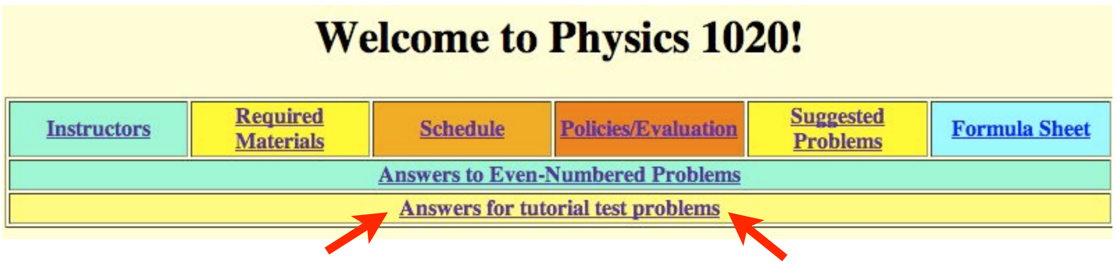
Answers for Tutorial Tests

A link to answers for the tests can be found on the PHYS 1020 home page:



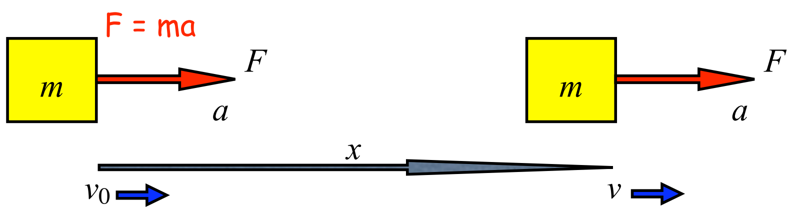
...but only for test 1 so far!

Chapter 6: Work and energy

- Work done by a constant force
- Work-energy theorem, kinetic energy
- Gravitational potential energy
- Conservation of mechanical energy
- Conservative and non-conservative forces
- Work-energy theorem and non-conservative forces
- Power
- Work done by a variable force

Work and Energy

Apply a constant force F to a mass m over a distance x



$v^2 = v_0^2 + 2ax$ and $a = F/m$ Newton's second law

So, $v^2 = v_0^2 + 2(F/m)x$

$(\times m/2)$

$$Fx = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

Work done = change in kinetic energy
Work-Energy theorem

$$Fx = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

Fx = "work" done by the force = force × displacement

The work changes the speed of the mass, increasing its *kinetic energy*

Initial kinetic energy: $KE_0 = mv_0^2/2$

Final kinetic energy: $KE = mv^2/2$

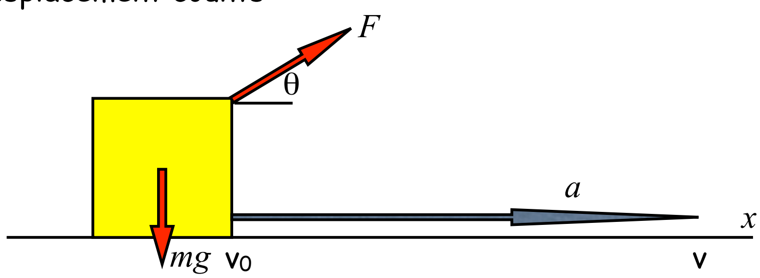
Work done $W = KE - KE_0 = \Delta KE$

Unit of work and energy: Joule (J)

1 J = 1 N.m

Work

Only the component of the force in the direction of the displacement counts –



$$a = F_x/m = F\cos\theta/m$$

$$\text{So, } v^2 = v_0^2 + 2(F\cos\theta/m)x \quad \rightarrow \quad Fx\cos\theta = mv^2/2 - mv_0^2/2$$

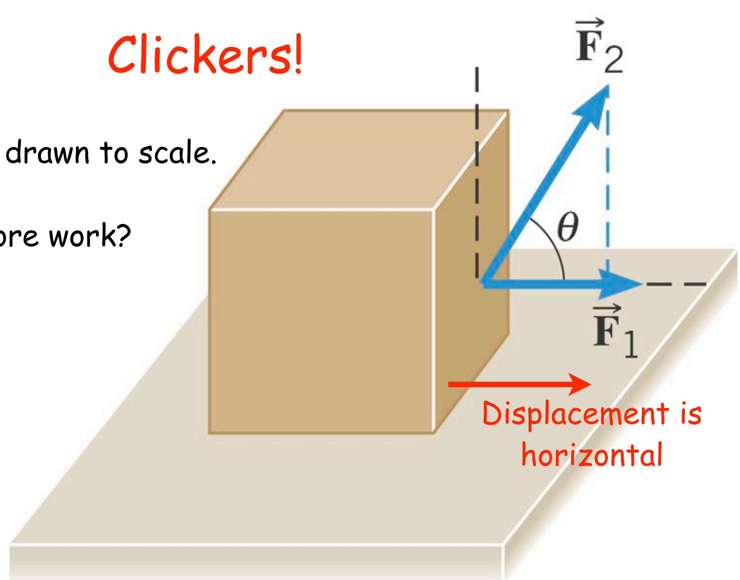
Work done, $W = Fx\cos\theta = \Delta KE$ **Work-Energy Theorem**

Work done = (force in direction of displacement) × (displacement)

Clickers!

6.C1: The forces are drawn to scale.

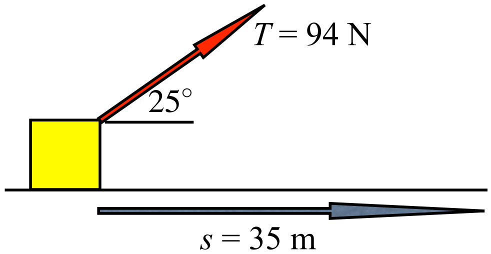
Which force does more work?



Work = (force in direction of displacement) × (displacement)

A) F_1 does more work, B) F_2 does more work
C) F_1 and F_2 do the same amount of work

6.4/3: How much work to pull the toboggan 35 m?



$$W = Ts\cos\theta$$

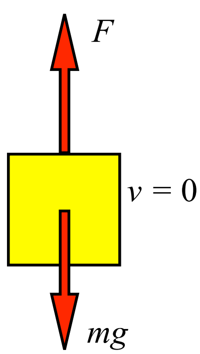
$$W = (94\text{ N}) \times (35\text{ m}) \times \cos 25^\circ = 2980\text{ J}$$

How much work if the rope is horizontal?

$$W = (94\text{ N}) \times (35\text{ m}) \times \cos 0^\circ = 3290\text{ J}$$

NB Less force is needed to move the toboggan when rope horizontal. Using the same force accelerates the toboggan more.

