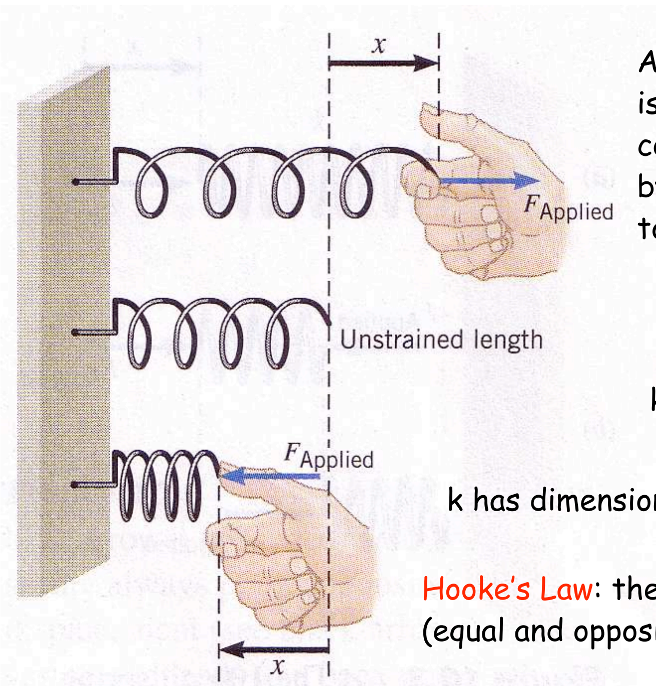
Chapter 10
Simple Harmonic Motion and Elasticity

- Hooke's Law, motion of a mass on a spring, simple harmonic motion
- Elastic potential energy – the return of the conservation of mechanical energy
- The pendulum and simple harmonic motion
- Read about: (10.5, 10.6, not covered in class)
  damped harmonic motion
  driven harmonic motion
  resonance
- Forget about: 10.7, 10.8, Elastic deformation, stress, strain...

The Ideal Spring



An equal amount of force is required to stretch or compress an ideal spring by the same amount – up to a point.

$$F_{applied} = k\,x$$

k is the spring constant

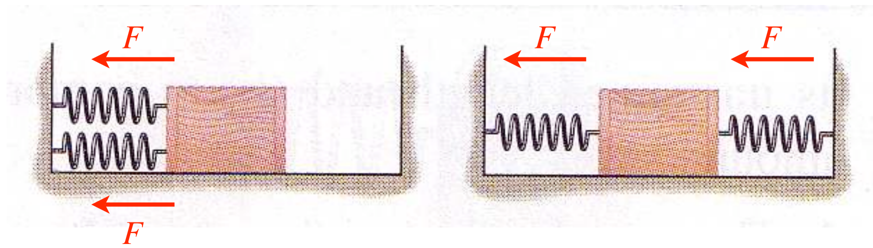
k has dimensions of force/length, N/m

Hooke's Law: the restoring force is –kx (equal and opposite to the applied force)

Clickers!

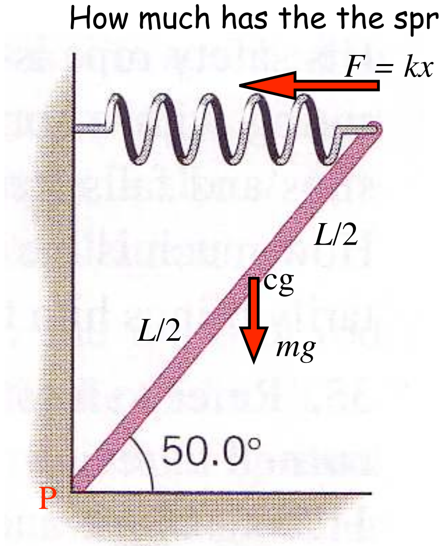
10.C2: The springs are identical and initially unstrained, as shown in the diagram.

The boxes are pulled to the right by the same distance and released. Which box feels the greater force from the springs?



A) box on left feels the greater force
B) box on right feels the greater force
C) the boxes feel the same force

10.8/10: A 10.1 kg uniform board is held in place by a spring. The spring constant is k = 176 N/m.
How much has the the spring stretched at equilibrium?



The length of the plank is L

Torques about P at the floor:

$$-mg(L/2)\cos 50^\circ + FL\sin 50^\circ = 0$$

$$F = \frac{mg}{2\tan 50^\circ} = kx$$

$$\text{So, } x = \frac{mg}{2k\tan 50^\circ} = \frac{10.1 \times 9.8}{2 \times 176 \tan 50^\circ}$$

$$\boxed{x = 0.24\ \text{m}}$$

Is a short spring easier to compress?

Imagine the long spring as two half-length springs joined together.

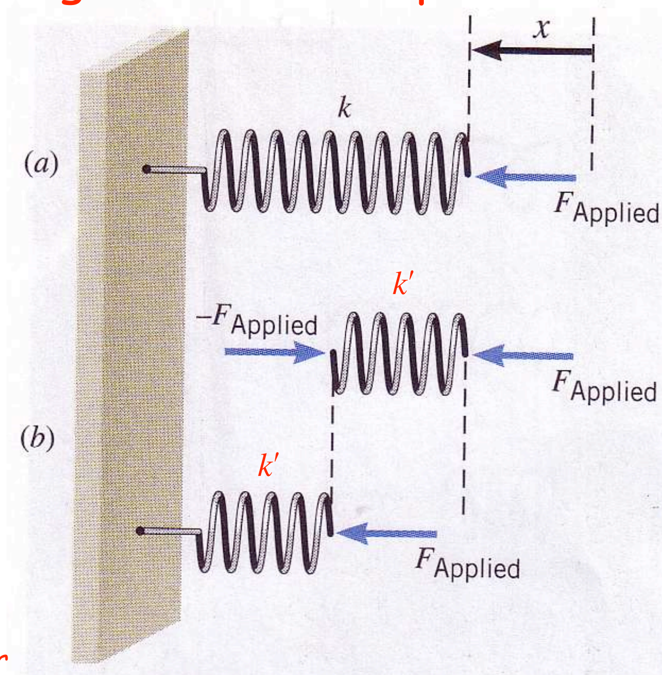
The applied force compresses each half spring by x/2.

The spring constant for the short springs is given by:

$F_{applied} = k' \times (x/2)$

So $k'x/2 = kx$, and $k' = 2k$

The shorter spring is stiffer



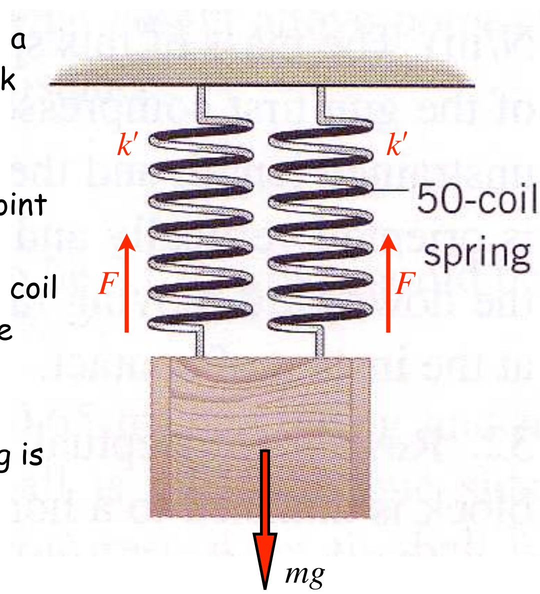
10.10/8: A mass, m, is attached to a 100 coil spring of spring constant k and its equilibrium position noted:

$x_0 = mg/k = 0.16$ m = equilibrium point

The spring is then cut into two 50 coil springs as shown. How much do the springs stretch?

