Chapter 3: Kinematics in Two Dimensions

Mastering Physics Assignment #1

The first assignment is available at the
Mastering Physics website for PHYS1020UM

It is due on Monday, September 24, at 5 pm

Register for Mastering Physics if you haven't done so already!

(5% of final grade for Mastering Physics assignments)

GENERAL PHYSICS I: PHYS 1020

Schedule - Fall 2007
(lecture schedule is approximate)

Week	Date		Lecture	Cutnell & Johnson	Topic	Labs/Tests (Tuesdays, Wednesdays, Thursdays)
1	F	Sept 7	1	Chapter 1	Introduction	No lab or tutorial
2	M	10	2			No lab or tutorial
	W	12	3	Chapter 2	Kinematics in one dimension	
	F	14	4			
3	M	17	5			Errors Lecture
	W	19	6	Chapter 3	Kinematics in two dimensions	
	F	21	7			
4	M	24	8			Experiment 1: Measurement of Length and Mass
	W	26	9	Chapter 4	Forces and Newton's laws	
	F	28	10			

This week in the lab: Errors Lecture

Next week: Experiment 1, measurement of length and mass

What's new in this chapter

- Displacement, velocity, acceleration extended to two dimensions
- Motion in x can be separated completely from motion in y, provided air resistance is negligible – treatment of projectile motion
- Relative velocity
- Not yet any physics as such!

Speed, Velocity and Acceleration in One Dimension

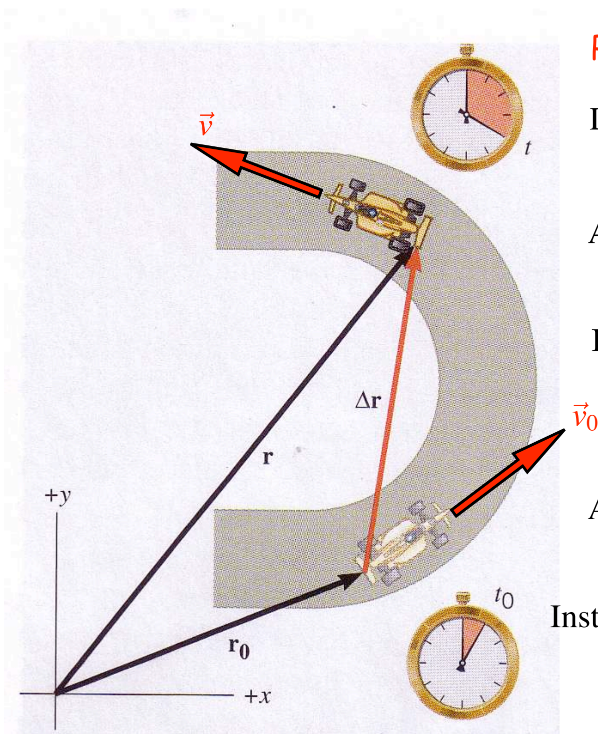
$$\text{Average speed } = \frac{\text{Distance}}{\text{Elapsed time}} = \frac{x - x_0}{t - t_0}$$

$$\text{Average velocity } = \frac{\text{Displacement}}{\text{Elapsed time}} = \frac{\Delta \vec{x}}{t - t_0}$$

$$\text{Instantaneous velocity } \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{elapsed time}} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$$

$$\text{Instantaneous acceleration} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$



Position vectors **r**, **r$_0$** at t, t$_0$

$$\text{Displacement } \Delta \vec{r} = \vec{r} - \vec{r}_0$$

$$\text{Average velocity } = \frac{\Delta \vec{r}}{t - t_0}$$

$$\text{Instantaneous velocity } = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\text{Average acceleration } = \frac{\vec{v} - \vec{v}_0}{t - t_0}$$

$$\text{Instantaneous acceleration } = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

There is an acceleration whenever there is a change of speed or direction

Clickers!

You drive 1500 m east in 2 minutes, then drive north the same distance in a further 2 minutes.

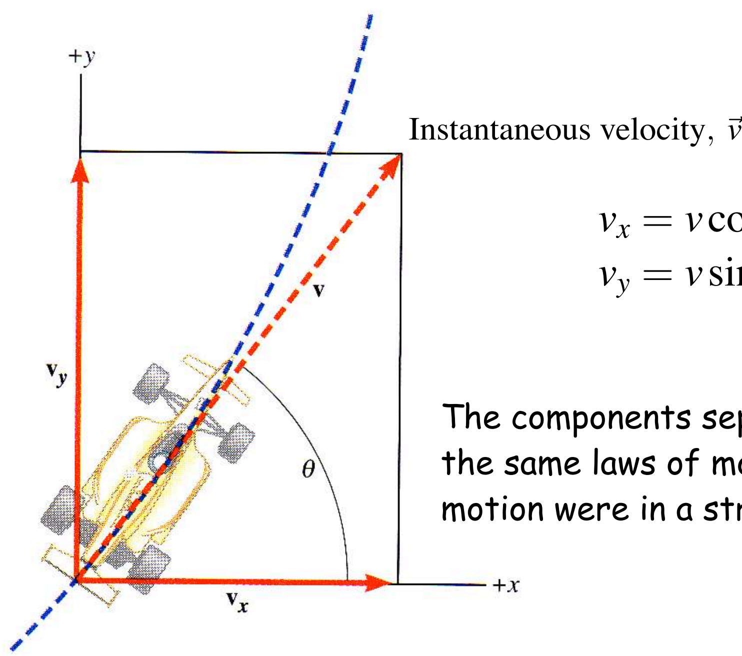
What can be said about the average speeds and velocities for the two parts of the trip?

a) The average speeds are the same, and the average velocities are the same.

b) The average speeds are the same, but the average velocities are different.

c) The average speeds are different, but the average velocities are the same.