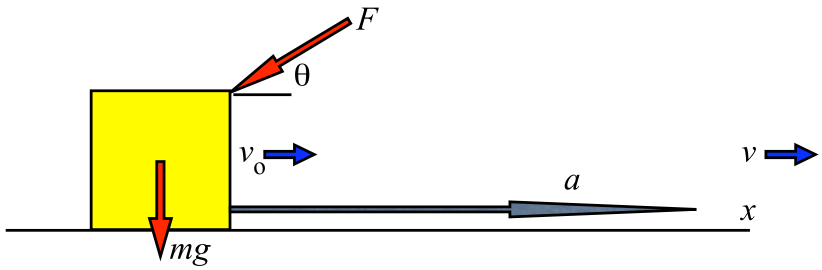
Work and Energy



No work is done in holding an object at rest:

- no displacement, no work
- also no net work in lifting an object up, then returning it to its starting point as the **net displacement is zero**

Negative Work



$a = -F\cos\theta/m, \quad v^2 = v_0^2 + 2ax$

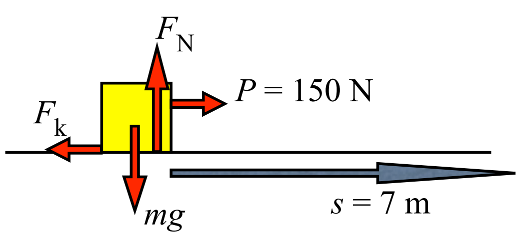
So, $v^2 = v_0^2 - 2(F\cos\theta/m)x$

Work done by force, $W = \frac{mv^2}{2} - \frac{mv_0^2}{2} = -Fx\cos\theta = \Delta KE$

where force in direction of displacement $= -F\cos\theta$

$\rightarrow$ negative work

6.10/8: A 55 kg box is pulled 7 m across the floor.



$\mu_k = 0.25$

How much work is performed by each of the 4 forces?

P: $W = Ps = (150\text{ N})\times(7\text{ m}) = 1050\text{ J}$

F_N: $W = F_N\times(\cos 90^o)\times(7\text{ m}) = 0$

mg: $W = mg\times(\cos 90^o)\times(7\text{ m}) = 0$

F_k: $W = -F_k\times(7\text{ m}) = -\mu_k\times mg\times(7\text{ m}) = -943\text{ J}$

Mastering Physics Assignment 3

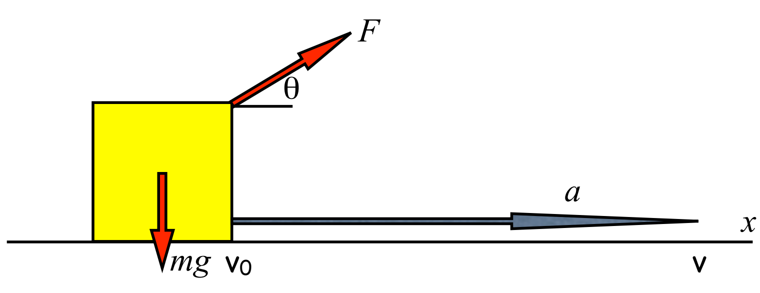
Assignment 3 is available on the Mastering Physics website

It is due Friday, October 26 at 11 pm

It covers material from chapters 4 and 5 as preparation for the term test on Tuesday

There are 7 questions for practice and 6 for credit

Work-Energy Theorem

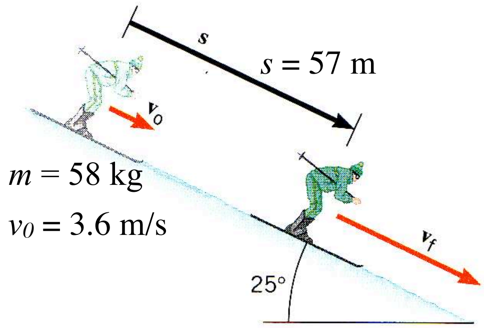


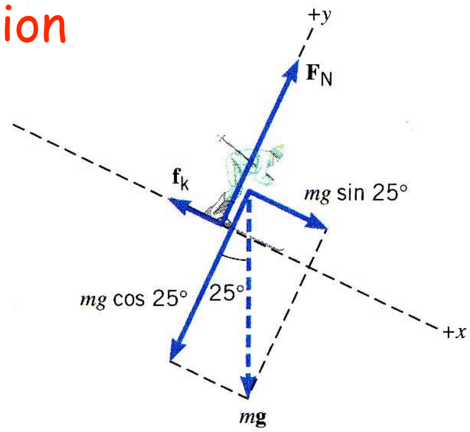
$$\text{Work done, } W = Fx\cos\theta = \frac{mv^2}{2} - \frac{mv_0^2}{2} = \Delta KE$$

Work done = (force in direction of displacement) × (displacement) = ΔKE

Unit of work: 1 Joule (J) = 1 N.m

Friction





A friction force f_k = 70 N acts on the skis.
Initial speed, v_0 = 3.6 m/s
Find final speed, v_f , after skiing 57 m down the slope

Work-energy theorem: W = F_{net} x displacement = ΔKE

F_{net} = mg $\sin 25^0$ - f_k = net force down the slope

Work-energy theorem: $W = F_{net} \times \text{displacement}$
$= \Delta KE = KE_f - KE_0$

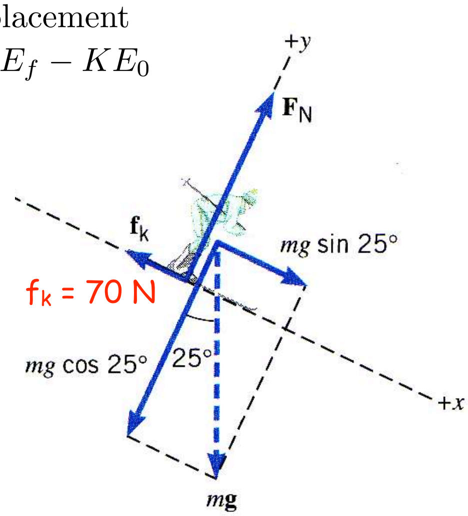
$F_{net} = mg \sin 25° - f_k$
$= (58\text{ kg}) \times g \sin 25° - 70\text{ N} = \underline{170\text{ N}}$