The spring constant of each spring is

$k' = 2k$

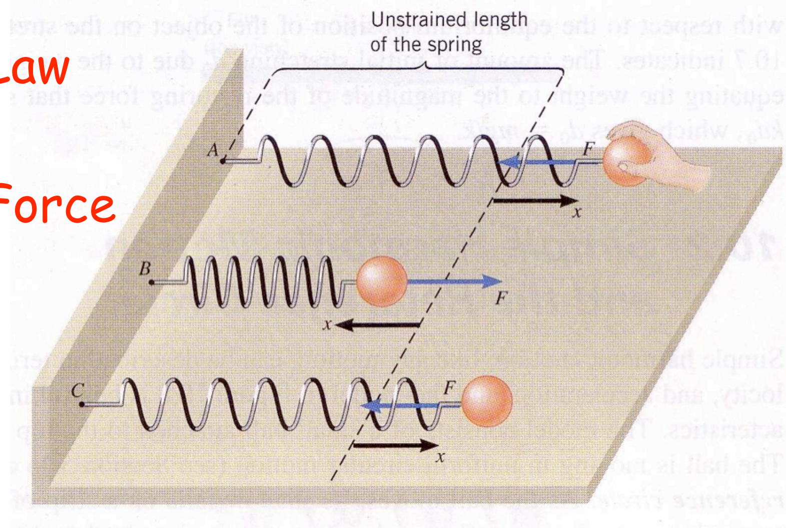


There are two of these springs, so the total restoring force is

$F_{tot} = 2 \times (k'x) = 2 \times (2kx) = mg$, so $x = mg/(4k) = x_0/4 = 0.04$ m

Hooke's Law

Restoring Force

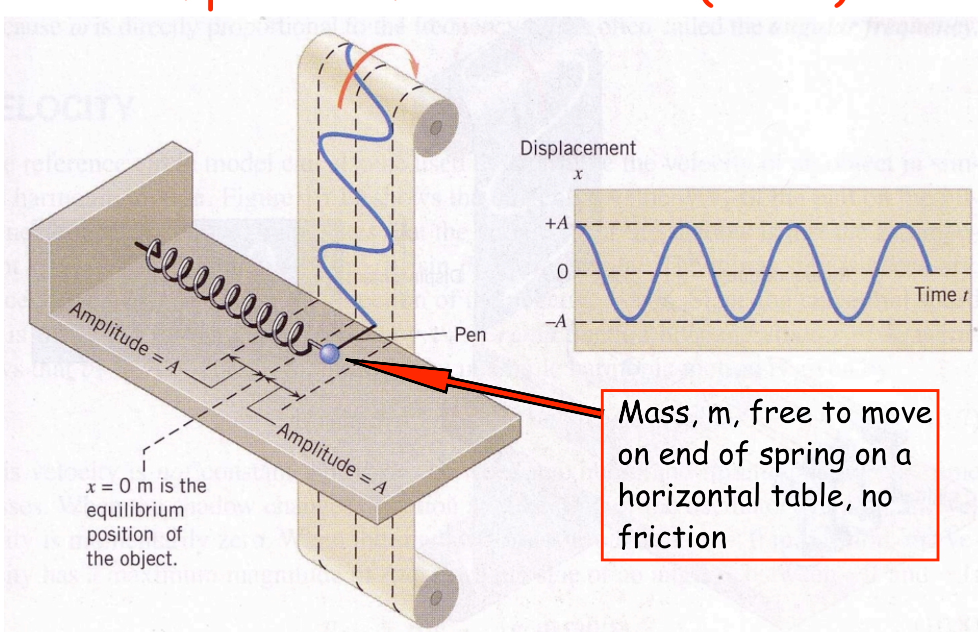


The **restoring force** is the force the spring exerts when stretched or compressed - tries to move spring back to equilibrium state.

Restoring force, $F = -F_{applied} = -kx$

$F = -kx$, Hooke's Law

Simple Harmonic Motion (SHM)



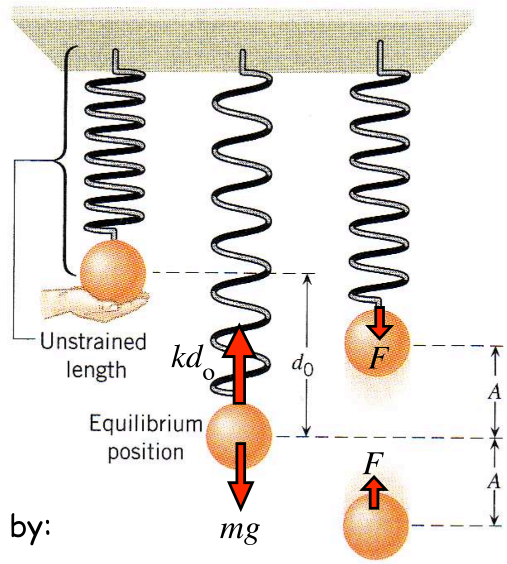
Pull the mass a distance A to the right, release, and observe motion...

Simple Harmonic Motion (SHM)

Simple harmonic motion also seen for a mass suspended from a spring.

SHM is characteristic motion when the restoring force is proportional to the displacement from the equilibrium position

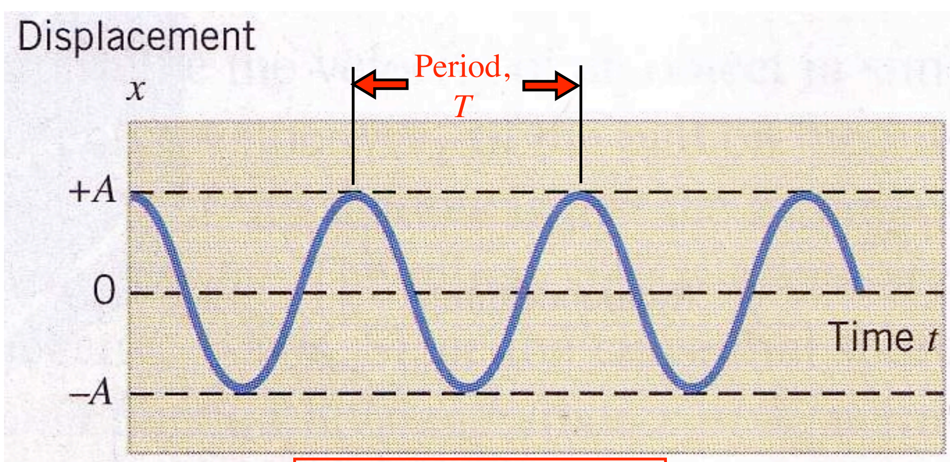
The equilibrium position corresponds to the spring stretched by an amount given by:



$mg = kd_0$, so $d_0 = mg/k$.

SHM is about $x = d_0$ with amplitude A.

Simple Harmonic Motion (SHM)