Vectors can be resolved into components



$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

The components separately follow the same laws of motion as if the motion were in a straight line

Equations of motion in two dimensions

Same as before, only with subscripts for each direction of motion

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

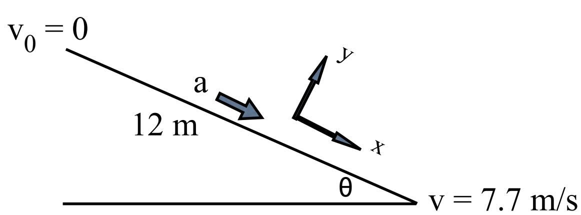
$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \qquad y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x) t \qquad y - y_0 = \frac{1}{2}(v_{0y} + v_y) t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

For projectile motion: $a_x = 0$, $a_y = -g$

3.8: A skateboarder rolls down a 12 m ramp, reaching a speed of 7.7 m/s at the bottom. What is her average acceleration?

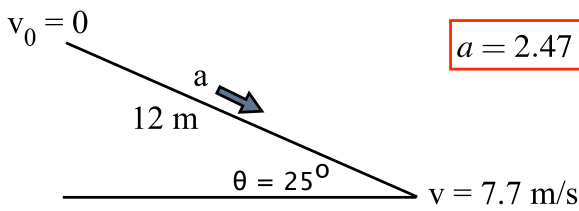


Tilt the x-axis to point down the slope

$$v^2 = v_0^2 + 2ax$$

$$7.7^2 = 0 + 2a \times (12\ \text{m}) \quad \rightarrow \underline{a = 2.47\ \text{m/s}^2}$$

If $\theta = 25^o$, what is the acceleration parallel to the horizontal?



$a = 2.47\ \text{m/s}^2$

Acceleration parallel to horizontal $= a\cos\theta = 2.47\cos 25^\circ = 2.24\ \text{m/s}^2$

A spacecraft is travelling with a velocity of v_{0x} = 5480 m/s along the +x direction. Two engines are fired for 842 seconds.

Engine one: a_x = 1.20 m/s^2
Engine two: a_y = 8.40 m/s^2

Find final v_x, v_y.

Acceleration in x direction: $v_x = v_{0x} + a_x t \ = 5480 + 1.2 \times 842 = 6490\ \text{m/s}$

Acceleration in y direction: $v_y = v_{0y} + a_y t \ = 0 + 8.4 \times 842 = 7073\ \text{m/s}$

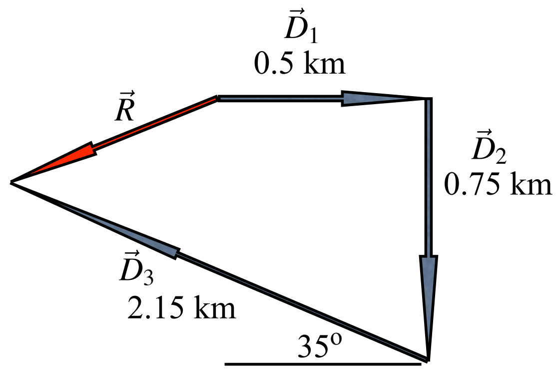
Final speed:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{6490^2 + 7073^2} = 9600\ \text{m/s}$$

3.10: A person walks 0.5 km east, 0.75 km south and 2.15 km at 35° north of west in 2.5 h.

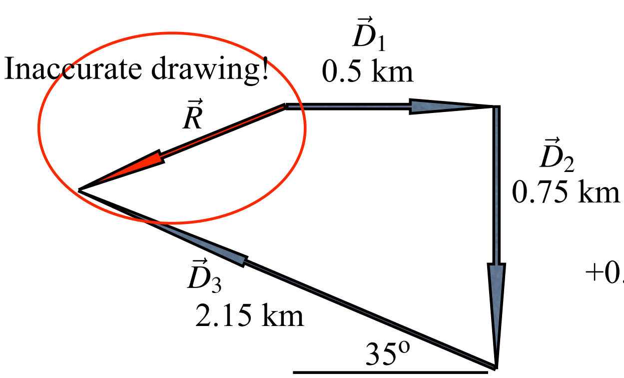
Find the displacement from the starting point and average velocity.

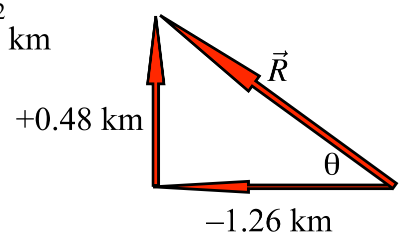
$\vec{R} = ?$



$$\vec{R} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 \qquad \rightarrow R_x = D_{1x} + D_{2x} + D_{3x}$$

$$R_y = D_{1y} + D_{2y} + D_{3y}$$





$$R_x = D_{1x} + D_{2x} + D_{3x} = 0.5 + 0 - 2.15\cos 35^\circ = -1.26 \text{ km}$$

$$R_y = D_{1y} + D_{2y} + D_{3y} = 0 - 0.75 + 2.15\sin 35^\circ = +0.48 \text{ km}$$

$$R = \sqrt{R_x^2 + R_y^2} = 1.35 \text{ km}$$

Angle to west direction: $\tan\theta = 0.48/1.26 \quad \rightarrow \theta = 20.9^\circ$ north of west

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{1.35 \text{ km at } 20.9^\circ \text{ north of west}}{2.5 \text{ h}}$$

$$= 0.54 \text{ km/h at } 20.9^\circ \text{ north of west}$$

Clickers!