Displacement, $\underline{s = 57\text{ m}}$

So $\underline{W = F_{net} \times s = 170 \times 57 = 9690\text{ J}}$

$m = 58$ kg, $v_0 = 3.6$ m/s

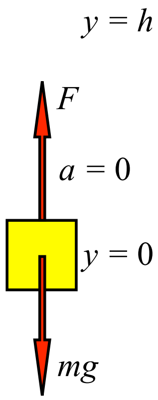
$\underline{KE_0 = mv_0^2/2 = 375.8\text{ J}}$



So, $W = 9690\text{ J} = KE_f - 375.8\text{ J} \rightarrow \underline{KE_f = 10{,}066\text{ J}}$ (= $mv_f^2/2$)
$\rightarrow \underline{v_f = 18.6\text{ m/s}}$

Check: $v_f^2 = v_0^2 + 2ax = v_0^2 + 2(F_{net}/m)x = 3.6^2 + 2(170/58)\times 57$
$v_f = 18.6$ m/s

Work done in lifting an object



A force F lifts the mass at constant speed through a height h.

The displacement is h.

The applied force in the direction of the displacement is:

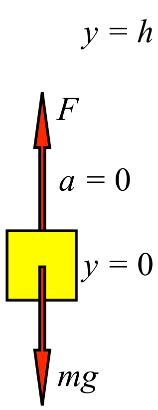
F = mg (no acceleration)

The work done by the force F is:

W = Fh = mgh

But the kinetic energy has not changed – the gravity force mg has done an equal amount of negative work so that the net work done on the mass by all forces (F and mg) is zero.

Work done in lifting an object



Alternative view: define a different form of energy –

Gravitational potential energy, PE = mgy

Define:

Mechanical energy = kinetic energy + potential energy

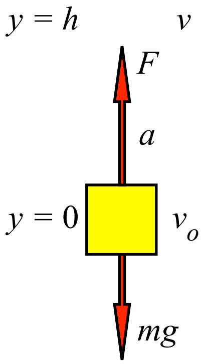
Mechanical energy, E = $mv^2/2$ + mgy

Then:

Work done **by applied force**, F, is (change in KE) + (change in PE)

So $W = Fh = \Delta\mathrm{KE} + \Delta\mathrm{PE}$

Check, using forces and acceleration



Net upward force on the mass is F – mg

Apply Newton's second law to find the acceleration:

F – mg = ma,

so, a = (F – mg)/m

One of famous four equations –

$$v^2 = v_0^2 + 2ah$$

$$\text{So, } v^2 = v_0^2 + \frac{2(F - mg)h}{m}$$

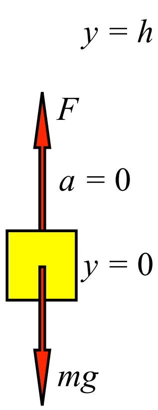
$(\times m/2)$

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = Fh - mgh$$

That is, $\Delta KE = Fh - \Delta PE$

Or, $W = Fh = \Delta KE + \Delta PE$

Two viewpoints



1) A net upward force (F - mg) does work and moves the mass upward and changes its kinetic energy:

(F - mg) x h = ΔKE (work-energy theorem)

2) An **applied force F** does work and changes the kinetic and potential energies of the mass:

W = Fh = ΔKE + mgh = ΔKE + ΔPE

The second is more powerful as it can be turned into a general principle that:

Work done by applied force = change in mechanical energy

If no work is done, then mechanical energy is conserved!

Conservation of Mechanical Energy

In the absence of applied forces and friction:

Work done by applied force = 0

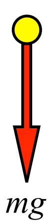
So, 0 = (change in KE) + (change in PE)

And KE + PE = E = mechanical energy = constant

Other kinds of potential energy:

- elastic (stretched spring)
- electrostatic (charge moving in an electric field)

6.28/26



$m = 0.6$ kg, $y_o = 6.1$ m

Ball is caught at $y = 1.5$ m

a) Work done on ball by its weight?

Weight force is in same direction as the displacement so,

Work = $mg \times$ displacement = $0.6g \times (6.1 - 1.5\text{ m}) = 27$ J

b) PE of ball relative to ground when released?

PE = $mgy_o = 0.6g \times (6.1\text{ m}) = 35.9$ J

c) PE of ball when caught?

PE = $mgy = 0.6\text{g} \times (1.5\text{ m}) = 8.8$ J

From last page:

$m = 0.6$ kg, $y_o = 6.1$ m

Ball is caught at $y = 1.5$ m

mg

b) PE of ball when released = $mgy_o = 35.9$ J

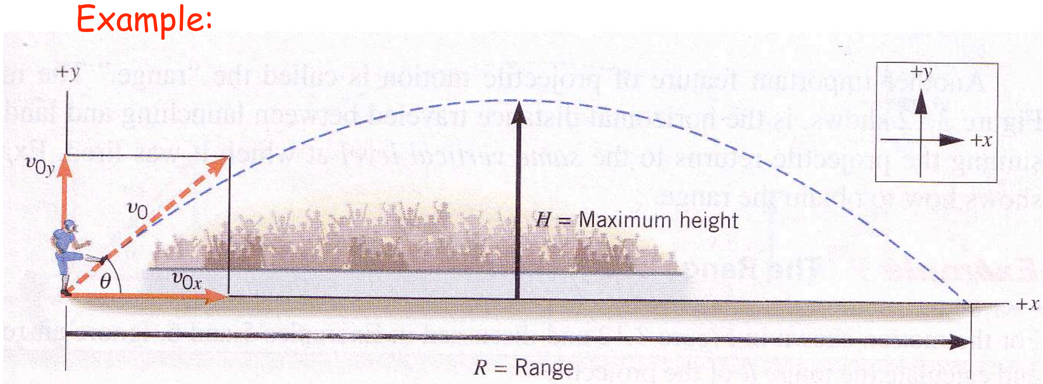
c) PE of ball when caught = $mgy = 8.8$ J

d) How is the change in the ball's PE related to the work done by its weight?