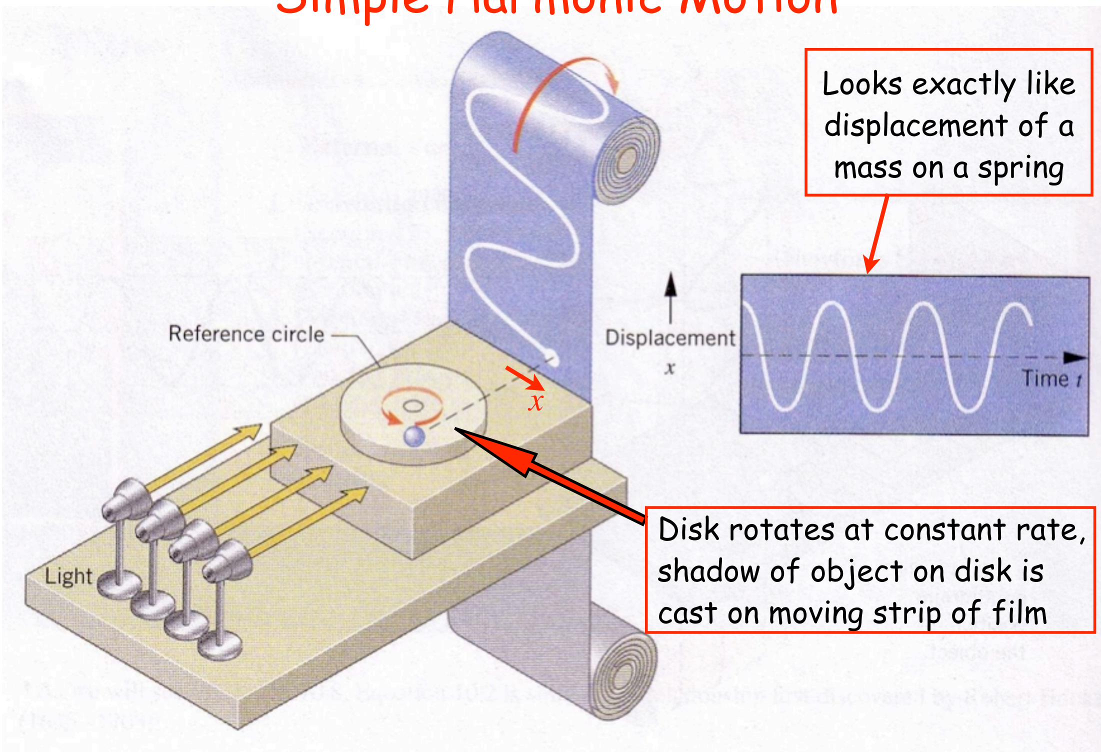
Displacement, $x = A\cos\omega t$

The time for one cycle is the period, T

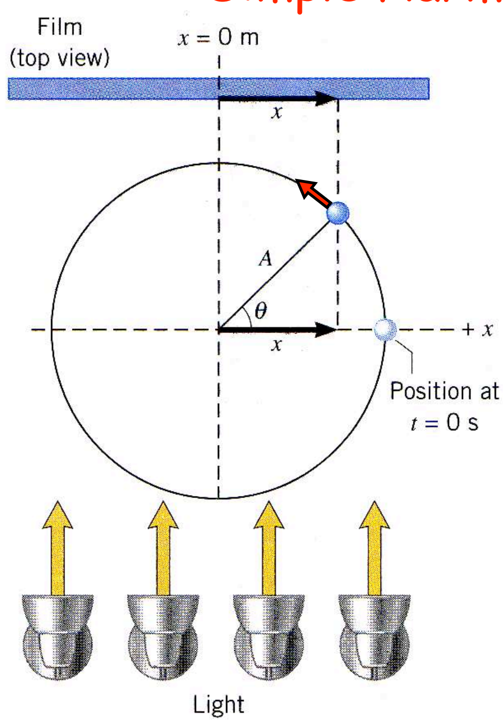
So $\omega T = 2\pi$ radians (i.e., 360^0, 1 cycle), and $\omega = 2\pi/T = 2\pi f$,

f = frequency of the SHM, cycles/second or Hertz (Hz)

Simple Harmonic Motion



Simple Harmonic Motion



$$x = A\cos\theta$$

and if $\theta = \omega t$

(rotation at constant angular velocity ω), then,

$$x = A\cos\omega t$$

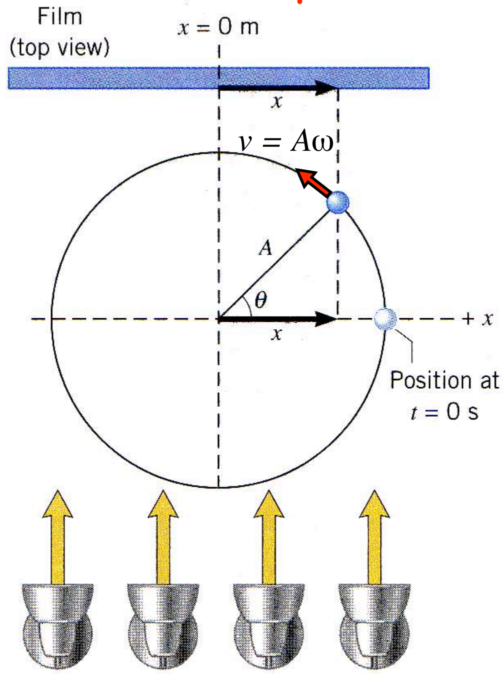
$\theta = \omega t = 2\pi$ (360^0) when $t = T$

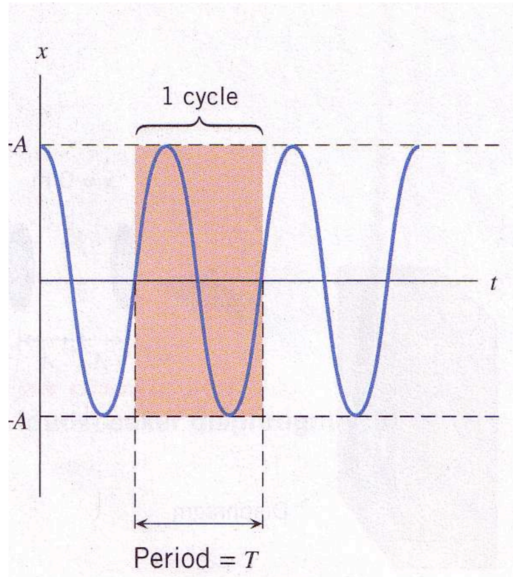
then $\omega T = 2\pi$

$\omega = 2\pi/T = 2\pi f$

T = period, f = frequency

Simple Harmonic Motion





Sinusoidal motion – just like the mass on a spring

Simple Harmonic Motion

- Simple harmonic motion results when the restoring force is proportional to the displacement –

  F = –kx

- For the mass on the spring, how is the period of the harmonic motion related to the spring constant, k?

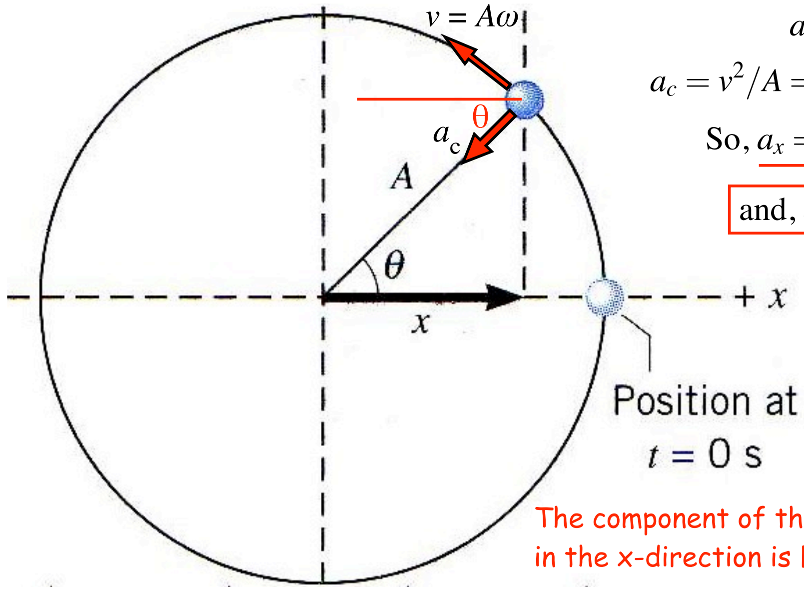
Clue:

- The motion of the mass on the spring looks just like the x component of the motion of the mass on the rotating disk.

Strategy – solve the easier problem:

- Look at the motion in a circle and find out what is the acceleration, a_x, as that can be related to a restoring force and an effective spring constant.

Simple Harmonic Motion
The "Reference Circle"



$$x = A\cos\theta$$

$$a_x = -a_c\cos\theta$$

$$a_c = v^2/A = (A\omega)^2/A = A\omega^2$$

$$\text{So, } a_x = -A\omega^2\cos\theta$$

$$\text{and, } a_x = -\omega^2 x$$

The component of the centripetal force in the x-direction is $F_x = ma_x = -m\omega^2x$.

Looks just like $F_x = -kx$ for the spring.

Rotating disk: $a_x = -\omega^2 x$ acceleration is proportional to displacement

Mass on a spring: the restoring force is: $F = -kx = ma_x$

That is, $a_x = -(k/m)x$ acceleration also proportional to displacement

COMPARE: The mass on the spring **moves in x in the same way** as a mass on a disk that is rotating with $\omega = [k/m]^{1/2}$.