A power boat, starting from rest, maintains a constant acceleration. After a certain time t, its displacement and velocity are $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$.

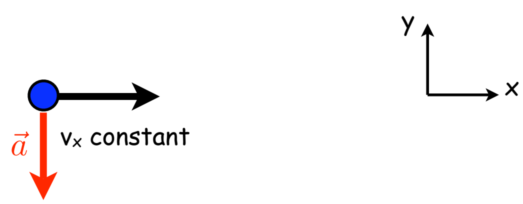
At time 2t, what would be its displacement and velocity, assuming the acceleration remains the same?

a) $2\vec{\mathbf{r}}$ and $2\vec{\mathbf{v}}$,
b) $2\vec{\mathbf{r}}$ and $4\vec{\mathbf{v}}$,
c) $4\vec{\mathbf{r}}$ and $2\vec{\mathbf{v}}$,
d) $4\vec{\mathbf{r}}$ and $4\vec{\mathbf{v}}$.

Projectile Motion

- Consider motion in x and y separately
- Ignore air resistance → velocity in x-direction is constant
- Write down positions in x and y as a function of time
- Remember that the projectile travels up and down (y) in the same time that it is travelling sideways (x)

Projectile Motion



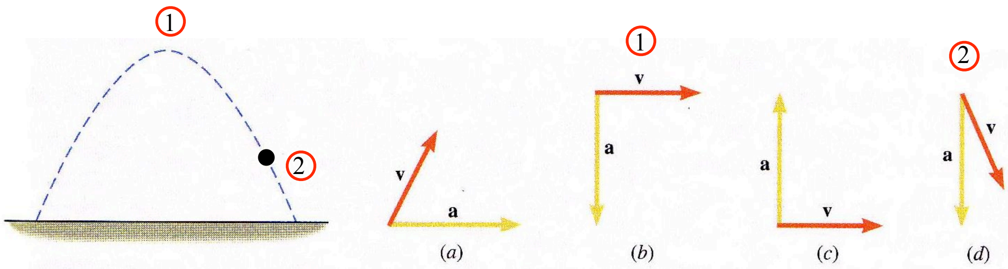
In absence of air resistance: no forces act in x-direction, so v_x, the speed in x-direction is constant throughout the path.

Speed changes in y-direction because of gravity.

Clickers!

The projectile has velocity $\vec{v}$ and acceleration $\vec{a}$

There is no air resistance



Which of (a), (b), (c) and (d) could **not** represent the directions of the vectors at any point of the trajectory?

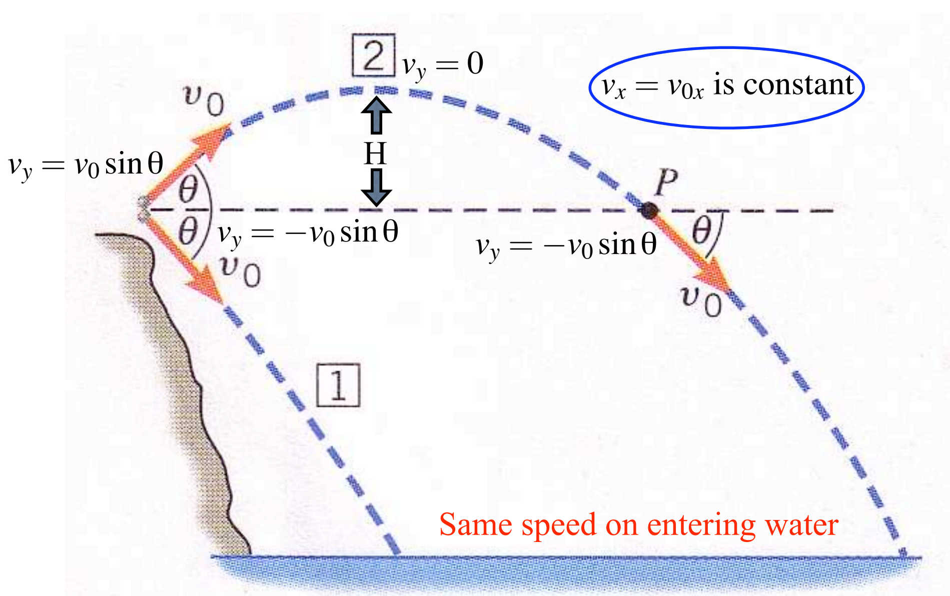
A: (a)

B: (b)

C: (c)

D: (d)

E: (a) and (c)



Stones 1 and 2 are thrown with the same speed, v_o, but at angles θ above and below the horizontal. Which hits the water with the greater speed?

Stone 2 at P has the same velocity as stone 1 at the start → same speed when they hit the water

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Answers to even-numbered problems

Username: PHYS1020

Password: xxxxxxxx

FALL 2007

Welcome to Physics 1020!

Instructors	Required Materials	Schedule	Policies/Evaluation	Suggested Problems	Formula Sheet
Answers to Even-Numbered Problems					

Mastering Physics Assignment # 1

Information on "Mastering Physics"

Combining Mastering Physics with Mastering Chemistry

GENERAL PHYSICS I: PHYS 1020

Schedule - Fall 2007
(lecture schedule is approximate)

3	M	17	5			
	W	19	6			Errors Lecture
	F	21	7	Chapter 3	Kinematics in two dimensions	
4	M	24	8			
	W	26	9			Experiment 1: Measurement of Length and Mass
	F	28	10	Chapter 4	Forces and Newton's laws	
5	M	Oct 1	11			
	W	3	12	Chapter 5	Uniform circular motion	Tutorial and Test 1 (chapters 1, 2, 3)
	F	5	13			

Next week

Experiment 1, measurement of length and mass

Week of October 1

Tutorial and test 1 on chapters 1, 2, 3

Projectile Motion

- Consider motion in x and y separately
- Ignore air resistance → velocity in x-direction is constant
- Write down positions in x and y as a function of time
- Remember that the projectile travels up and down (y) in the same time that it is travelling sideways (x)

Clickers!