Change in PE = $mg(y - y_o)$ (final minus initial)

Work done by weight = $mg \times$(displacement) = $mg(y_o - y) = -\Delta$PE

Example:



No applied (i.e. external) forces

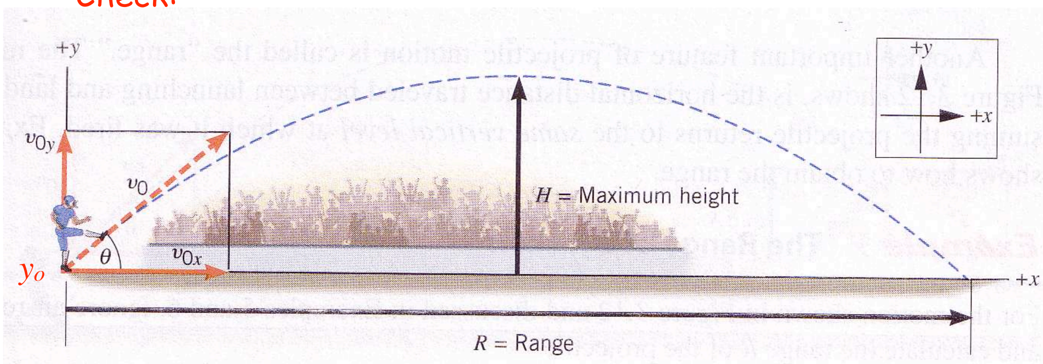
$E = KE + PE =$ constant

$KE = mv^2/2$

$PE = mgy$

So $E = mv^2/2 + mgy =$ constant, until the ball hits the ground

Check:



$$v_x = v_0 \cos\theta = \text{constant, in absence of air resistance}$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) = (v_0 \sin\theta)^2 - 2g(y - y_0) \text{ object at height } y$$

$$v^2 = v_x^2 + v_y^2 = (v_0 \cos\theta)^2 + (v_0 \sin\theta)^2 - 2g(y - y_0)$$

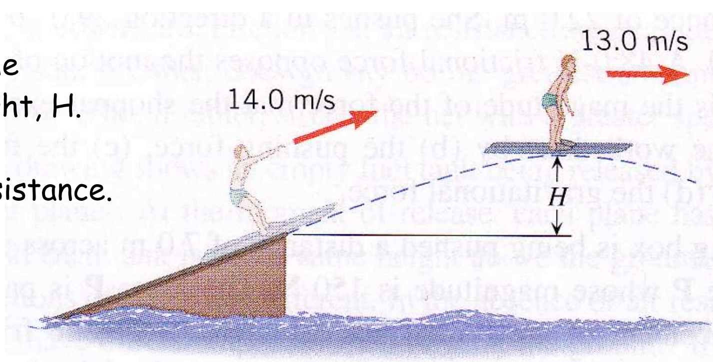
$$v^2 = v_0^2 - 2g(y - y_0), \text{ as } \sin^2\theta + \cos^2\theta = 1$$

$(\times m/2)$ (= constant)

$$mv_0^2/2 + mgy_0 = mv^2/2 + mgy \text{ and } KE_0 + PE_0 = KE + PE$$

6.-/34: Find the maximum height, H.

Ignore air resistance.



Conservation of mechanical energy: KE + PE = constant

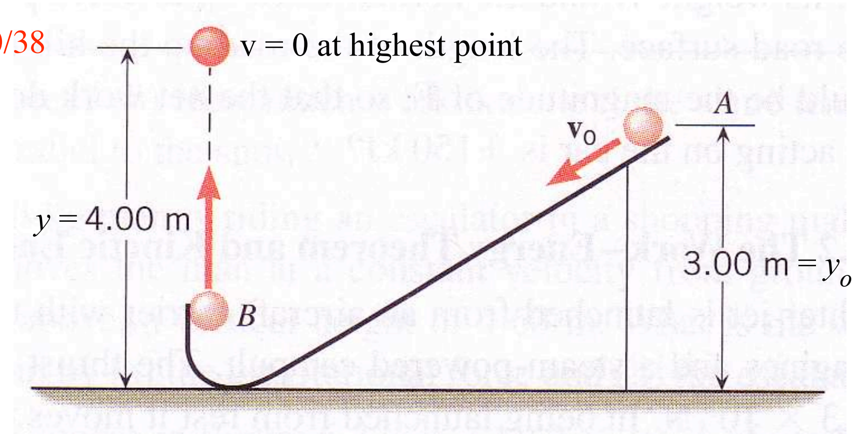
At take-off, set $y = 0$: $E = mv_0^2/2 + 0$

At highest point, $y = H$: $E = mv^2/2 + mgH$

So, $E = mv_0^2/2 = mv^2/2 + mgH$

$$H = \frac{(v_0^2 - v^2)/2}{g} = \frac{(14^2 - 13^2)/2}{9.8} = 1.38 \text{ m}$$

6.40/38



Find the speed of the particle at A (v_o). There is no friction.

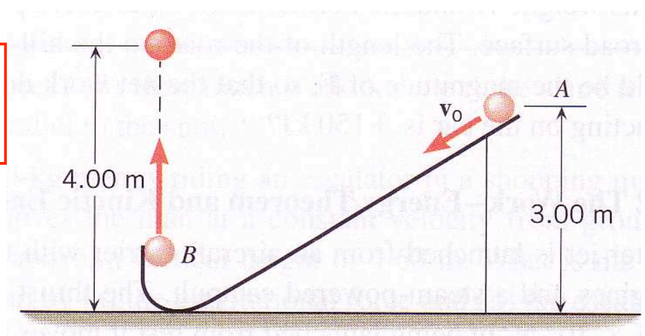
Conservation of mechanical energy: E = KE + PE = constant

At A: $E = mv_0^2/2 + mgy_0 = mv_0^2/2 + 3mg$

At highest point: $E = KE + mgy = 0 + 4mg$

At A: $E = mv_0^2/2 + 3mg$

At highest point: $E = 4mg$



So, $E = mv_0^2/2 + 3mg = 4mg$

$mv_0^2/2 = mg$

$v_0 = \sqrt{2g} = 4.43$ m/s

What happens at B doesn't matter, provided there is no loss of energy due to friction!

