torque about point P:

1) Torque = (force) × (lever arm): $\tau = Fl = F \times d\sin\theta$

2) Torque = $\mathbf{F}_\perp \times d$: $\tau = F_\perp d = F\sin\theta \times d$

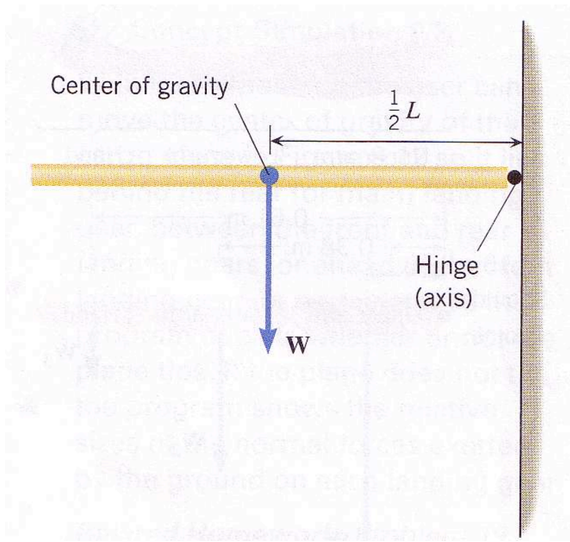
Choose the method that is the more convenient...

Centre of Gravity

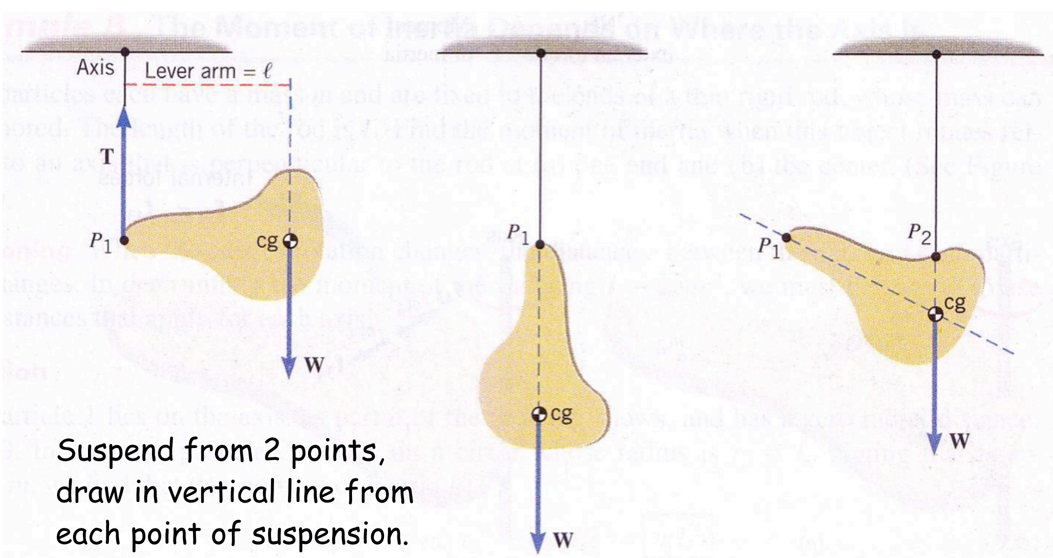
The point at which the whole weight of a solid object can be considered to act.

Torque about hinge:

$$\tau = W \times \frac{L}{2}$$



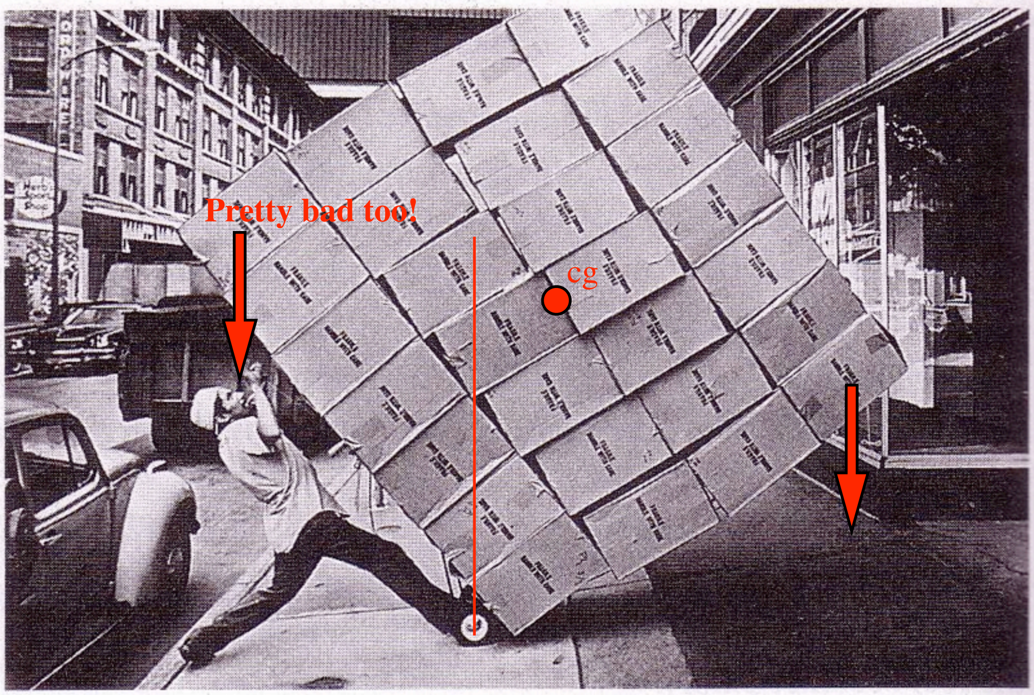
Finding the Centre of Gravity



Suspend from 2 points, draw in vertical line from each point of suspension.

Where the lines meet is the centre of gravity.

9.C24 Torque: the worst box?



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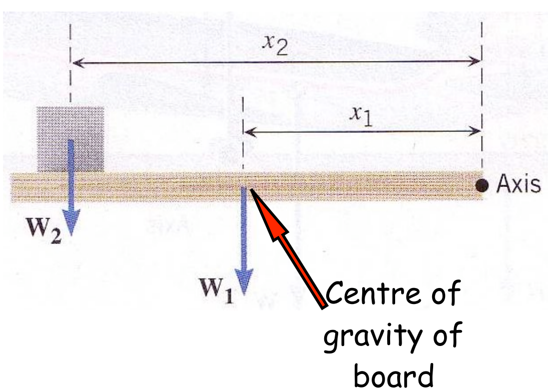
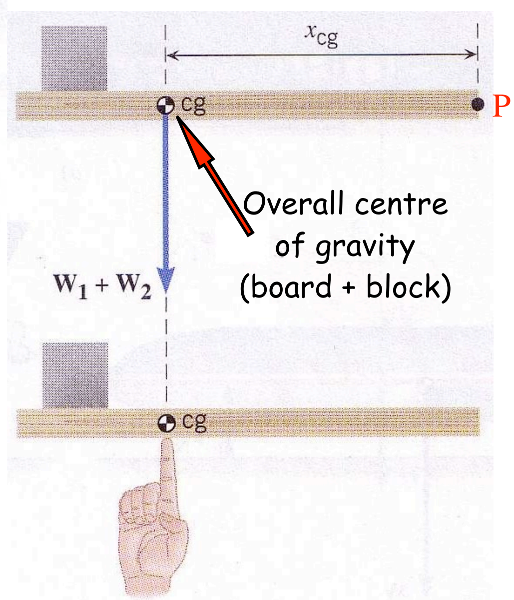


Centre of Gravity



The whole weight concentrated at the overall centre of gravity should give the correct torque –

Torques about P at the right:

$$(W_1 + W_2)x_{cg} = W_1x_1 + W_2x_2$$

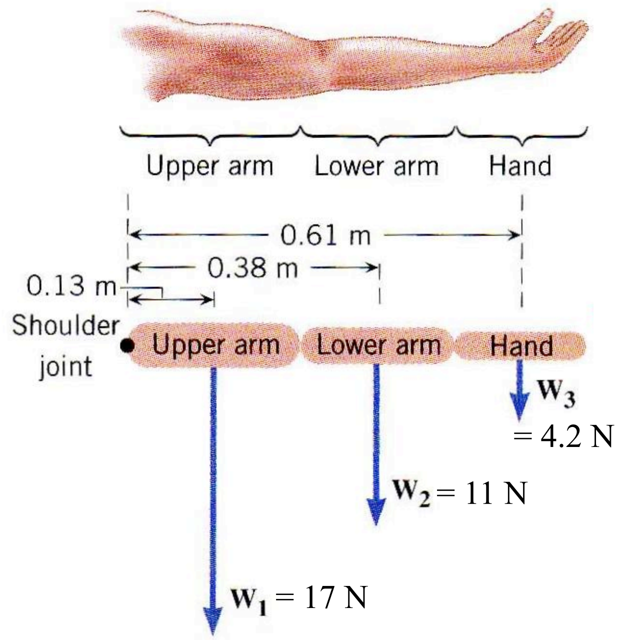
$$\boxed{x_{cg} = \frac{W_1x_1 + W_2x_2}{W_1 + W_2}}$$

Centre of gravity of an arm

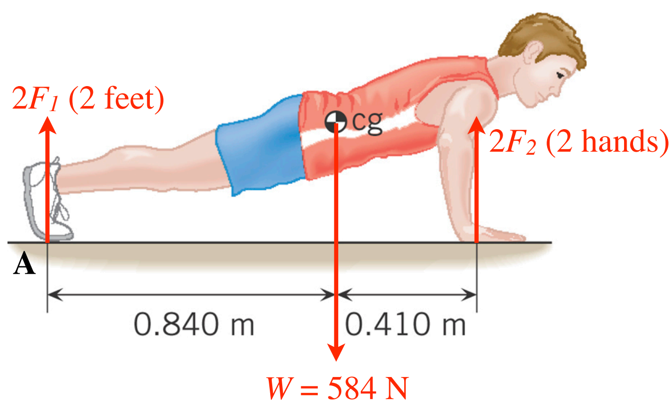
$$x_{cm} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3}{W_1 + W_2 + W_3}$$

$W_1 = 17\ \text{N}, x_1 = 0.13\ \text{m}$
$W_2 = 11\ \text{N}, x_2 = 0.38\ \text{m}$
$W_3 = 4.2\ \text{N}, x_3 = 0.61\ \text{m}$

$x_{cm} = 0.28\ \text{m}$



9.11: Find the normal force exerted by the floor on each hand and each foot.

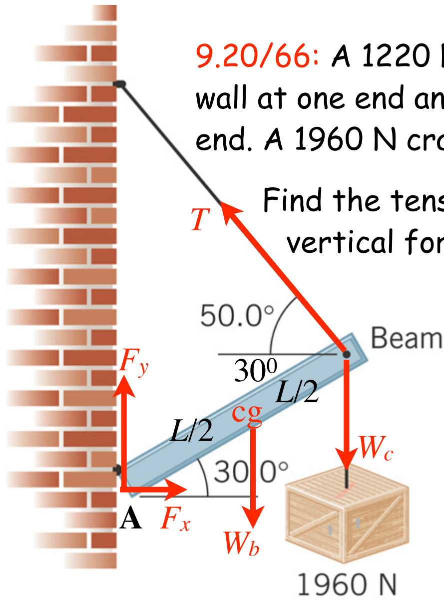


Torques about A: $-W \times 0.84 + 2F_2 \times (0.84+0.41) = 0 \rightarrow F_2 = 196$ N

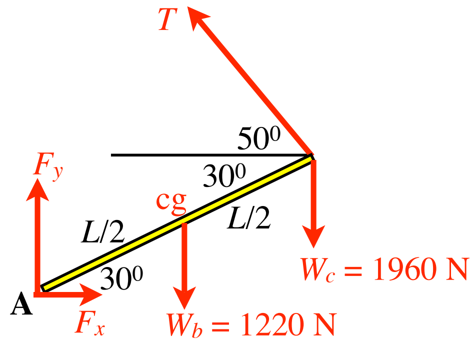
Forces: $2F_1 + 2F_2 - W = 0 \rightarrow F_1 = 96$ N

9.20/66: A 1220 N uniform beam is attached to a vertical wall at one end and is supported by a cable at the other end. A 1960 N crate hangs from the far end of the beam.

