This Week

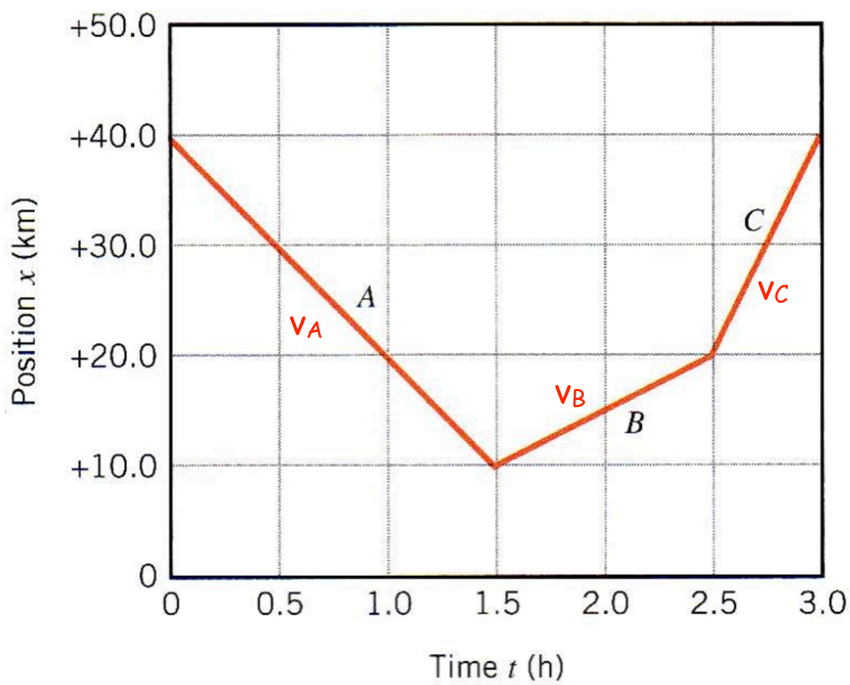
Tutorial and Test 1, in the lab
(chapters 1 and 2)

Next Week

Experiment 1:
Measurement of Length and Mass

WileyPLUS Assignment 1 now available
Due Monday, October 5 at 11:00 pm
Chapters 2 & 3

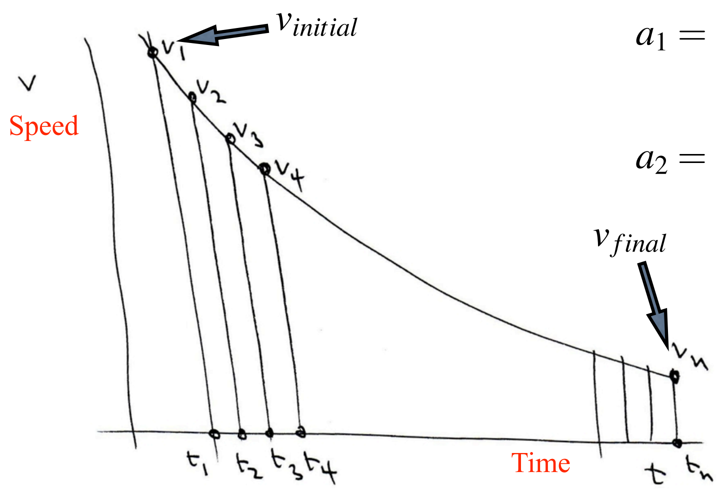
Question about average acceleration



Average acceleration $= \dfrac{v_C - v_A}{T}$ What about v_B?