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There are 8 questions for practice and 6 for credit

OPUS Tutoring Schedule (room 105C Allen)

Monday	Tuesday	Wednesday	Thursday	Friday
8:30	8:30	8:30	8:30	8:30
9:30 Erica		9:30 Eric H		9:30 Charles
	10:00		10:00	
10:30		10:30		10:30 Andrei
11:30	11:30	11:30 Nikki	11:30 Liz	11:30
12:30 Adam		12:30 Eric R		12:30 Trevor
	1:00 Andrew		1:00 Ryan	
1:30		1:30		1:30
2:30 Todd	2:30 Kyle	2:30 Dan	2:30	2:30
3:30		3:30 Mitchell	3:30 Mike	3:30
	4:00			
4:30		4:30		4:30

Conservation of Mechanical Energy

Mechanical energy = KE + PE = $mv^2/2 + mgy$

In the absence of applied forces and friction:

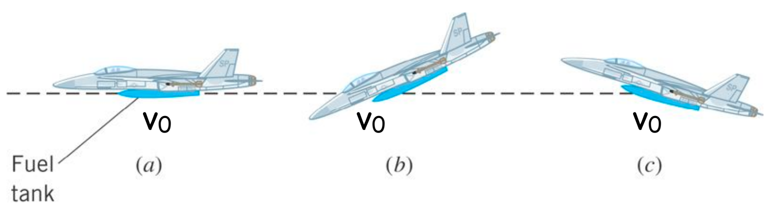
(change in KE) + (change in PE) = 0

so mechanical energy is conserved.

KE + PE = constant

Note: the weight mg is included in the PE so do not add in the work done by gravity, it's already there.

Clickers!

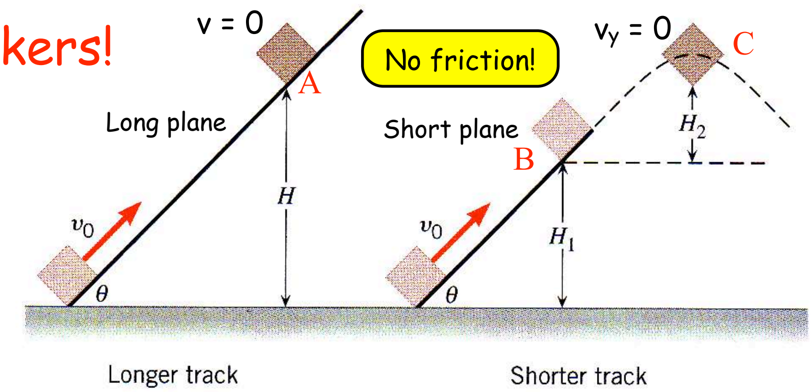


6.C17: An empty fuel tank is released by three different planes. At the moment of release, each plane has the same speed and each tank is at the same height above the ground.

In the absence of air resistance, do the tanks have different speeds when they hit the ground?

A) The tank in (b) has the highest speed when it hits the ground.
B) The tank in (c) has the highest speed as it reaches a greater height before falling.
C) All three tanks hit the ground at the same speed.

Clickers!



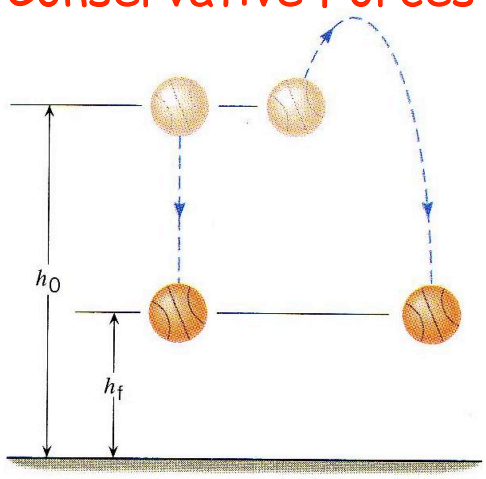
6.C87: Initial mechanical energy the same for the two blocks.

Block at left reaches highest point on plane at A, at y = H.

Block at right leaves shorter plane at B, at $y = H_1$
flies through air to highest point at C, at $y = H_1 + H_2$.

A) A is higher than C
B) A is lower than C
C) A and C are at the same height

Conservative Forces



Gravitational potential energy depends only on height

The difference in PE, $mg(h_o - h_f)$ is **independent of path taken**

→ Gravity is a "conservative force"

Conservative Forces

Alternative definitions of conservative forces:

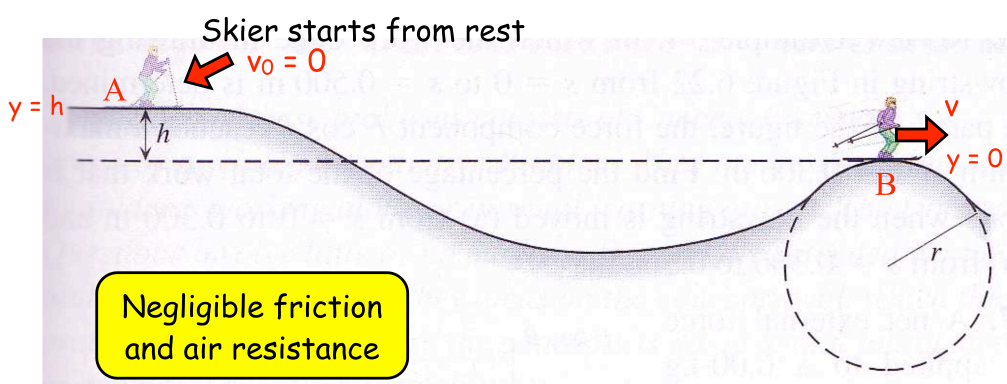
• The work done by a conservative force in moving an object is independent of the path taken.
(Compare pushing a crate across the floor – the most direct path requires the least work. Friction is **not** a conservative force).

• A force is conservative when it does no net work in moving an object around a closed path, ending up where it started.

In either case, the potential energy due to a conservative force depends only on position.