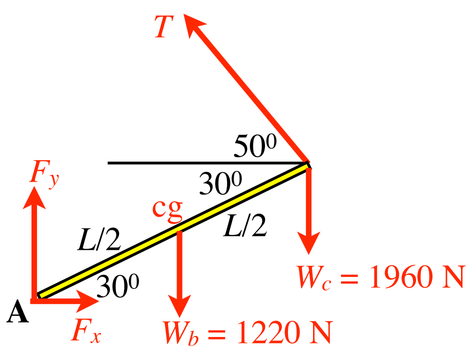
Find the tension in the cable and the horizontal and vertical forces exerted on the left end of the beam by the wall.



The length of the beam is L. It is uniform, so its cg is half way along



Free-body diagram - forces on the beam



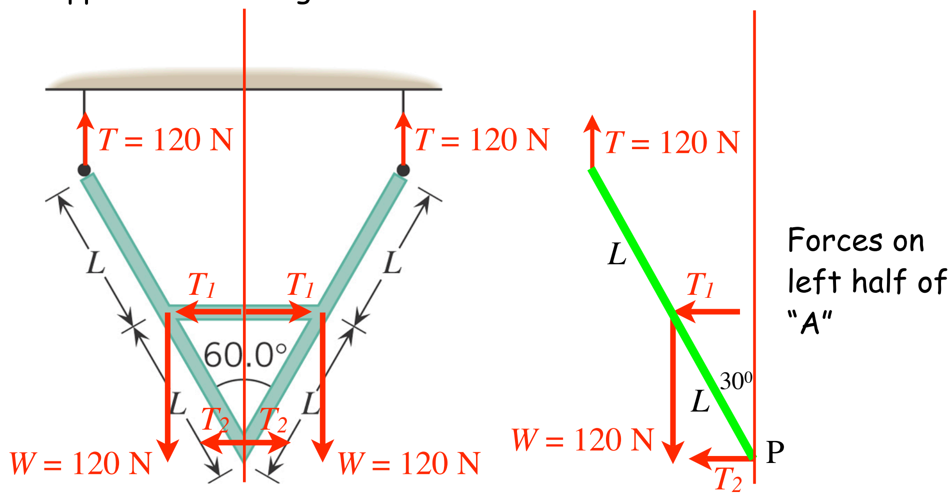
Torques about A: $TL\sin 80^\circ - W_b(L/2)\times\cos 30^\circ - W_c L\cos 30^\circ = 0$

$$\rightarrow T = \frac{(W_c + W_b/2)\cos 30^\circ}{\sin 80^\circ} = 2260\ \text{N}$$

Forces in x: $F_x = T\cos 50^\circ = 1453\ \text{N}$

Forces in y: $F_y + T\sin 50^\circ - W_b - W_c = 0 \rightarrow F_y = 1449\ \text{N}$

9.25/71: Each leg of the "A" has weight 120 N. The horizontal crossbar has negligible weight. Find the force that the crossbar applies to each leg.



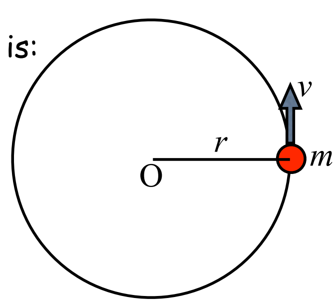
Torques about P: $T_1 L \cos 30^\circ + W L \sin 30^\circ = T \times 2L \sin 30^\circ$

$$\rightarrow T_1 = \tan 30^\circ [2T - W] = 69 \text{ N}$$

Angular Momentum

Angular momentum is conserved if the net torque acting on an object is zero.

The angular momentum of the mass about O is:



L = mvr

With v = rω, angular velocity ω,

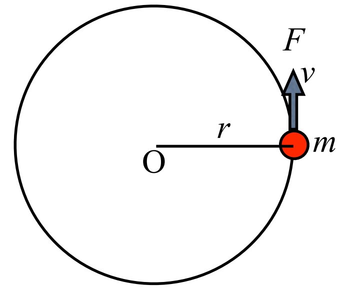
L = m(rω)r = mr$^2\omega$

Define **I = mr^2 = "moment of inertia"** about centre of circle

Angular momentum, L = mvr = Iω

Conservation of Angular Momentum

Apply a force, F, in the direction of the instantaneous velocity of the mass. The mass is constrained to move in a circular path (eg by a string).



