

Chapter 8, Rotational Kinematics

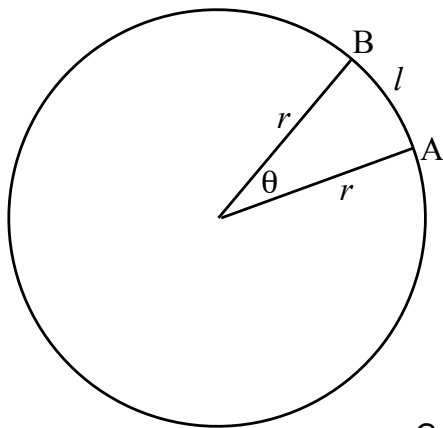
Sections 1 - 3 only

- Rotational motion and angular displacement
- Angular velocity and angular acceleration
- Equations of rotational kinematics

Wednesday, October 31, 2007

1

Angular Displacement



The length of the arc of the circle of radius r from A to B is:

$$l = r\theta$$

where θ is in radians.

$$\theta \text{ (radians)} = \frac{\text{arc length}}{\text{radius}} = \frac{l}{r}$$

One complete revolution corresponds to:

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians} \quad (\text{rad})$$

$$\pi \text{ radians} = 180^\circ$$

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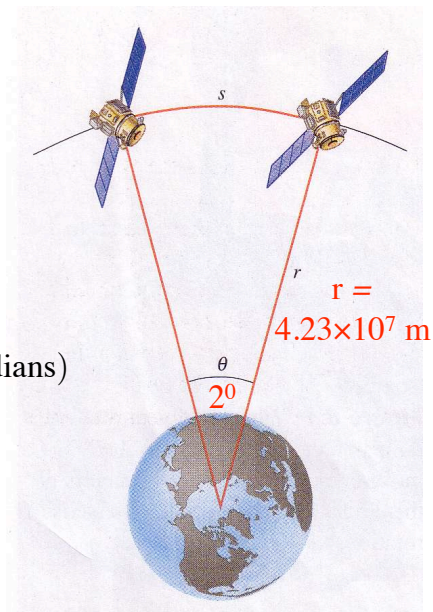
2

Two geostationary satellites are at an altitude of 4.23×10^7 m. Their angular separation θ is 2° .

Find the arc length s .

$$s = r \theta = (4.23 \times 10^7 \text{ m}) \times (2\pi/180 \text{ radians})$$