3.C8: A rifle, at a height H above the ground, fires a bullet parallel to the ground.

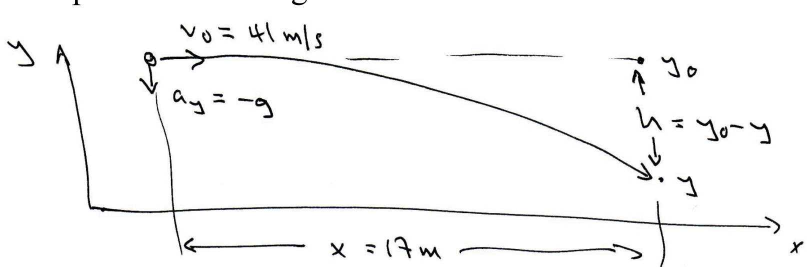
At the same instant and at the same height, a second bullet is dropped from rest.

In the absence of air resistance, which bullet strikes the ground first?

A) The bullet that is dropped strikes the ground first
B) The bullet fired from the rifle strikes the ground first
C) The bullets strike the ground at the same time
D) Impossible to say without knowing the speed of the bullet

Projectile Motion

3.24/26: A ball is thrown horizontally at 41 m/s. How much does it drop while travelling a horizontal distance of 17 m?



Motion in x direction:

$v_x = v_0 = 41$ m/s (constant in absence of air resistance)

Time to travel 17 m in x direction: $t = \frac{17 \text{ m}}{41 \text{ m/s}} = 0.4146$ s

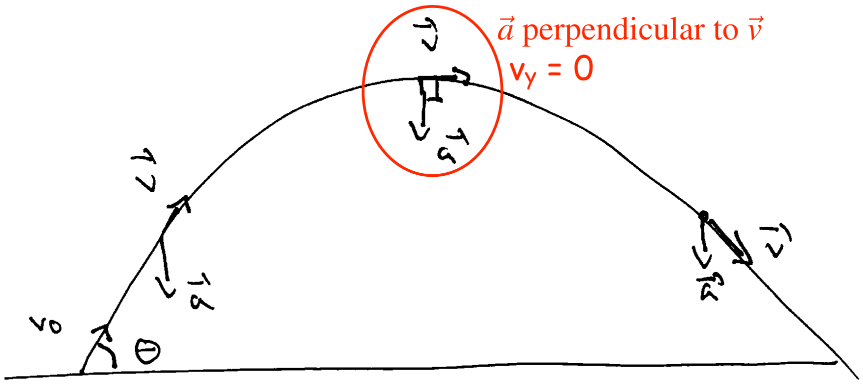
Motion in y: $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

Ball drops by: $h = y_0 - y = 0 + \frac{1}{2}g \times 0.4146^2 = 0.84$ m

3.C2: An object is thrown up in the air at an angle θ less than 90^0.

a) Is there a point where the acceleration and velocity are perpendicular?

b) Is there any point where velocity and acceleration are parallel?

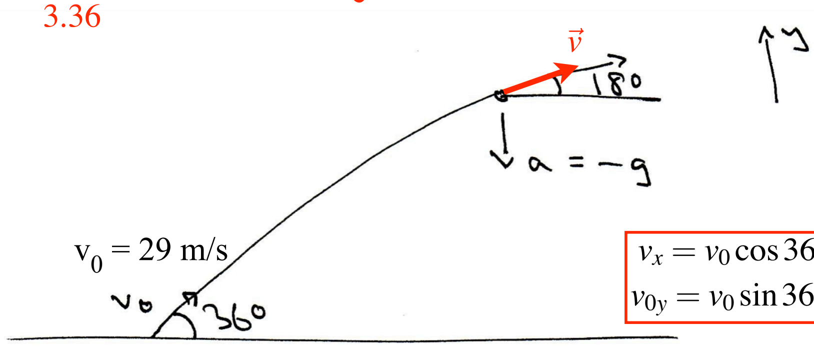


Acceleration is always downward – gravity always pulls downward

Velocity is always tangent to the trajectory

Projectile Motion

3.36



$v_x = v_0 \cos 36^\circ$ (constant)
$v_{0y} = v_0 \sin 36^\circ$

A projectile is launched with initial speed v_0 = 29 m/s at 36^0 to the horizontal. When does the path make an angle of 18^0 to the horizontal?

The angle to the horizontal is given by: $\tan\theta = \frac{v_y}{v_x}$

So need to find when: $v_y = v_x \tan 18^\circ$

So need to find when: $v_y = v_x \tan 18^\circ$

$v_0 = 29$ m/s

v_x

So, $v_y = v_0 \cos 36^\circ \tan 18^\circ = 0.2629 v_0$

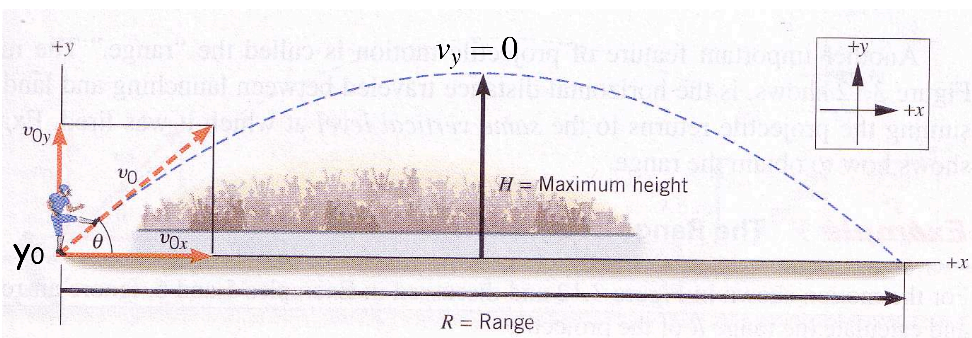
Famous equation for v_y:

$$v_y = v_{0y} - gt$$

$$\text{Therefore, } t = \frac{v_{0y} - v_y}{g} = \frac{v_0 \sin 36^\circ - 0.2629 v_0}{g}$$

$$t = (29 \text{ m/s}) \times \frac{(\sin 36^\circ - 0.2629)}{9.8} = 0.961 \text{ s}$$

Projectile Motion - Range, Maximum Height



$v_x = v_{0x} = v_0 \cos\theta$ $\quad$ $v_y = v_{0y} + a_y t = v_0 \sin\theta - gt$

In a time t, the projectile travels a distance R (range) to the right

$R = v_x t \rightarrow t = R/v_x$ $\quad$ and v_x is constant

In the same time, the projectile falls back to the ground, at $y = y_0$

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$0 = v_{0y}t - \frac{1}{2}gt^2 \rightarrow t = 2v_{0y}/g$$

$$t = \frac{R}{v_x} = \frac{2v_{0y}}{g} \rightarrow R = \frac{2v_x v_{0y}}{g}$$

Range of a projectile

$$R = \frac{2v_x v_{0y}}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$[2\sin\theta\cos\theta = \sin 2\theta]$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Maximum range when $\theta = 45^\circ$

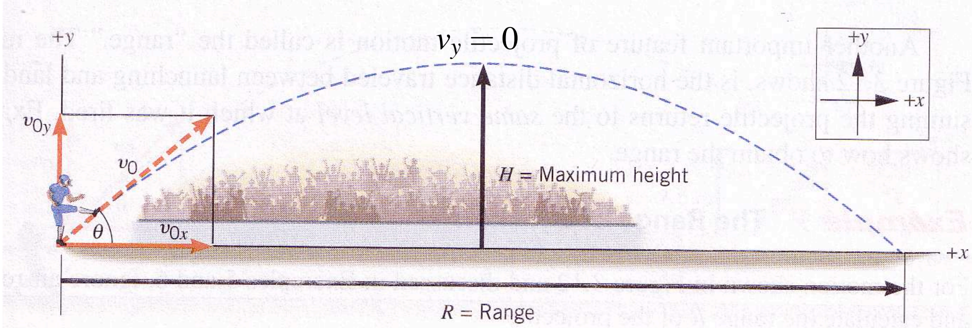
Projectile reaches maximum height, H, when $v_y = 0$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \qquad \text{so, } 0 = (v_0 \sin\theta)^2 - 2gH$$

$$\text{Therefore } H = \frac{(v_0 \sin\theta)^2}{2g}$$

eg $v_0 = 100$ m/s, $\theta = 30^\circ \rightarrow R = 884$ m, $H = 128$ m

Projectile Motion – Range, Maximum Height



The projectile travels the horizontal distance R in the same time that is travels up to height H and back down to the initial height.

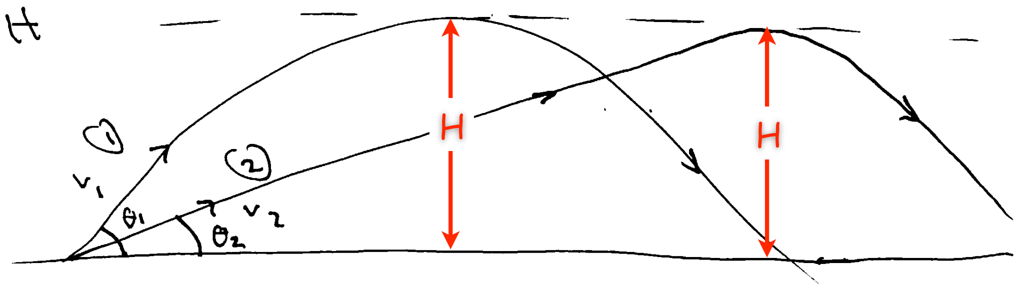
$$R = \frac{v_0^2 \sin 2\theta}{g}, \quad \text{greatest range when } \theta = 45^\circ$$

$$H = \frac{(v_0 \sin\theta)^2}{2g}$$

Clickers!

3.C12: Balls 1 and 2 are launched from the same spot at different angles to the ground. They both reach the same maximum height, H, but ball 2 has the greater range.

Decide which ball, if either, has the greater initial speed.



A) Ball 1 is faster, B) Ball 2 is faster

As they reach the same maximum height, they must have the same initial speed in upward direction, $v_{oy1} = v_{oy2}$, so $v_1 \sin\theta_1 = v_2 \sin\theta_2$

Answers to even-numbered problems

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Password: xxxxxx

FALL 2007

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Week of September 24
Experiment 1, measurement of length and mass

Week of October 1
Tutorial and test 1 on chapters 1, 2, 3

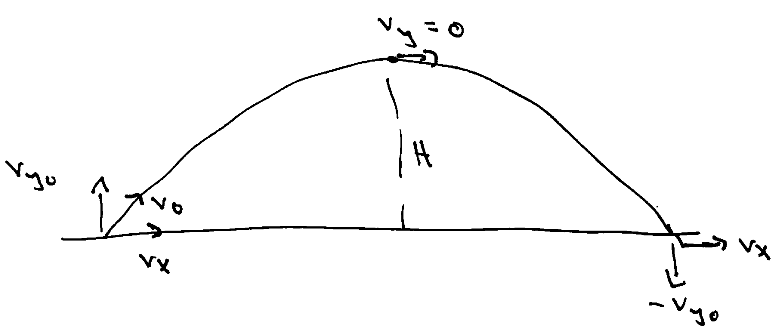
Projectile Motion

3.C5: A tennis ball is hit upward into the air and moves along an arc.