Then: $\omega = 2\pi f = \frac{2\pi}{T}$

So, $T = 2\pi\sqrt{\frac{m}{k}}$ for the mass on the spring

A new physics course for students of the Biological Sciences:

PHYS 2270 Physical Topics For Biologists A (3 hrs)

There is no lab
Prerequisites: not updated on Aurora - see Dr. Sharma for permission to register:
509 Allen, 474-9817

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Mastering Physics Assignment 4

Is due Monday, November 12 at 11 pm

Covers material from chapters 6 and 7

There are 8 questions for practice and 6 for credit

The Final Exam Schedule is Now Final!

PHYS 1020: Monday, December 17, 6 - 9 pm
Frank Kennedy Brown & Gold Gyms
The whole course
30 multiple choice questions
Formula sheet provided

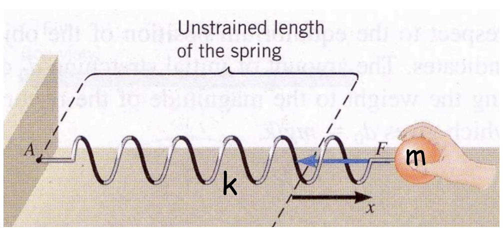
Useful for experiment 4:

10.11/7: A small ball is attached to one end of a spring that has an unstrained length of 0.2 m. The spring is held by the other end, and the ball is whirled around in a horizontal circle at a speed of 3 m/s. The spring remains nearly parallel to the ground and is observed to stretch by 0.01 m. By how much would the spring stretch if it were attached to the ceiling and the ball allowed to hang straight down, motionless?

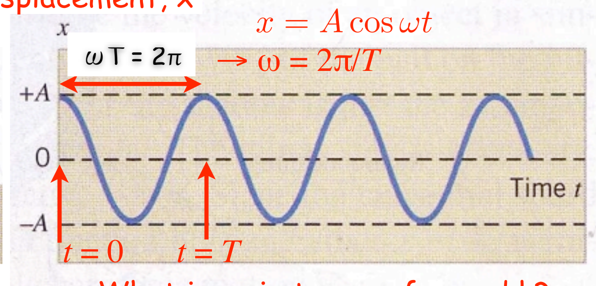
Find the spring constant from the amount the spring stretches when the ball is whirled around in a circle.

→ Find the stretch when the ball is suspended.

Mass on a spring

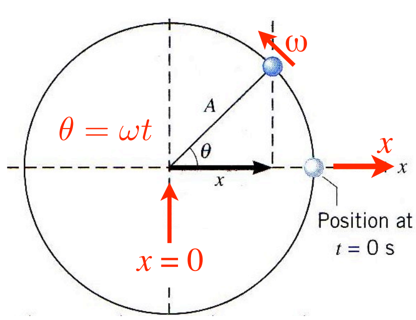


Displacement, x

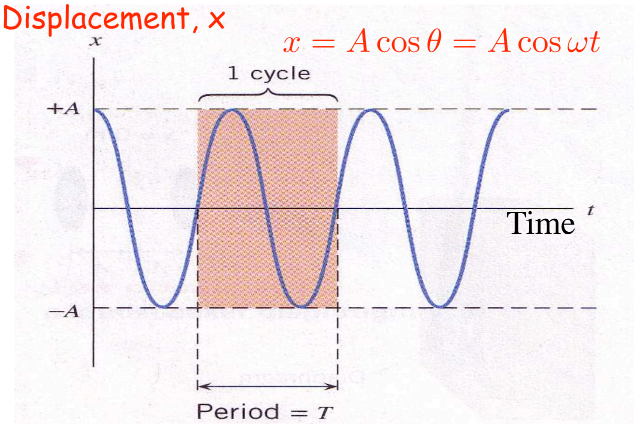


What is ω in terms of m and k?

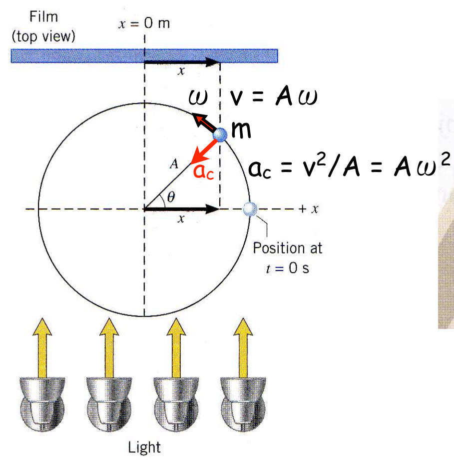
Circular motion

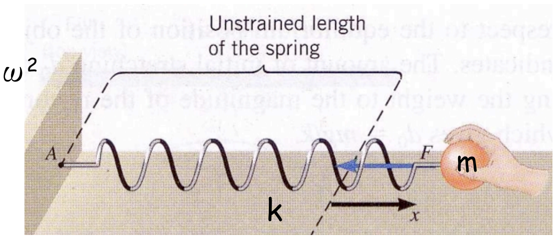


Displacement, x



Simple Harmonic Motion





Mass on a rotating disk
$F_x = -m\omega^2 x$

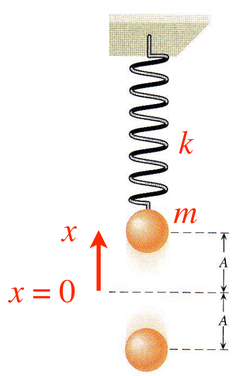
Mass on a spring
$F_x = -kx$

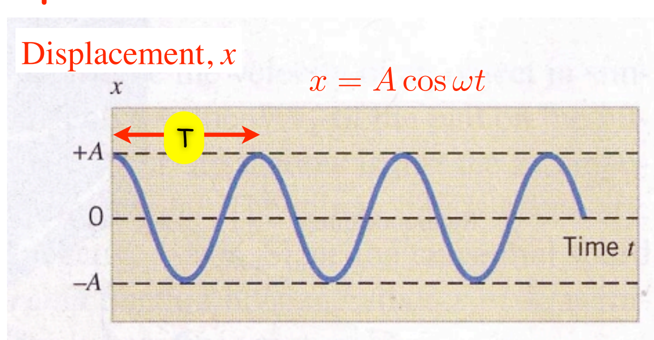
Simple harmonic motion in both cases - restoring force ∝ displacement

COMPARE: if $m\omega^2 = k$, then motions in x are **exactly the same**,

so $\omega^2 = k/m$ for the mass on the spring, and $x = A\cos(\omega t)$

Simple Harmonic Motion





- SHM results when the restoring force is proportional to displacement:

  F = -kx

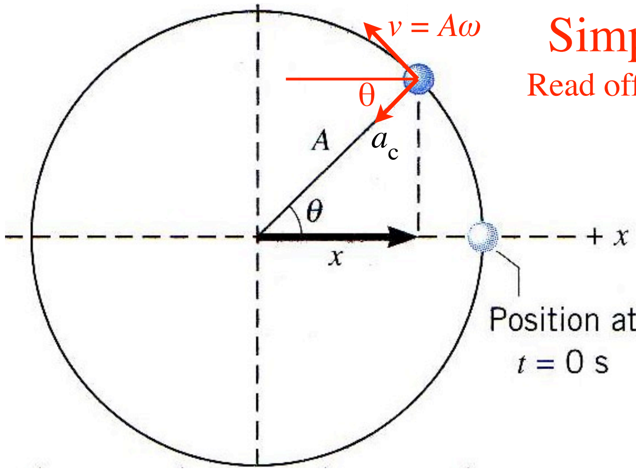
- The resulting motion is:

$$x = A\cos\omega t, \quad (\text{or } x = A\sin\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} \text{ and } T = 2\pi\sqrt{\frac{m}{k}}$$