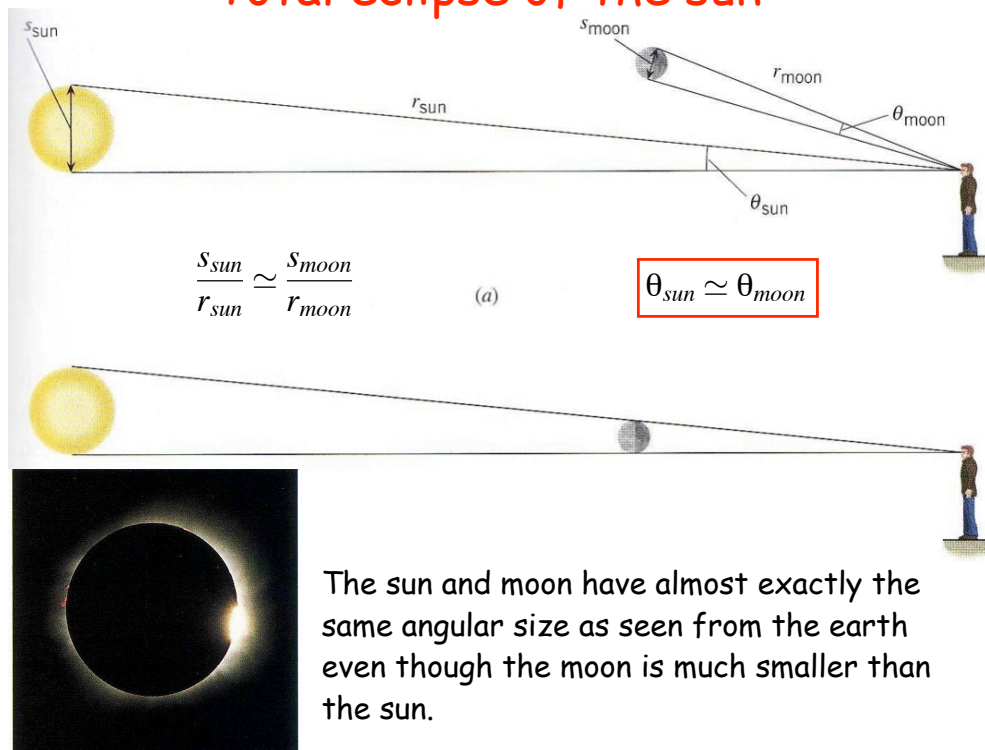
$$= 1.48 \times 10^6 \text{ m}$$



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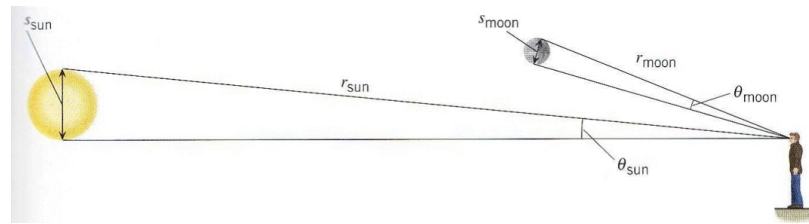
Total eclipse of the sun



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8.68/14



Moon: diameter = 3.48×10^6 m,
distance from earth = 3.85×10^8 m

Sun: diameter = 1.39×10^9 m,
distance from earth = 1.50×10^{11} m

a) Angular size of moon = $(3.48 \times 10^6 \text{ m}) / (3.85 \times 10^8 \text{ m}) = 0.00904 \text{ rad}$

Angular size of sun = $(1.39 \times 10^9 \text{ m}) / (1.50 \times 10^{11} \text{ m}) = 0.00927 \text{ rad}$

b) The moon has slightly smaller angular size than the sun.

c) Apparent area of moon = $(0.00904 / 0.00927)^2 \times$ apparent area of sun
= 95.1% of apparent area of sun.

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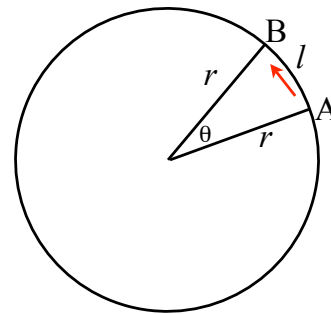
5

Angular velocity - motion around a circular path

Angular velocity is:

$$\bar{\omega} = \frac{\theta_B - \theta_A}{t_B - t_A} \text{ rad/s} \quad (\text{average})$$

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ rad/s} \quad (\text{instantaneous})$$



The instantaneous angular velocity is measured over a vanishingly small time interval.

If the motion from A to B takes time t, then

$$v = \frac{l}{t} = \frac{r\theta}{t} = r\omega$$

That is, $v = r\omega$

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Angular acceleration

Angular acceleration is the rate of change of angular velocity:

$$\text{Average angular acceleration } \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{\Delta\omega}{\Delta t}$$

$$\text{Instantaneous angular acceleration } \alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

Equations for rotational motion

Analogous to the equations for linear motion.

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ \theta &= \frac{1}{2}(\omega_0 + \omega)t \\ \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta\end{aligned}$$

$$\begin{aligned}v &= v_0 + at \\ x &= \frac{1}{2}(v_0 + v)t \\ x &= v_0 t + \frac{1}{2}at^2 \\ v^2 &= v_0^2 + 2ax\end{aligned}$$

Mastering Physics Assignment 4

Is due Monday, November 12 at 11 pm

Covers material from chapters 6 and 7

There are 8 questions for practice and 6 for credit

The Final Exam Schedule is Now Final!

PHYS 1020: Monday, December 17, 6 - 9 pm

Frank Kennedy Brown & Gold Gyms

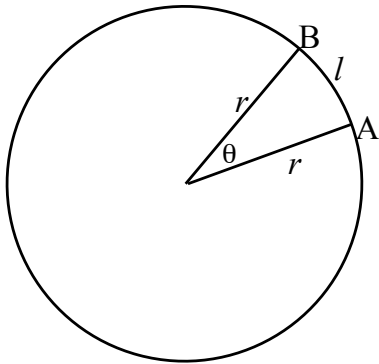
The whole course

30 multiple choice questions

Formula sheet provided

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Angular Displacement

The length of the arc of the circle of radius r from A to B is:

$$l = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\pi \text{ radians} = 180^\circ$$

Angular Velocity

$$\bar{\omega} = \frac{\theta_B - \theta_A}{t_B - t_A} \text{ rad/s} \quad (\text{average})$$

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ rad/s} \quad (\text{instantaneous})$$

$$\text{And, } v = r\omega$$

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Equations for rotational motion

Analogous to the equations for linear motion.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega \Leftrightarrow v$$

$$\alpha \Leftrightarrow a$$

$$\theta \Leftrightarrow x$$

$$v = v_0 + at$$

$$x = \frac{1}{2}(v_0 + v)t$$

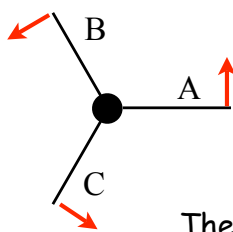
$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax$$

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8.12: A stroboscope is a light that flashes on and off at a constant rate to illuminate a rotating object. If the flashing rate is adjusted properly, the rotating object appears to be stationary.