Neglecting air resistance, where along the arc is the speed of the ball

a) a minimum?

b) a maximum?



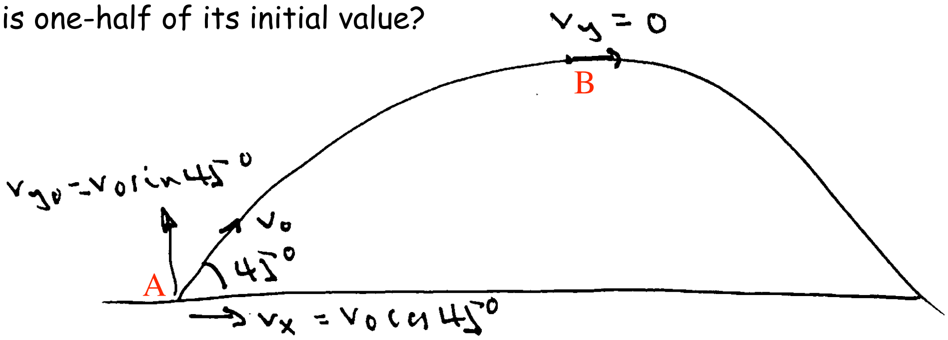
v_x is constant, v_y varies

$v^2 = v_x^2 + v_y^2$

$v_y = 0$ at highest point, and so, the smallest v...

3.C11: A leopard springs upward at a 45^0 angle and then falls back to the ground.

Does the leopard, at any point on its trajectory, ever have a speed that is one-half of its initial value?



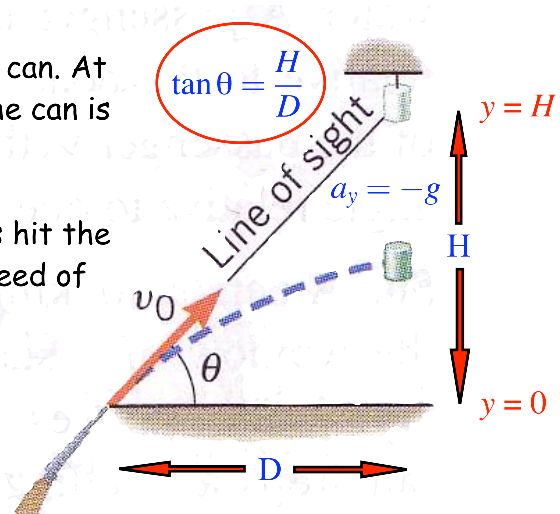
At any point: $v = \sqrt{v_x^2 + v_y^2}$, and $v_x = v_0 \cos 45° = v_0/\sqrt{2}$

At B: $v_y = 0$, so $v = v_x = v_0/\sqrt{2} \approx 0.7 v_0$, which is the smallest v

→ The speed never falls to half its initial value.

3.46: A rifle is aimed at a small can. At the instant the rifle is fired, the can is released.

Show that the bullet will always hit the can, regardless of the initial speed of the bullet.



Bullet:

$$y_b = v_{0y} t - \frac{1}{2} g t^2$$

Can:

$$y_c = H - \frac{1}{2} g t^2$$

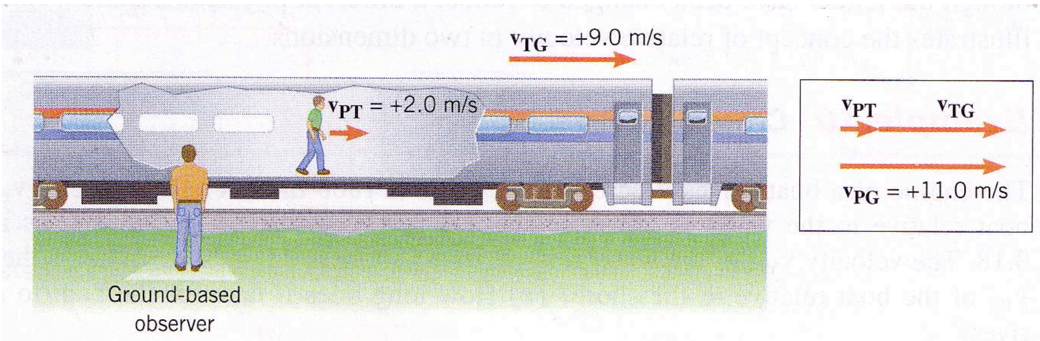
$$x = v_x t = v_0 \cos\theta \frac{H}{v_0 \sin\theta} = \frac{H}{\tan\theta}$$

$$x = D$$

Bullet and can meet when $y_b = y_c$:

$$v_{0y} t - \frac{1}{2} g t^2 = H - \frac{1}{2} g t^2 \rightarrow t = \frac{H}{v_{0y}}$$

Relative Velocity



$\vec{v}_{PT} =$ velocity of passenger relative to train

$\vec{v}_{TG} =$ velocity of train relative to ground $\qquad \vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$

$\vec{v}_{PG} =$ velocity of passenger relative to ground

Relative Velocity

If: A moves at velocity $\vec{v}_A$ (relative to the ground)

and: B moves at velocity $\vec{v}_B$ (relative to the ground)

then, the velocity of ***B* relative to *A*** is:

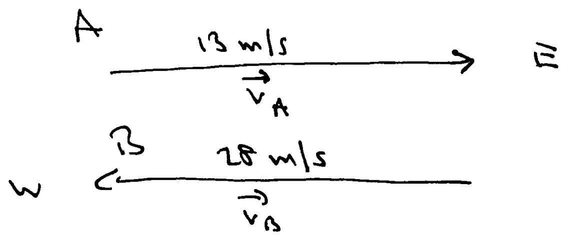
$$\boxed{\vec{v}_{BA} = \vec{v}_B - \vec{v}_A}$$

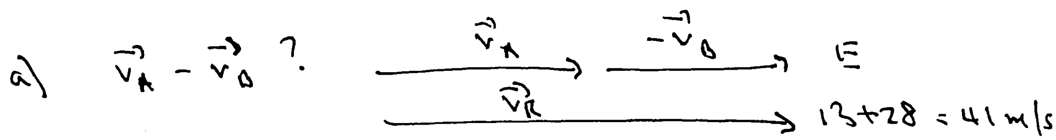
This is the velocity of B as seen by A.

3.51/47: Two trains are passing each other. Train A is moving east at 13 m/s, train B is travelling west at 28 m/s.

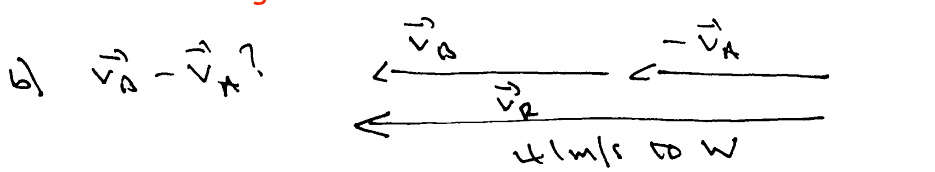
a) What is the velocity of train A relative to train B?

b) What is the velocity of train B relative to train A?



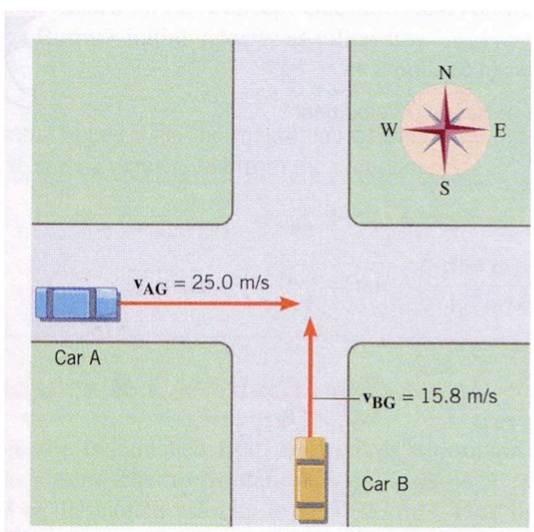


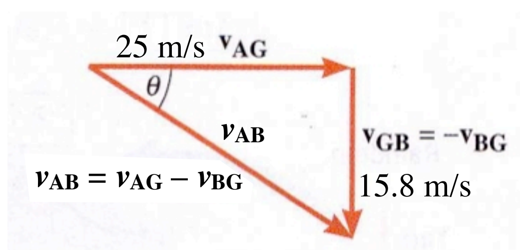
B sees A travelling to E at 41 m/s relative to it



Relative Velocity

Velocity of car A relative to car B is $\vec{v}_{AB} = \vec{v}_{AG} - \vec{v}_{BG}$



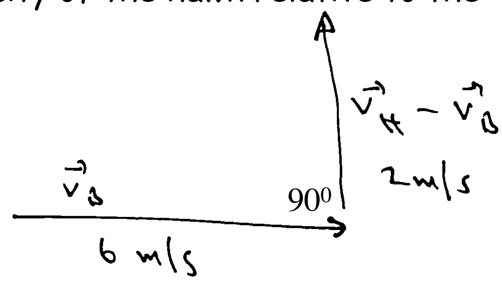


$$v_{AB}^2 = v_{AG}^2 + v_{BG}^2 = 25^2 + 15.8^2$$
$$v_{AB} = 29.6 \text{ m/s}$$

$$\tan\theta = 15.8/25, \ \theta = 32.3^\circ$$

Relative to B, A is travelling at 29.6 m/s at 32.3^0 south of east

3.53: A hot air balloon is moving relative to the ground at 6 m/s due east. A hawk flies at 2 m/s due north **relative to the balloon**. What is the velocity of the hawk relative to the ground?

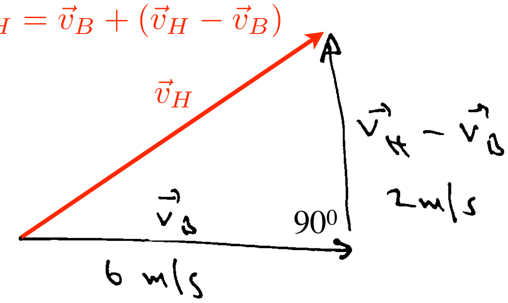


Break into components, or...

$$\vec{v}_H = \vec{v}_B + (\vec{v}_H - \vec{v}_B)$$

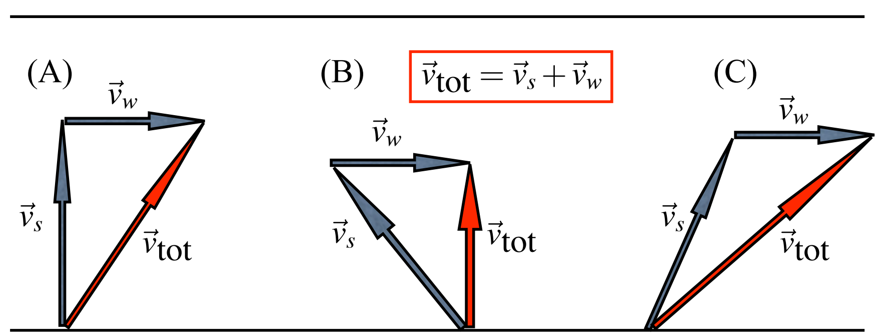
$$v_H = \sqrt{6^2 + 2^2} = 6.3 \text{ m/s}$$

$$\tan\theta = \frac{2}{6},\ \theta = 18.4^\circ \text{ N of E}$$



Clickers!