rad/s, Hz, s

$\omega = 2\pi f = 2\pi/T$



Simple Harmonic Motion

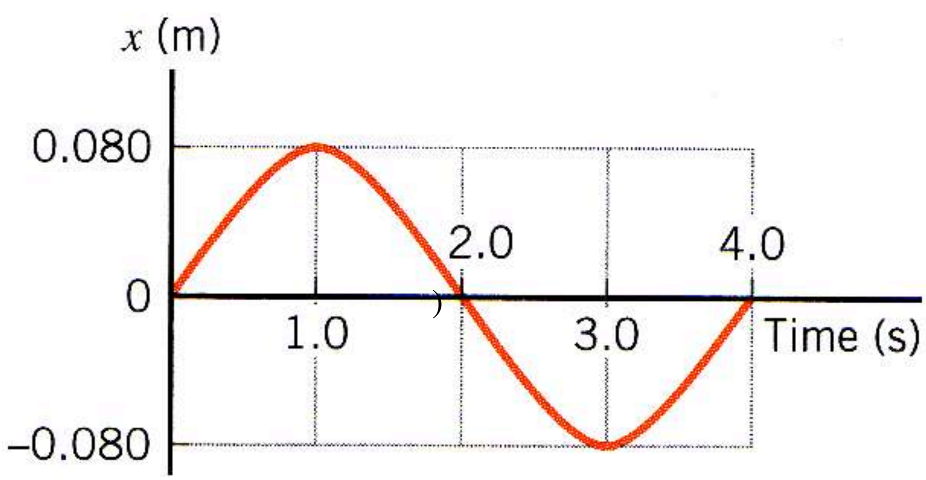
Read off motion from the reference circle

$$\theta = \omega t$$
$$x = A\cos\omega t$$
$$v_x = -A\omega\sin\omega t$$
$$a_x = -A\omega^2\cos\omega t = -\omega^2 x$$

Also by differentiating

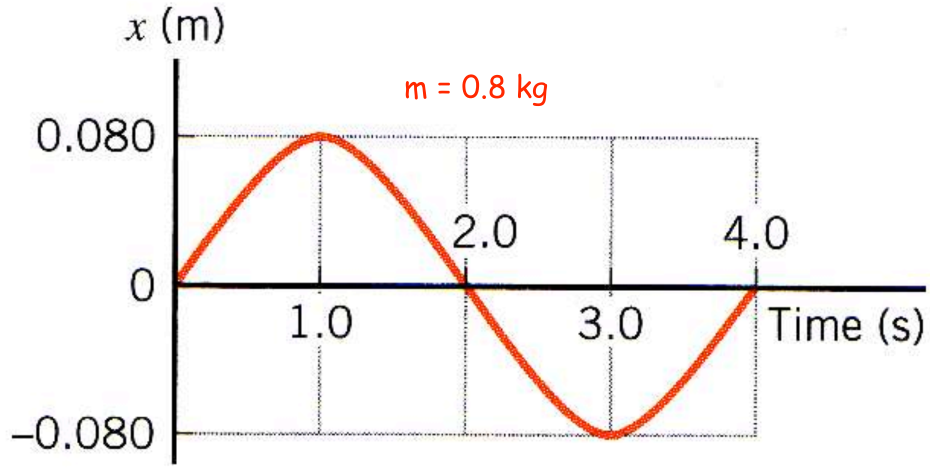
$$\text{Maximum } v_x = \pm A\omega \text{ when } x = 0$$
$$\text{Maximum } a_x = \mp A\omega^2 \text{ when } x = \pm A$$
$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

10.17: A 0.8 kg mass is attached to a spring.



a) Find the amplitude, A, of the motion.

A = maximum displacement from the equilibrium position = 0.08 m

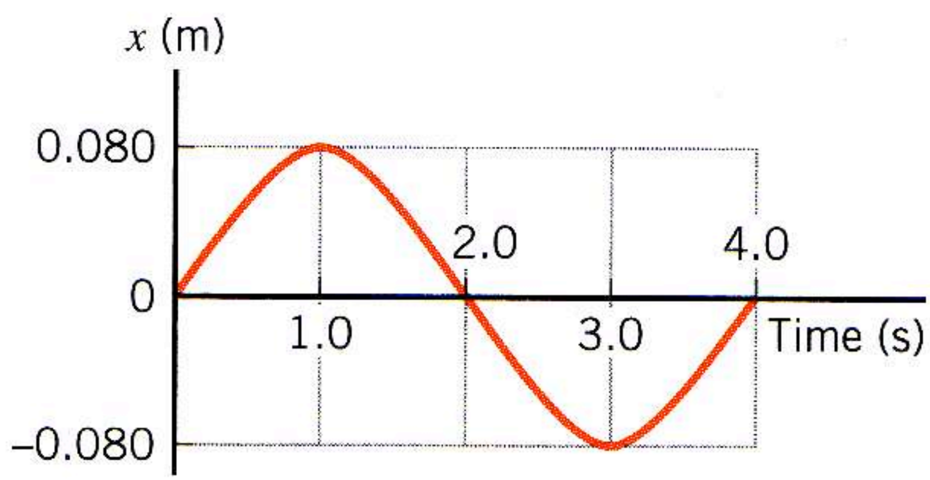


b) Find the angular frequency, ω

$\omega = 2\pi/T$ and T = 4 s, so $\omega = 2\pi/4 = 1.57$ rad/s

c) Find the spring constant, k.

$\omega^2 = k/m$, so $k = m\omega^2 = (0.8\text{ kg})(1.57\text{ rad/s})^2 = 1.97$ N/m



d) Find the speed of the object at t = 1 s.
v = 0 (has reached maximum x and has come momentarily to rest)

e) Find the magnitude of the acceleration at t = 1 s.
$a = -\omega^2 x = -(1.57\text{ rad/s})^2 \times (0.08\text{ m}) = -0.20\text{ m/s}^2$

(Check: $a = F/m = -kx/m = -1.97 \times 0.08/0.8 = -0.20\text{ m/s}^2$)

10.-/20: When a mass m_1 is hung from a vertical spring and set into vertical simple harmonic motion, its frequency is 12 Hz. When another object of mass m_2 is hung on the spring along with m_1, the frequency of motion is 4 Hz. Find m_2/m_1.

$$\omega = \sqrt{k/m} = 2\pi f \qquad \text{that is, } f \propto 1/\sqrt{m}$$

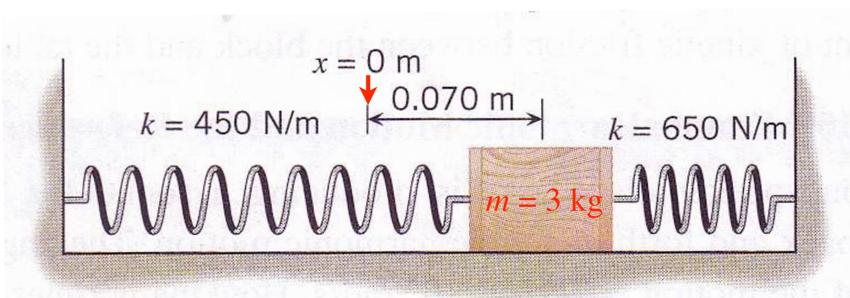
For mass m_1: $f_1 = 12$ Hz

For mass $m_1 + m_2$: $f_2 = 4$ Hz

$$\frac{f_1}{f_2} = \sqrt{\frac{m_1 + m_2}{m_1}} = \sqrt{1 + m_2/m_1}$$

$$\text{And, } \frac{12}{4} = \sqrt{1 + m_2/m_1}$$

$$\frac{m_2}{m_1} = 3^2 - 1 = 8$$



10.22: A 3 kg block is placed between two horizontal springs. The springs are neither strained nor compressed when the block is at x = 0. The block is displaced to x = 0.07 m and is released.

Find the speed of the block when it passes back through x = 0 and the angular frequency of the system.

Question: What is the effective spring constant? That is, what is the restoring force when the block is displaced unit distance?

When the block is moved x to the right, the restoring force is:

$F = -(450 + 650)x = -1100x$ N.

F = -1100x N

The effective spring constant is 1100 N/m.

When the block passes through x = 0, it has its maximum speed,

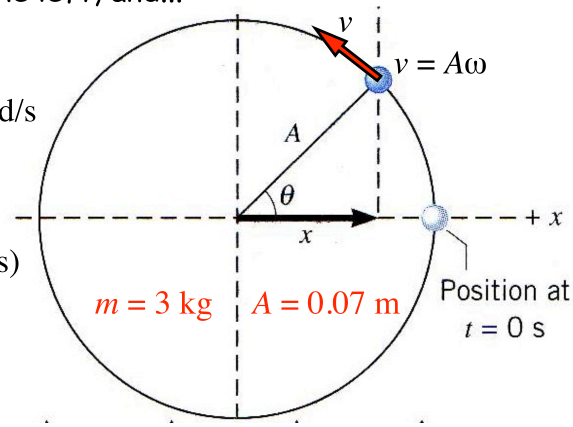
$v_x = -A\omega$ when travelling to the left, and...