A three blade propeller is rotating at 16.7 revolutions per second.

What should be the shortest time between flashes?

The propeller appears to be stationary if it rotates from A to B between flashes.

This is 1/3 of a revolution, which takes the propeller a time:

$$t = \frac{1}{3} \times \frac{1}{16.7} = 0.020 \text{ seconds}$$

For the next shortest time, the propeller rotates from A to C in 0.040 s.

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8.-/20: The angular speed of the rotor in a centrifuge increases from 420 to 1420 rad/s in 5 s.

a) Through what angle does the rotor move in this time?

$$\text{Average angular speed} = (420 + 1420)/2 = 920 \text{ rad/s.}$$

In 5 s, rotor turns through $920 \times 5 = 4600$ radians.

$$\text{or, } \frac{4600}{2\pi} = 732.1 \text{ revolutions}$$

b) What is the angular acceleration?

$$\omega = \omega_0 + \alpha t \quad (\text{think of } v = v_0 + at)$$

$$\text{So, } \alpha = \frac{(1420 - 420) \text{ rad/s}}{5 \text{ s}} = 200 \text{ rad/s}^2$$

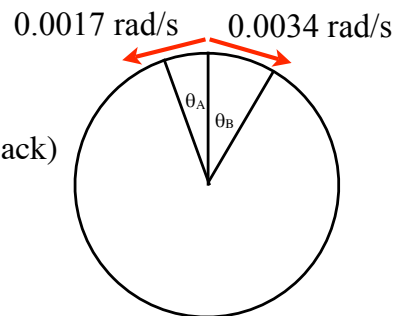
8.10/64: Two people start at the same place and walk around a circular lake in opposite directions. One has angular speed 0.0017 rad/s, the other 0.0034 rad/s. How long before they meet?

$$\text{A: } \theta_A = 0.0017t$$

$$\text{B: } \theta_B = 2\pi - 0.0034t \quad (\text{starts at } 360^\circ, \text{ walks back})$$

$$\theta_A = \theta_B \text{ when } 0.0017t = 2\pi - 0.0034t$$

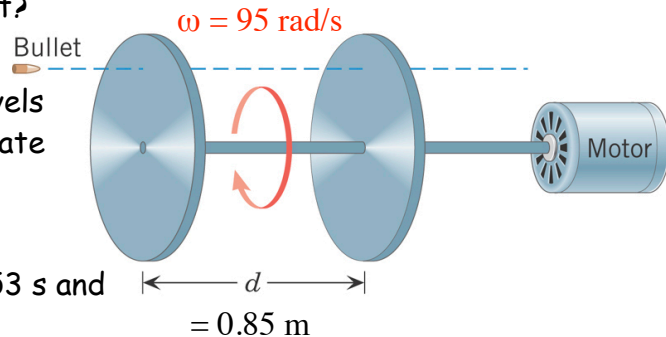
$$t = 2\pi/(0.0034 + 0.0017) = 1232 \text{ s.}$$



8.13/11: The speed of a bullet can be measured with the apparatus shown. The bullet passes through two disks that are rotating together. The disks rotate as the bullet travels from one disk to the other, so the holes in the disks do not line up.

The angular displacement between the bullet holes is 0.24 rad. What is the speed of the bullet?

In time t , the bullet travels 0.85 m and the disks rotate 0.24 rad at 95 rad/s.



So, $t = 0.24/95 = 0.00253 \text{ s}$ and $d = 0.85 \text{ m}$

$v = (0.85 \text{ m})/(0.00253 \text{ s}) = 336 \text{ m/s}$

8.20/-: The wheels of a bicycle have an angular velocity of 20 rad/s. The brakes are applied, bringing the bicycle to a uniform stop. During braking, the angular displacement of the wheels is 15.92 revolutions.