The mass accelerates:

$$F = ma = m\frac{\Delta v}{\Delta t}$$

The torque applied by F is $\tau = Fr = mr\frac{\Delta v}{\Delta t}$

As angular momentum is $L = mvr$ and $\Delta L = mr\Delta v$

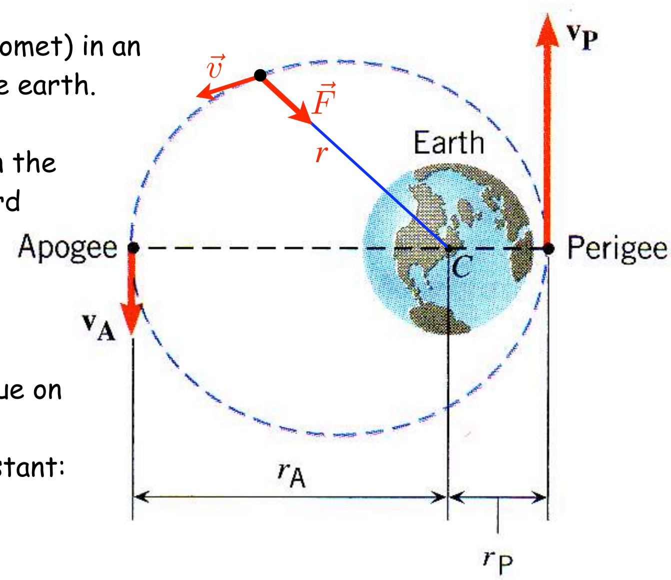
$$\tau = \frac{\Delta L}{\Delta t} = I\frac{\Delta \omega}{\Delta t} \text{ as } L = I\omega, \text{ so } \Delta L = I\Delta\omega$$

Angular momentum is constant if net torque is zero
Rate of change of angular momentum = torque applied

Conservation of Angular Momentum

Motion of a satellite (or comet) in an eccentric orbit around the earth.

The gravitational force on the satellite is directed toward the earth, the axis of rotation - zero torque.



As gravity exerts no torque on the satellite, its angular momentum about C is constant:

$$L = mvr$$

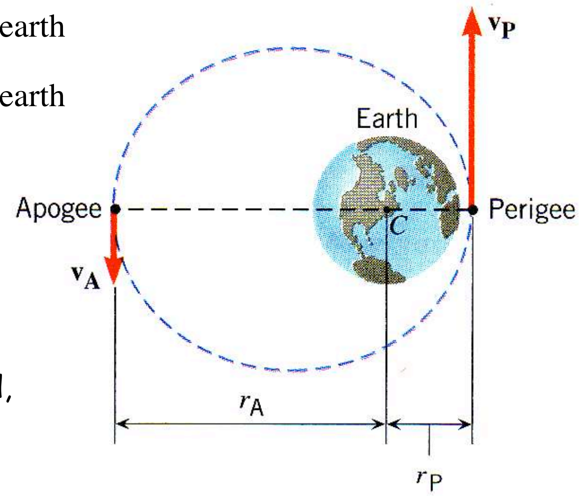
is constant around the orbit. → $v_A r_A = v_P r_P$

Example: comets speed up as they approach the sun

$r_A = 25.1 \times 10^6$ m from centre of earth

$r_P = 8.37 \times 10^6$ m from centre of earth

If $v_P = 8450$ m/s, find v_A.



Angular momentum is conserved,

$$m\, v_A\, r_A = m\, v_P\, r_P$$

$$\text{So, } v_A = \frac{v_P \times r_P}{r_A}$$

$$= \frac{8450 \times 8.37 \times 10^6}{25.1 \times 10^6} = 2818 \text{ m/s}$$

Conservation of Angular Momentum

The skater pulls in her arms, moving mass closer to the axis of rotation and decreasing her moment of inertia (divide body into chunks of mass m_i at distance r_i from axis of rotation, add up all of the $m_i r_i^2$).

As $L = I\omega$ is constant, the rotation rate increases as I decreases.

9.-/52: A woman is standing at the centre of a rotating platform that is turning at 5 rad/s. The total moment if inertia of woman and platform is 5.4 kg.m^2.

By pulling in her arms, she reduces her moment of inertia to 3.8 kg.m^2.

Find the new angular speed.