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1100}{3}} = 19.15 \text{ rad/s}$$

So, $v_x = -A\omega$

$= -(0.07 \text{ m})\times(19.15 \text{ rad/s})$

$= -1.3 \text{ m/s}$



Motion of mass on spring by reference to the reference circle

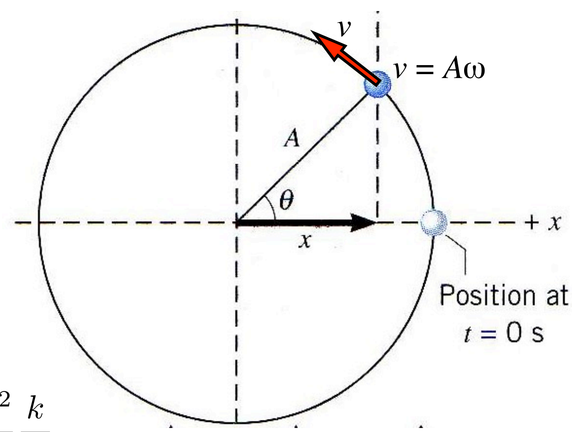
$$\omega = \sqrt{\frac{k}{m}}$$

At $x = 0$, $v_x = -A\omega$

The KE of the mass at $x = 0$ is

$$\text{KE} = \frac{mv_x^2}{2} = \frac{mA^2\omega^2}{2} = \frac{mA^2}{2}\frac{k}{m}$$

$$\text{KE} = \frac{kA^2}{2}$$



Conservation of mechanical energy: when the spring is stretched to x = A, it has PE = $kA^2/2$, which is converted entirely to KE when x = 0.

Energy and Simple Harmonic Motion

Elastic Potential Energy:

Energy is stored in a spring when it is stretched or compressed. The potential energy is released when the spring is released.

The restoring force exerted by the spring when stretched by x is:

$F = -kx$

The work done by the restoring force when the spring is stretched from x_0 to x_f is:

$$W_{elastic} = F_{average} \times (x_f - x_0) = \frac{-k(x_f + x_0)}{2} \times (x_f - x_0)$$

$$W_{elastic} = \frac{-k(x_f + x_0)}{2} \times (x_f - x_0)$$

$$W_{elastic} = \frac{1}{2}kx_0^2 - \frac{1}{2}kx_f^2$$ = work done **by the restoring force**

Initial elastic PE — Final elastic PE

$$PE_{elastic} = \frac{1}{2}kx^2$$

The total mechanical energy is now:

$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

Kinetic energy due to rotation could be included too, but we do not cover that.

Mechanical Energy

Elastic Potential Energy:

$$PE_{elastic} = \frac{1}{2}kx^2$$

Mechanical Energy:

$$E = KE + PE_{grav} + PE_{elastic}$$
$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

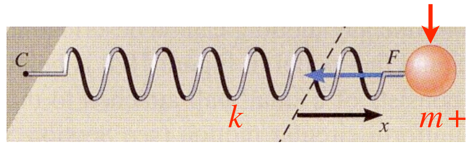
Mechanical energy is conserved in the absence of nonconservative (applied and friction) forces:

$$W_{nc} = \Delta E = \Delta KE + \Delta PE_{grav} + \Delta PE_{elastic}$$

10.C6: A block is attached to a horizontal spring and slides back and forth in simple harmonic motion on a frictionless horizontal surface. A second identical block is suddenly attached to the first block at the moment the block reaches its greatest displacement and is at rest.

Explain how a) the amplitude, b) the frequency, c) the maximum speed of the oscillation change.

$v = 0, x = A$



$m + m$

a) Amplitude unchanged

b) $\omega = \sqrt{\frac{k}{m}} \rightarrow \sqrt{\frac{k}{2m}}$ frequency reduced by factor of $\sqrt{2}$

c) Mechanical energy: at $x = A$: KE = 0, PE = $kA^2/2$ – unchanged

At $x = 0$: $KE_{max} = kA^2/2 = mv_{max}^2/2$ is unchanged

If m is doubled, v_{max} is reduced by factor of $\sqrt{2}$

Mechanical Energy

An object of mass 0.2 kg is oscillating on a spring on a horizontal frictionless table. The spring constant is k = 545 N/m.

The spring is stretched to x_0 = 4.5 cm, then released from rest.

Find the speed of the mass when (a) x_f = 2.25 cm, (b) x_f = 0 cm.

Conservation of mechanical energy: $E_f = E_0$, so

$$\frac{1}{2}mv_o^2 + mgh_o + \frac{1}{2}kx_o^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

$h_f = h_o$ and $v_o = 0$, so:

$$\frac{1}{2}kx_o^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{\cancel{2}}kx_o^2 = \frac{1}{\cancel{2}}mv_f^2 + \frac{1}{\cancel{2}}kx_f^2$$

$$v_f = \sqrt{\frac{k(x_o^2 - x_f^2)}{m}}$$

$x_o = 4.5$ cm, $k = 545$ N/m, $m = 0.2$ kg

(a) $x_f = 2.25$ cm

$$v_f = \sqrt{\frac{545(0.045^2 - 0.0225^2)}{0.2}} = 2.03 \text{ m/s}$$

(b) $x_f = 0$ cm

$$v_f = \sqrt{\frac{545(0.045^2 - 0.0)}{0.2}} = 2.35 \text{ m/s}$$

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Mechanical Energy

Mechanical energy, conserved in the absence of nonconservative (applied and friction) forces:

$$E = KE + PE_{grav} + PE_{elastic}$$
$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

In the presence of nonconservative forces:

$$W_{nc} = \Delta E = \Delta KE + \Delta PE_{grav} + \Delta PE_{elastic}$$

10.24: An archer pulls the bowstring back 0.47 m. The bow and string act like a spring with spring constant k = 425 N/m.

What is the elastic potential energy of the drawn bow?

$$E = \frac{1}{2}kx^2 = \frac{1}{2} \times 425 \times 0.47^2 = 46.9 \text{ J}$$

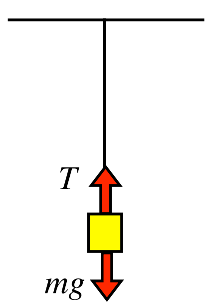
The arrow has a mass m = 0.03 kg. How fast will it travel when it leaves the bow?

$$E = \frac{1}{2}kx^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$46.9 \text{ J} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.03v^2$$

$$v = \sqrt{2 \times 46.9/0.03} = 55.9 \text{ m/s}$$

10.30: A 3.2 kg block hangs stationary from the end of a vertical spring attached to the ceiling. The elastic potential energy of the spring/mass system is 1.8 J. What is the elastic potential energy when the 3.2 kg mass is replaced by a 5 kg mass?



At equilibrium, mg = T = kx,

where x is the amount the spring is stretched.

So, x = mg/k.

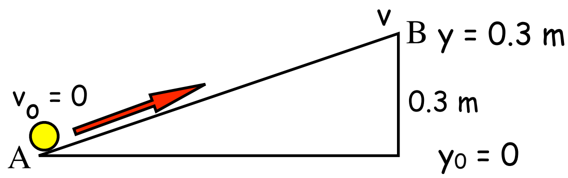
The elastic potential energy is

$PE_{elastic} = kx^2/2 = k(mg/k)^2/2$.