- How much time does it take to stop?
- What is the angular acceleration of the wheels?

$$\text{a) } \theta = \bar{\omega}t, \quad \bar{\omega} = (\omega_0 + \omega)/2 = \omega_0/2 = 10 \text{ rad/s}$$

$$\text{So, } t = \frac{\theta}{\bar{\omega}} = \frac{15.92 \times 2\pi}{10} = 10 \text{ s}$$

$$\begin{aligned} \text{b) } \omega &= \omega_0 + \alpha t \\ 0 &= \omega_0 + \alpha t \rightarrow \alpha = \frac{-\omega_0}{t} = -\frac{20}{10} = -2 \text{ rad/s}^2 \end{aligned}$$

8.27: A child is running around a stationary merry-go-round at 0.25 rad/s. At the moment he sees his favourite horse, one quarter turn away, the merry-go-round starts to move in the same direction as the child is running, accelerating at 0.01 rad/s².

What is the shortest time it takes the child to catch up with the horse?

What are angular positions of child and horse?

Child: $\theta_c = \omega_c t = 0.25 t$

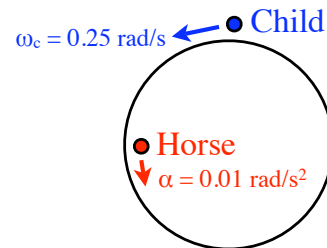
Horse: $\theta_h = \theta_o + \cancel{\omega_o t} + \alpha t^2/2 = \pi/2 + 0 + 0.01 t^2/2$

They meet when $\theta_c = \theta_h$: $0.25 t = \pi/2 + 0.01 t^2/2$

$$0.005t^2 - 0.25t + \pi/2 = 0 \rightarrow t = \frac{0.25 \pm \sqrt{0.25^2 - 4 \times 0.005 \times \pi/2}}{0.01}$$

$\rightarrow t = 7.37 \text{ s, or } 42.6 \text{ s}$

Horse passes child



8.21: A spinning top is made to spin by pulling on a string that is wrapped around it. The length of the string is $L = 64 \text{ cm}$ and the radius of the top is $r = 2 \text{ cm}$. The string is pulled so that the angular acceleration of the top is 12 rad/s^2 .

What is the final angular velocity of the top?

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad (\text{think } v^2 = v_0^2 + 2ax)$$

What is θ ?

The number of turns on the top is $N = L/(2\pi r) = 5.09$

$$\omega^2 = 0 + 2 \times 12 \times 5.09 \times 2\pi = 768 \text{ (rad/s)}^2$$

$$\omega = 28 \text{ rad/s}$$

8.14/68: A golf ball passes through a windmill, which has 8 blades and rotates at $\omega = 1.25 \text{ rad/s}$.

The opening between successive blades is equal to the width of a blade.

A golf ball is of diameter $d = 0.045 \text{ m}$. What must be the minimum speed of the golf ball so that it passes through an opening between blades?

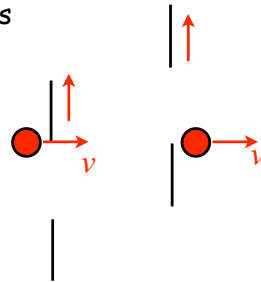
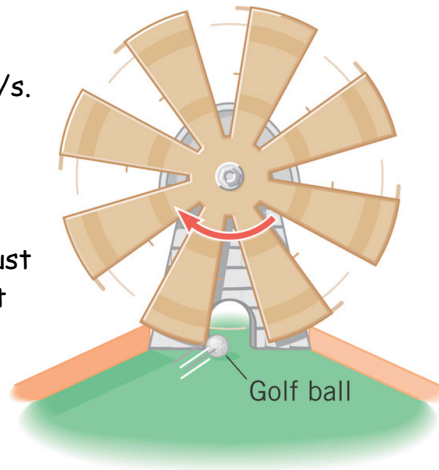
There are 8 blades and 8 gaps between blades. The angular width of each gap is $2\pi/16 \text{ rad}$.

The golf ball must travel at least a distance equal to its diameter while the blade moves the width of one gap.

That is, $t = \frac{d}{v} = \frac{2\pi/16}{\omega} \rightarrow v = \frac{8d\omega}{\pi}$

$v = 0.143 \text{ m/s}$

$t = \theta/\omega$



Equations for rotational motion

Analogous to the equations for linear motion.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega \Leftrightarrow v$$

$$\alpha \Leftrightarrow a$$

$$\theta \Leftrightarrow x$$

$$v = v_0 + at$$

$$x = \frac{1}{2}(v_0 + v)t$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax$$

arc length, $l = r\theta$

speed, $v = r\omega$

$\pi \text{ radians} = 180^\circ$