3.C16: Strategies for swimming across the river in the shortest time. Which is fastest? The swimmers swim at the same speed v_s relative to the water. The water flows at speed v_w.

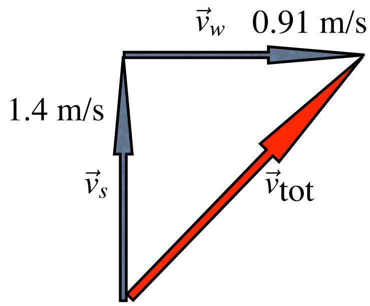


(A) Head straight across, get carried downstream

(B) Head upstream to end at closest point on far bank

(C) Head downstream to take advantage of the current

Relative Velocity

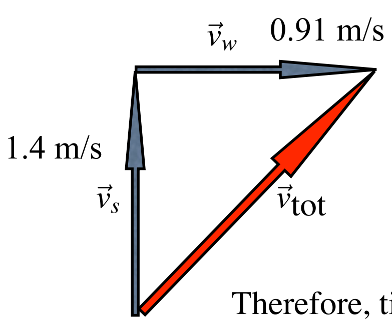


$\vec{v}_{tot} = \vec{v}_s + \vec{v}_w$

3.47/51: A swimmer swims directly across a river that is 2.8 km wide. He can swim at 1.4 m/s in still water (v_s), i.e. at 1.4 m/s relative to the water. The river flows at 0.91 m/s (v_w), i.e. at 0.91 m/s relative to the riverbank.

How long to cross the river?
Where does he end up on the other bank?

Relative Velocity contd



As he's swimming directly across the river, his speed toward the other bank is 1.4 m/s.

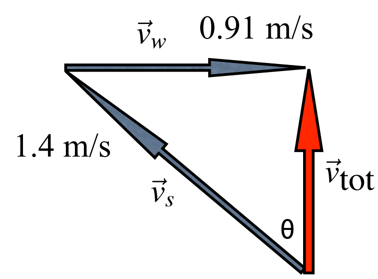
Therefore, time to cross the river is $\frac{2800 \text{ m}}{1.4 \text{ m/s}} = 2000 \text{ s}$

In this time, the current will carry him downstream by:

$$(0.91 \text{ m/s}) \times (2000 \text{ s}) = 1820 \text{ m}$$

Relative Velocity contd

Alternative strategy: he heads upstream a little so as to swim directly across the river – the most direct route.



$$\vec{v}_{tot} = \vec{v}_s + \vec{v}_w$$

Pythagoras:

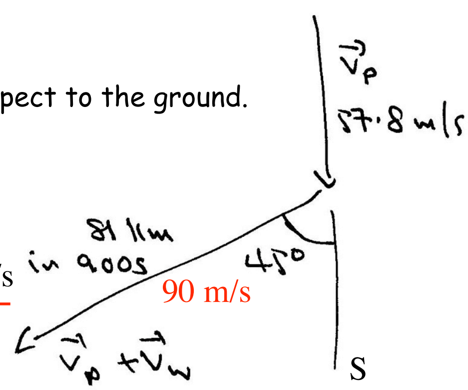
$$v_s^2 = v_w^2 + v_{tot}^2$$

So, $v_{tot} = \sqrt{v_s^2 - v_w^2} = \sqrt{1.4^2 - 0.91^2} = 1.064$ m/s

Takes $\dfrac{2800 \text{ m}}{1.064 \text{ m/s}} = 2630$ s to cross the river $\qquad \sin\theta = \dfrac{0.91}{1.4} \rightarrow \theta = 41^\circ$

3.69/57: An aircraft is headed due south with a speed of 57.8 m/s relative to still air. Then, for 900 s a wind blows the plane so that it moves in a direction 45⁰ west of south, even though the plane continues to point due south. The plane travels 81 km with respect to the ground in this time.

Determine the velocity of the wind with respect to the ground.



To south:

$$(v_p)_S + (v_w)_S = 90\cos 45^\circ$$
$$57.8 + (v_w)_S = 63.64 \rightarrow (v_w)_S = 5.84 \text{ m/s}$$

To west:

$$(v_p)_W + (v_w)_W = 90\sin 45^\circ$$
$$0 + (v_w)_W = 63.64 \rightarrow (v_w)_W = 63.64 \text{ m/s}$$

$v_w = \sqrt{5.84^2 + 63.64^2} = 63.9$ m/s $\qquad \tan\theta_w = \dfrac{63.84}{5.84} \rightarrow \theta_w = 84.8^\circ$ west of south

Chapter 3 Summary

- The laws of motion can be applied separately to motions in x and y (negligible air resistance).
- The time for a projectile to move up and down is the same as the time for it to sideways.
- Relative velocity is an application of the subtraction of vectors covered in chapter 1.
- If A travels at $\vec{v}_A$ and B travels at $\vec{v}_B$ the velocity of B relative to A is $\vec{v}_B - \vec{v}_A$