Examples of conservative forces:

• Gravity, elastic spring force, electric force



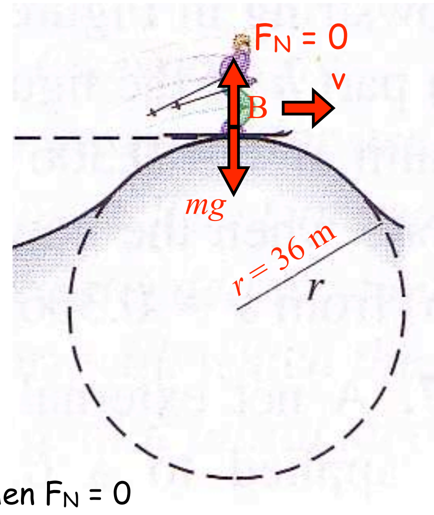
6.42: What must be the height h if the skier just loses contact with the snow at the crest of the second hill at B?

Mechanical energy is conserved, so $E_A = E_B$.

That is, $\underbrace{0 + mgh}_{\text{At A}} = \underbrace{mv^2/2 + 0}_{\text{At B}}$

So, $v^2 = 2gh$

$v^2 = 2gh$



If skier loses contact with snow at B, then $F_N = 0$

So, centripetal acceleration toward centre of curved slope is provided only by the skier's weight, mg.

That is, $mg = mv^2/r$ and $v^2 = rg$

So $v^2 = rg = 2gh$, and $h = r/2 = 18$ m

Non-conservative Forces

The work done by a non-conservative force depends on the path taken (as in pushing a crate across the floor...).

The longer the path taken, the more the (negative) work is done by the friction force.

A potential can be defined only for a conservative force.

Examples of non-conservative forces

- Static and kinetic friction forces
- Air resistance
- Tension, or any applied force
- Normal force
- Propulsion force in a rocket

Work-energy theorem revisited

From earlier, work done by an **applied force** is:

$$W = \Delta KE + \Delta PE$$

Identify this applied force as an example of a non-conservative force, and state that, for any non-conservative force:

$$W_{nc} = \Delta KE + \Delta PE$$

This is the work-energy theorem in terms of non-conservative forces

The important point is that a non-conservative force does not conserve mechanical energy

6.-/46: A 0.6 kg ball is pitched from a height of 2 m above the ground at 7.2 m/s. The ball travels at 4.2 m/s when it is 3.1 m above the ground.

How much work is done by air resistance, a non-conservative force?

$$\begin{aligned} W_{nc} &= \Delta KE + \Delta PE \qquad \text{(work-energy theorem)} \\ &= \frac{m(v_f^2 - v_0^2)}{2} + mg(y_f - y_0) \\ &= \frac{0.6(4.2^2 - 7.2^2)}{2} + 0.6g(3.1 - 2) = -14.1 \text{ J} \end{aligned}$$

Air resistance does -14.1 J of work

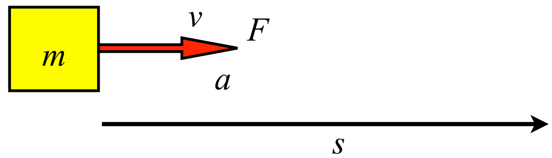
Power

Power is the rate of doing work, or the rate at which energy is generated or delivered.

Unit: 1 watt (W) = 1 J/s

$$\text{Power, } P = \frac{W}{t} = \frac{Fs}{t} = F \times \frac{s}{t} = Fv \qquad \text{(speed = distance/time)}$$

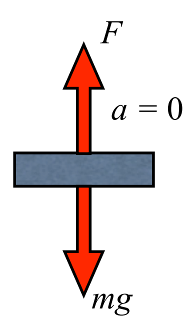
P = Fv



Kilowatt-hour (kWh): the energy generated or work done when 1 kW of power is supplied for 1 hour.
1 kWh = (1000 J/s)×(3600 s) = 3,600,000 J = 3.6 MJ

6.-/58: A 300 kg piano is being lifted by a crane at a steady speed to a height of 10 m. The crane produces a steady power of 400 W.

How much time does it take to lift the piano?



Power $P = Fv$ (F = mg, force to lift piano at constant speed)

So $P = mgv$ and $v = P/(mg)$

$$v = \frac{(400\ \text{W})}{(300\ \text{kg}) \times g} = 0.136\ \text{m/s}$$

$$\text{Takes time } \frac{h}{v} = \frac{10\ \text{m}}{0.136\ \text{m/s}} = 73.5\ \text{s}$$

6.59/74: The cheetah can accelerate from rest to 27 m/s (97 km/h) in 4 s. If its mass is 110 kg, determine the average power developed by the cheetah while it is accelerating.

Power = rate of doing work

Work done = $\Delta KE = KE_{final} - KE_{initial}$

$= m(v_f^2 - v_i^2)/2$

$= 110 \times (27^2 - 0^2)/2 = 40,095$ J

This work is done in 4 s, so the average power developed is:

P = 40,095/4 = 10,000 W = 10 kW (13.4 hp)

Mastering Physics Assignment 3

Assignment 3 is available on the Mastering Physics website

It is due Friday, October 26 at 11 pm

It covers material from chapters 4 and 5 as preparation for the term test on Tuesday

There are 8 questions for practice and 6 for credit

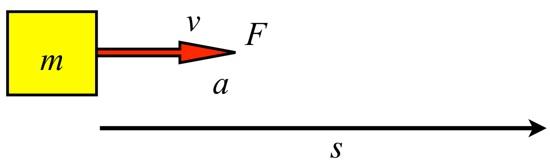
Power

Power is the rate of doing work, or the rate at which energy is generated or delivered.