That is, $PE_{elastic} \propto m^2$

So $PE_{elastic} = (5/3.2)^2 \times 1.8 = 4.4$ J

10.-/26: The spring in a pinball machine (k = 675 N/m) is compressed 0.065 m. The ball (m = 0.0585 kg) is at rest against the spring at point A. When the spring is released, the ball slides to point B, which is 0.3 m higher than point A. How fast is the ball moving at B? (no friction)



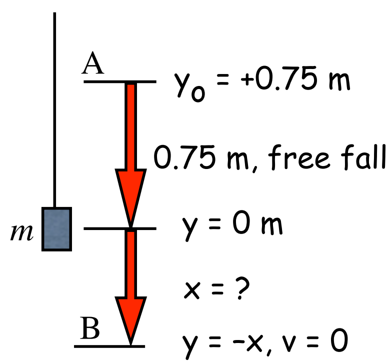
Conservation of mechanical energy:

At A: $E_A = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}kx_0^2$

$= 0 + 0 + \frac{1}{2} \times 675 \times 0.065^2 = 1.426 \text{ J}$

At B: $E_B = \frac{1}{2}mv^2 + 0.3mg + 0 = 1.426 \text{ J} \rightarrow v = 6.55 \text{ m/s}$

10.76/34: An 86 kg climber is scaling the vertical wall of a mountain. His safety rope when stretched acts like a spring with spring constant k = 1200 N/m. He falls 0.75 m before the rope becomes taut. How much does the rope stretch when it breaks his fall and momentarily brings him to rest?



Mechanical energy is conserved:

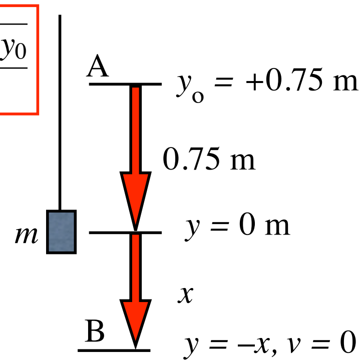
At A: $E = mgy_0$

At B: $E = mgy + kx^2/2$
$= -mgx + kx^2/2$

So, $mgy_0 = -mgx + kx^2/2$

$kx^2/2 - mgx - mgy_0 = 0$ so, $x = \frac{mg \pm \sqrt{m^2g^2 + 2kmgy_0}}{k}$

$$\text{so, } x = \frac{mg \pm \sqrt{m^2g^2 + 2kmgy_0}}{k}$$



$$x = \frac{86g \pm \sqrt{86^2g^2 + 2 \times 1200 \times 86g \times 0.75}}{1200}$$

x = 1.95 m, or −0.54 m, corresponding to:

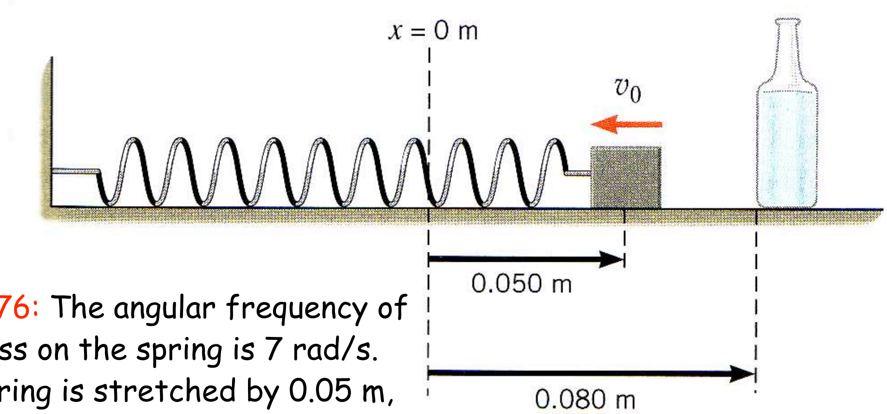
so, y = −x = −1.95 m, or +0.54 m.

y = +0.54 m means the rope is stretched by a negative amount!

So, y = −1.95 m and the rope is stretched 1.95 m

The angular frequency of the climber on the end of the rope is

$$\omega = \sqrt{k/m} = \sqrt{(1200 \text{ N/m})/(86 \text{ kg})} = 3.74 \text{ rad/s} \qquad (T = 1.7 \text{ s})$$



10.34/76: The angular frequency of the mass on the spring is 7 rad/s. The spring is stretched by 0.05 m, as shown, and the block is thrown to the left.

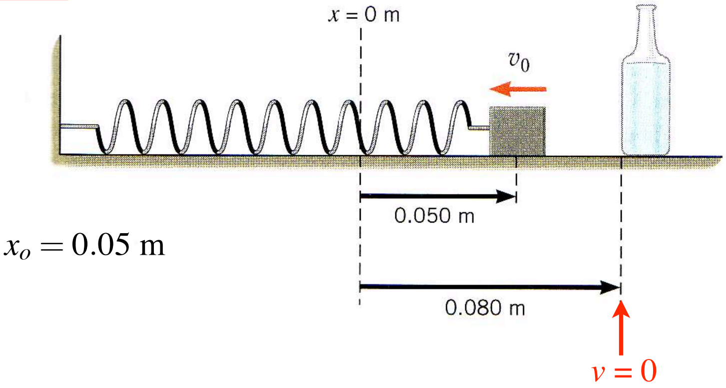
Find the minimum speed v_0 so that the bottle gets hit (ignore width of block).

Conservation of mechanical energy:

$PE_0 + KE_0 = PE + KE$

$PE_0 + KE_0 = PE + KE$

$\omega = 7$ rad/s



$$PE_o = \frac{1}{2}kx_o^2, \text{ with } x_o = 0.05 \text{ m}$$

$$KE_o = \frac{1}{2}mv_o^2$$

$$PE = \frac{1}{2}kx^2, \text{ with } x = 0.08 \text{ m}$$

$$KE = 0 \text{ (just reaches the bottle)}$$

$$\frac{1}{2}k \times 0.05^2 + \frac{1}{2}mv_o^2 = \frac{1}{2}k \times 0.08^2 + 0$$

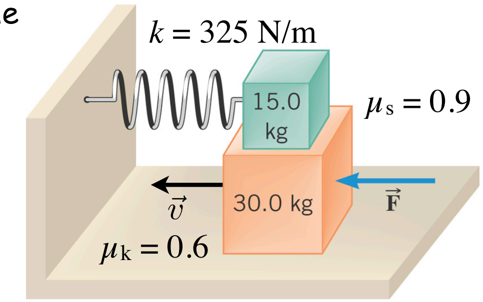
$$\frac{1}{2}k \times 0.05^2 + \frac{1}{2}mv_o^2 = \frac{1}{2}k \times 0.08^2 + 0$$

$$\omega = \sqrt{\frac{k}{m}} = 7 \text{ rad/s, so, } k = 49m$$

$$\text{Therefore, } mv_0^2 = 49m(0.08^2 - 0.05^2)$$

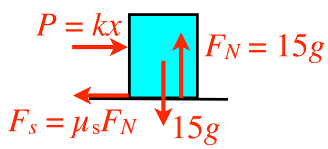
$$v_0 = 0.44 \text{ m/s}$$

10.12: A horizontal force F is applied to the lower block in such a way that the blocks move at constant speed. At the point where the upper block begins to slip, determine a) the amount by which the spring is compressed and b) the magnitude of the force, F.