Conservation of angular momentum:

$$\text{Angular momentum, } L = I_0\, \omega_0 = I_f\, \omega_f$$

$$\text{Therefore, } \omega_f = \frac{I_0\, \omega_0}{I_f} = \frac{5.4 \times 5}{3.8} = 7.1 \text{ rad/s}$$

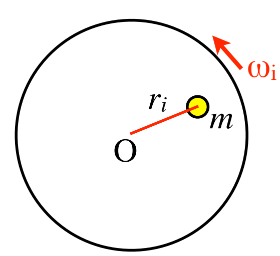
9.56/-: A playground carousel is free to rotate about its centre without friction or air resistance. The carousel has moment of inertia 125 kg.m^2.

When a person is standing at 1.5 m from the centre, the carousel has angular velocity ω_i = 0.6 rad/s. As this person moves inward to a point 0.75 m from the centre, the angular velocity increases to ω_f = 0.8 rad/s. What is the person's mass?

The moment of inertia of the person about the centre of the carousel is mr_i^2 (compare satellite orbiting the earth). The total moment of inertia of carousel plus person is:

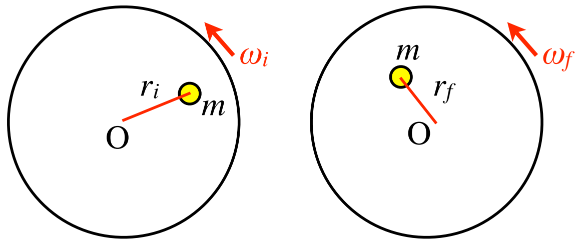
$$I_i = I_{carousel} + mr_i^2$$

and the angular momentum is $L = I_i \omega_i$.



$$I_i = I_{carousel} + mr_i^2$$

Angular momentum is $L = I_i\omega_i$.



When the person moves inward to a distance r_f from the centre of the carousel, the moment of inertia decreases, but the angular momentum is unchanged, so the angular velocity increases to ω_f.

$$L = I_i\omega_i = I_f\omega_f$$

That is, $L = (I_{carousel} + mr_i^2)\omega_i = (I_{carousel} + mr_f^2)\omega_f$

$$m = \frac{I_{carousel}(\omega_f - \omega_i)}{r_i^2\omega_i - r_f^2\omega_f}$$

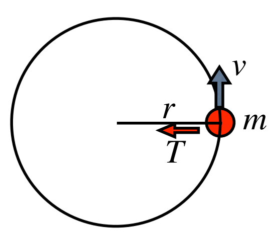
$I_{carousel} = 125$ kg.m^2
$r_i = 1.5$ m, $\omega_i = 0.6$ rad/s
$r_f = 0.75$ m, $\omega_f = 0.8$ rad/s

$m = 28$ kg

9.60: A small 0.5 kg object moves on a frictionless horizontal table in a circular path of radius 1 m. The angular speed is 6.28 rad/s.

The string is shortened to make the radius of the circle smaller without changing the angular momentum.

If the string breaks when its tension is 105 N, what is the radius of the smallest possible circle in which the object can move?



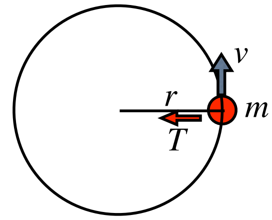
Angular momentum conserved:

$$L = m\,v\,r = \text{ constant}$$

$$v_f\,r_f = v_i\,r_i, \text{ so, } v_f = \frac{v_i\,r_i}{r_f} = \frac{r_i^2\,\omega_i}{r_f}$$

$$v_f = \frac{1^2 \times 6.28}{r_f}$$

$$v_f = \frac{6.28}{r_f}$$



The tension in the string is:

$$T = \frac{m\, v_f^2}{r_f} = 105 \text{ N} \quad \text{(the maximum before the string breaks)}$$

$$\frac{m}{r_f}\left[\frac{6.28}{r_f}\right]^2 = 105 \text{ N}$$

$$r_f^3 = \frac{(0.5 \text{ kg}) \times 6.28^2}{105 \text{ N}}$$

$$r_f = 0.57 \text{ m}$$

Summary

- Torque, calculated two ways

- First and second conditions of equilibrium
  (1) forces add to zero
  (2) torques add to zero about any point

- Centre of gravity (centre of mass)
  - the point at which the weight of an object may be considered to act
  - torques can be calculated by concentrating the whole mass at the centre of gravity

- Angular momentum is conserved in the absence of a net torque