Unit: 1 watt (W) = 1 J/s

$$\text{Power, } P = \frac{W}{t} = \frac{Fs}{t} = F \times \frac{s}{t} = Fv \qquad \text{(speed = distance/time)}$$

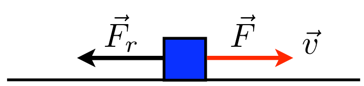
P = Fv



Kilowatt-hour (kWh): the energy generated or work done when 1 kW of power is supplied for 1 hour.
1 kWh = (1000 J/s)×(3600 s) = 3,600,000 J = 3.6 MJ

6.60: A motorcycle (mass of cycle + rider = 250 kg) is travelling at a steady speed of 20 m/s. The force of air resistance on cycle + rider is 200 N. Find the power necessary to maintain this speed if a) the road is level and b) slopes upward at 37^0.

a)

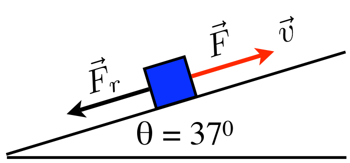


Work-energy theorem: $W_{nc} = \Delta KE + \Delta PE$, and $\Delta KE = \Delta PE = 0$

The force supplied by the engine $F = F_r = 200$ N

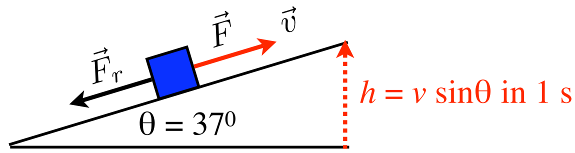
Power needed, $P = Fv = 200 \times 20 = 4000$ W (5.4 hp)

b)



The motorcycle gains potential energy, so an extra amount of energy must be supplied by the engine.

b)



Work-energy theorem: $W_{nc} = \Delta KE + \Delta PE$, and $\Delta KE = 0$

In 1 s, cycle goes up an amount $h = v \sin\theta$ (travels distance v in 1 s)

So, extra work done by engine in 1 s is given by $\Delta PE = mgv \sin\theta$

So, $P = 4000 + mgv \sin\theta$

$= 4000 + 250 \times g \times 20 \sin 37^0$

$= 33{,}500$ W (45 hp)

Other Forms of Energy

There are many forms of energy:

- Electrical
- Elastic (eg energy stored in a spring)
- Chemical
- Thermal
- Nuclear

Energy is conserved overall:

Energy may be converted from one form to another, but the total amount of energy is conserved.

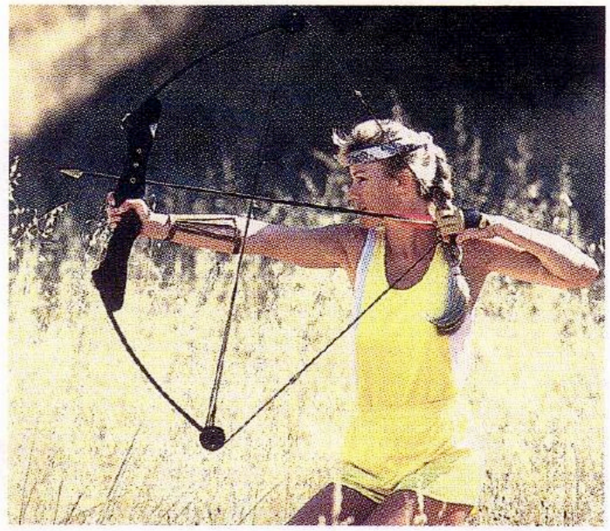
Work done by a variable force

Example: compound bow

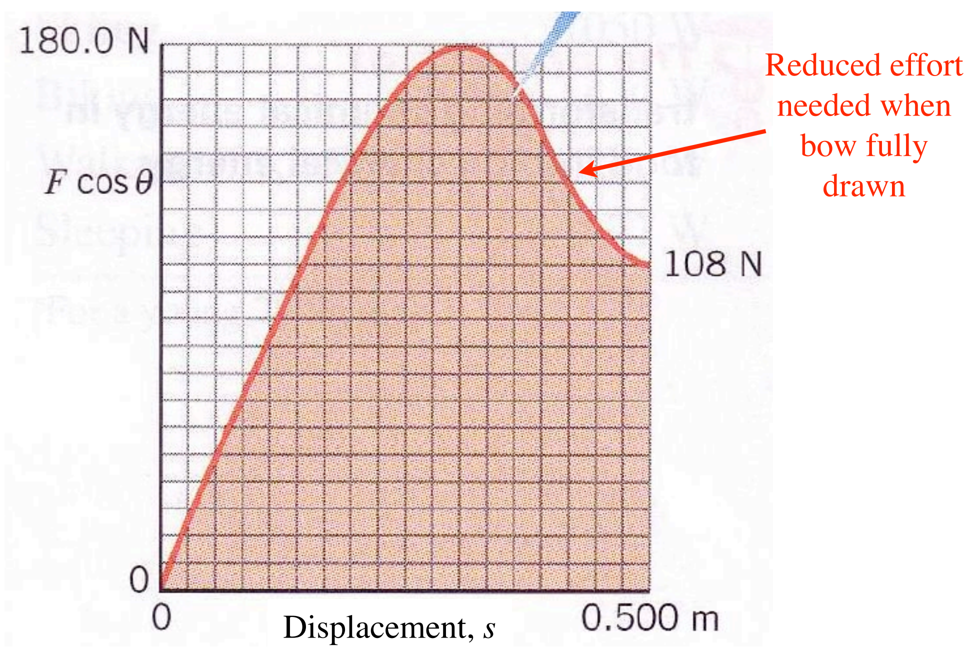
- a number of pulleys and strings

• maximize the energy stored in the bow for finite effort

• reduced force with bow fully drawn.

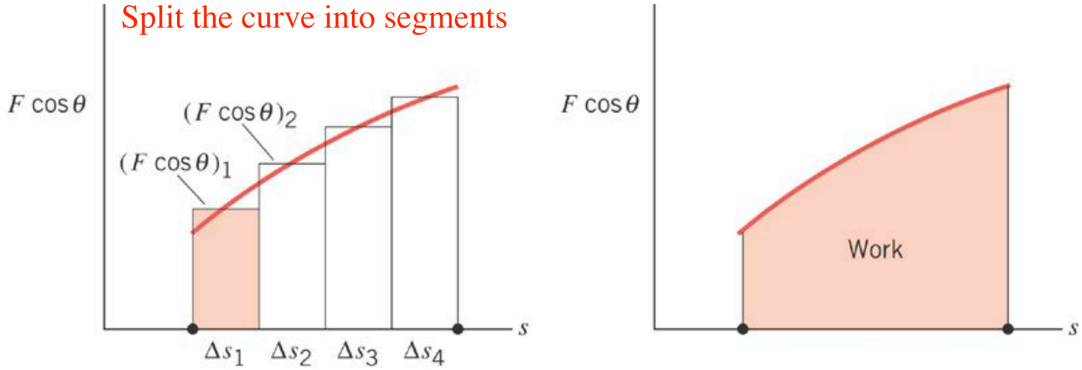


Force to draw the bow



How much work is needed to draw the bow?