There is no acceleration, so the net force on each block is zero.

a) Forces on the upper block:



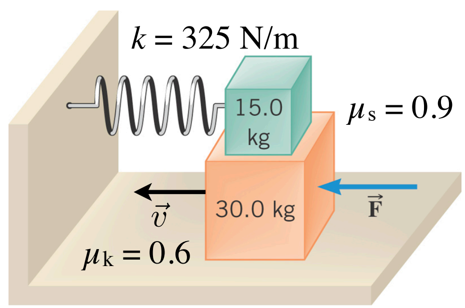
Block slips when P = $(F_s)_{max} = \mu_s F_N$

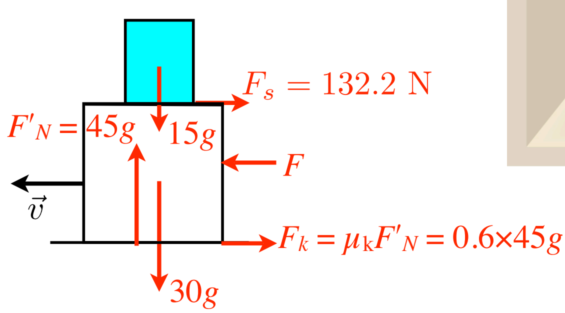
That is: kx = μ_s × 15g

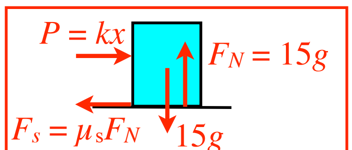
$$\rightarrow x = \frac{15\mu_s g}{k} = \frac{15 \times 0.9g}{325} = 0.407 \text{ m}$$

$$\text{and } F_s = 15\mu_s g = 132.2 \text{ N}$$

b) Forces on the lower block:







Forces on the upper block

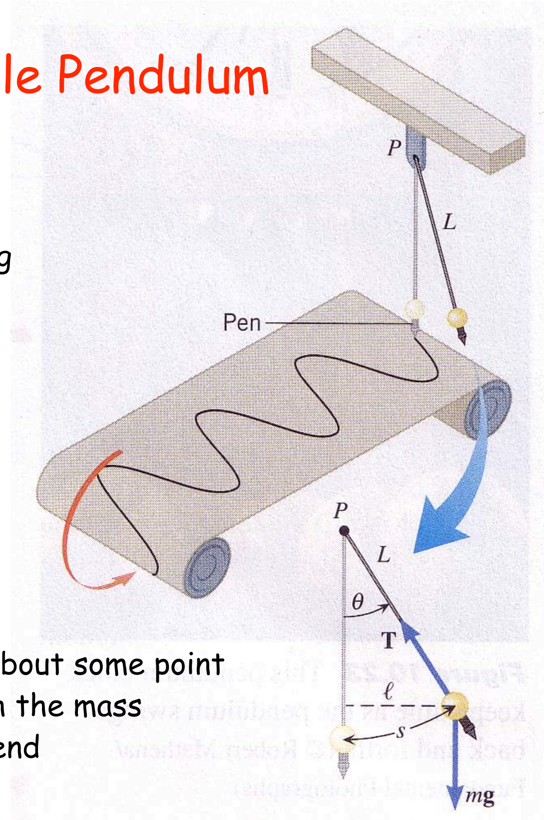
Acceleration = 0, so net force on block = 0

$$F = F_s + F_k = 132.2 + 0.6 \times 45g = 397 \text{ N}$$

The Simple Pendulum

Simple Pendulum:

- a mass on the end of a string
- executes SHM for small displacements



"Physical Pendulum"
(not covered)
- an extended mass pivoting about some point
- example, a solid bar in which the mass is not concentrated at one end

Simple Pendulum

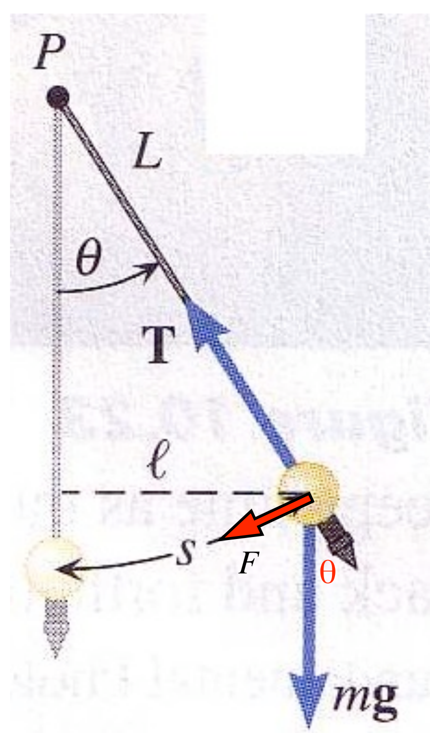
The restoring force along the arc s along which the mass moves is:

$$F = -mg\sin\theta \simeq -mg\theta \text{ for small angles}$$

$$\text{and } \theta = \frac{s}{L} \text{ radians}$$

$$\text{So, } F = -\left(\frac{mg}{L}\right)s$$

Force pulls mass back to θ = 0



This is of the same form as for a mass on a spring:

F = -kx, with s taking the place of x and with an effective spring constant:

k = mg/L

Effective spring constant, $k = mg/L$

Then, the angular frequency for the motion is:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}}$$

As, $\omega = 2\pi f = 2\pi/T$, the period is

$$T = 2\pi\sqrt{\frac{L}{g}}$$ Period of a simple pendulum

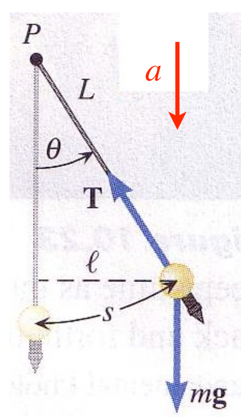
"Physical pendulum": an extended object pivoting about a point

Not covered!

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

I = moment of inertia
L = distance from pivot to centre of gravity

Clickers!



You have a simple pendulum in an elevator that is accelerating downward with acceleration a.

Does the pendulum swing more slowly, more quickly, or at the same rate as it does when the elevator is at rest?

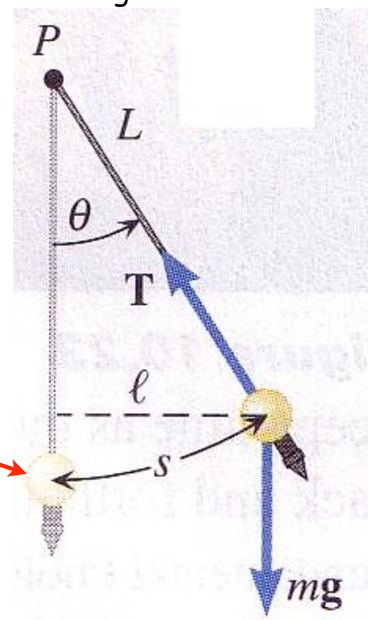
A) The pendulum swings more slowly

B) The pendulum swings more quickly

C) The pendulum swings at unchanged rate

The tension in a string from which a mass is suspended is m(g - a), as if the acceleration due to gravity has been reduced...

10.40: A pendulum consists of a ball on the end of a string 0.65 m long. The ball is pulled to one side through a small angle and released. How long does it take the ball to reach its greatest speed?



Conservation of mechanical energy:

$$PE_o + KE_o = PE + KE\text{, or}$$

$$PE_o + 0 = 0 + KE$$

At lowest point, PE = 0,
so KE = maximum value

Time to reach lowest point is T/4:

$$t = \frac{1}{4} \times 2\pi\sqrt{L/g} = 0.40 \text{ s}$$

10.-/42: A pendulum clock acts as a simple pendulum of length 1 m. It keeps accurate time at a location where the acceleration due to gravity is 9.83 m/s^2. What must be the length of the pendulum to keep accurate time if the local acceleration due to gravity is 9.78 m/s^2?

$$\text{Period, } T = 2\pi\sqrt{\frac{L}{g}}$$

For a fixed period, L/g = constant.

$$\text{So, } \frac{L_1}{g_1} = \frac{L_2}{g_2} \text{ to keep time}$$

$$L_2 = L_1 \times \frac{g_2}{g_1} = (1 \text{ m}) \times \frac{9.78}{9.83}$$

$$L_2 = 0.995 \text{ m}$$

10.69/41: Astronauts on a distant planet set up a simple pendulum of length 1.2 m. The pendulum executes simple harmonic motion and makes 100 complete swings in 280 s. What is the acceleration due to gravity on the planet?

$$\text{Period, } T = 2\pi\sqrt{\frac{L}{g}} = \frac{280}{100} = 2.8 \text{ s}$$

$$g = L\left[\frac{2\pi}{T}\right]^2 = 1.2\left[\frac{2\pi}{2.8}\right]^2 = 6.0 \text{ m/s}^2$$

Simple Harmonic Motion

- The restoring force has the form: F = -kx
- The motion is: x = A cos(ωt), or x = A sin(ωt)
- The angular frequency is: $\omega = \sqrt{\frac{k}{m}}$

$$\omega = 2\pi f = 2\pi/T \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

- Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$