Work done is force × distance...



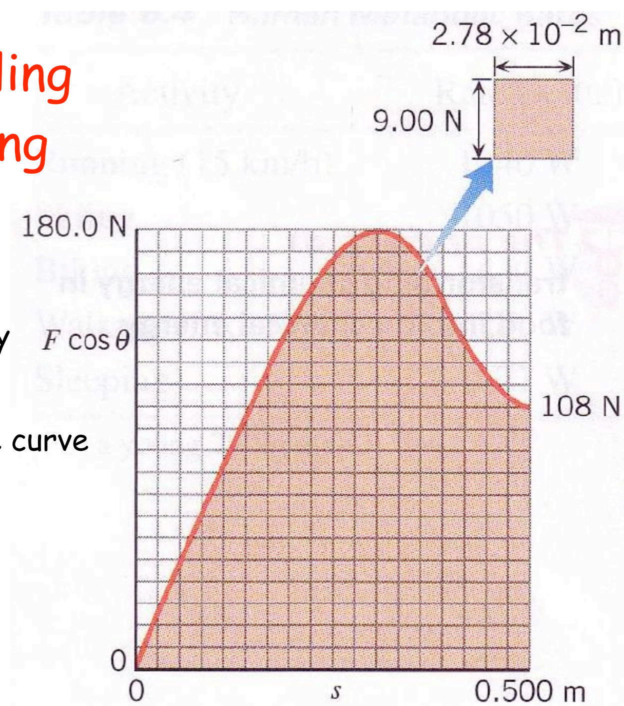
(a) (b)

$$W \simeq (F\cos\theta)_1\Delta s_1 + (F\cos\theta)_2\Delta s_2 + \ldots$$

= sum of force × distance

Becomes exactly the area under the curve when the slices become vanishingly narrow → integral calculus

Work done in pulling back the bowstring



Work done in drawing the bow = area under the curve

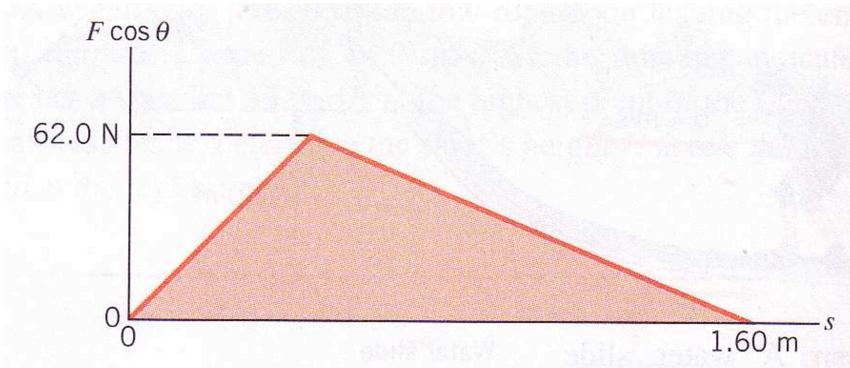
Count the squares, multiply by area of each.

Number of squares under the curve ≈ 242.
Area of each square is:

(9 N) × (0.0278 m)
= 0.25 N.m = 0.25 J.

So, work done is W = 242 × 0.25 = 60.5 J

6.66/64



Work done = area under triangular curve

$$= \frac{1}{2} \times (\text{base}) \times (\text{height})$$

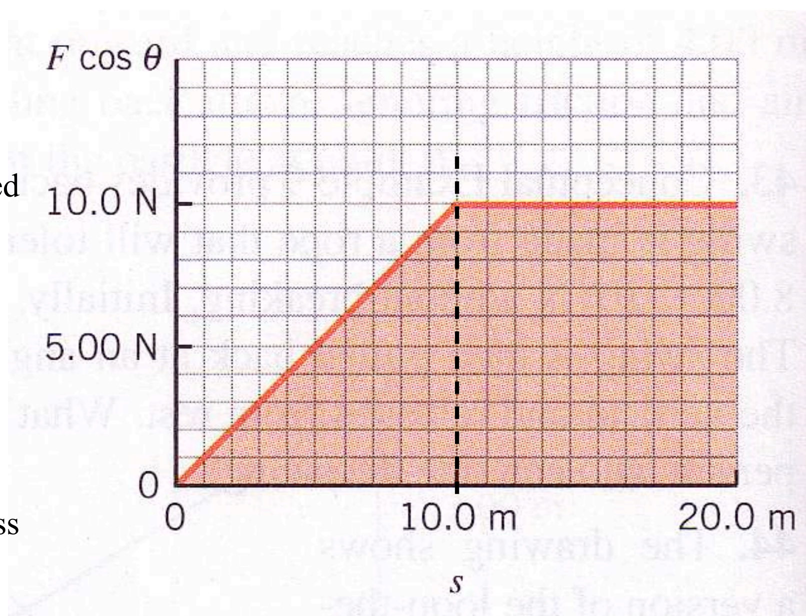
$$W = 0.5 \times (1.6\ \text{m}) \times (62\ \text{N}) = 49.6\ \text{J}$$

6.67

A force is applied to a 6 kg mass initially at rest.

a) How much work is done by the force?

b) What is the speed of the mass at $s = 20$ m?



a) Work done = area under the force-displacement curve

$$W = \frac{1}{2} \times (10\ \text{m}) \times (10\ \text{N}) + (20 - 10\ \text{m})(10\ \text{N}) = 150\ \text{J}$$

b) What is the speed of the mass at $s = 20$ m?

$$W_{nc} = \Delta KE + \Delta PE = mv^2/2 + 0 \quad = 150 \text{ J}$$

$$v = \sqrt{2W_{nc}/m} \;=\; \sqrt{2 \times 150/6} = 7.07 \text{ m/s}$$

Summary

In absence of non-conservative forces:

Conservation of mechanical energy: E = KE + PE = constant

When non-conservative forces are present:

Work-energy theorem: $W_{nc} = \Delta KE + \Delta PE$

Power = rate of doing work (1 W = 1J/s)

P = Fv

Work done by a variable force = area under the force versus displacement curve