Question about average acceleration

Average acceleration is $\bar{a} = \dfrac{v_{final} - v_{initial}}{T}$



$$a_1 = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_2 - v_1}{\Delta t},$$

$$a_2 = \frac{v_3 - v_2}{t_3 - t_2} = \frac{v_3 - v_2}{\Delta t}, \text{ etc}$$

Work out the acceleration at each point,
make all the time intervals equal

Question about average acceleration

$$a_1 = \frac{v_2 - v_1}{\Delta t}$$

$$a_2 = \frac{v_3 - v_2}{\Delta t}$$

$$a_3 = \frac{v_4 - v_3}{\Delta t} \text{ etc}$$

Calculate the average acceleration:

$$\bar{a} = [a_1 + a_2 + \ldots + a_n] \times \frac{1}{n}$$

$$= \frac{(\cancel{v_2} - v_1) + (v_3 - \cancel{v_2}) + \ldots}{\Delta t} \times \frac{1}{n}$$

All of the speeds cancel apart from v_1 and v_n, the first and last values

The total elapsed time is $T = n\Delta t$

So, the average acceleration is $\bar{a} = \dfrac{v_n - v_1}{T} = \dfrac{v_{final} - v_{initial}}{T}$

The intermediate speeds cancel from the average

Speed, Velocity and Acceleration

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}} = \frac{x - x_0}{t - t_0}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}} = \frac{\Delta\vec{x}}{t - t_0}$$

$$\text{Instantaneous velocity } \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t}$$

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{elapsed time}} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$$

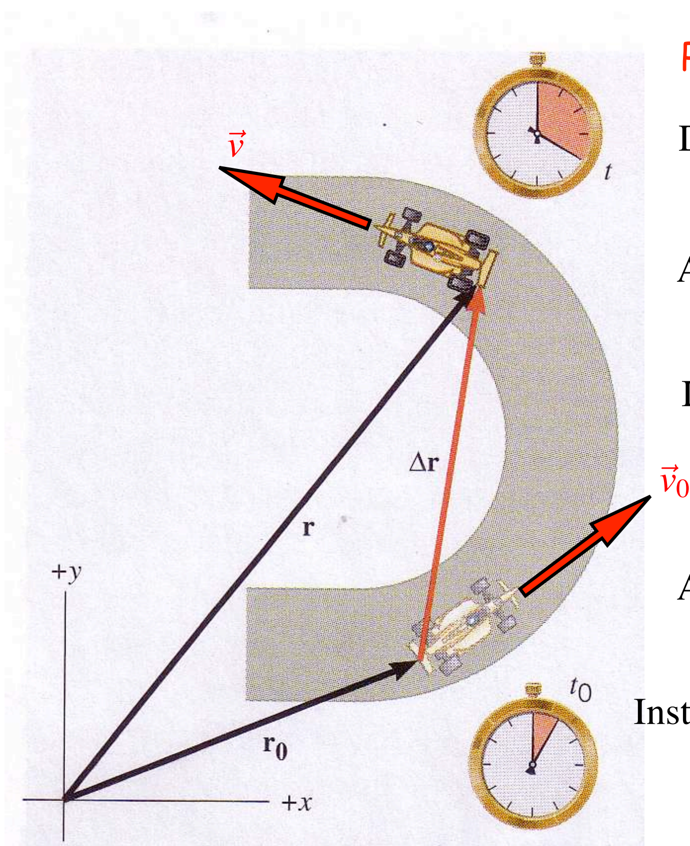
$$\text{Instantaneous acceleration} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

Chapter 3: Kinematics in Two Dimensions

What's new in this chapter

- The same as chapter 2, only for motion in **two dimensions** – the displacement is no longer in a straight line
- In the absence of air resistance, motion in x and y can be separated:

  • velocity in x-direction is constant
  • free fall in y-direction, a_y = -g (y-axis pointing up)
- Relative velocity
- Not yet any physics as such!



Position vectors **r**, **r**$_0$ at t, t_0

Displacement $\Delta\vec{r} = \vec{r} - \vec{r}_0$

Average velocity $= \dfrac{\Delta\vec{r}}{t - t_0}$

Instantaneous velocity $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t}$

Average acceleration $= \dfrac{\vec{v} - \vec{v}_0}{t - t_0}$

Instantaneous acceleration $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t}$

There is an acceleration whenever there is a change of speed or direction

Clicker Question

You drive 1500 m east in 2 minutes, then drive north the same distance in a further 2 minutes.

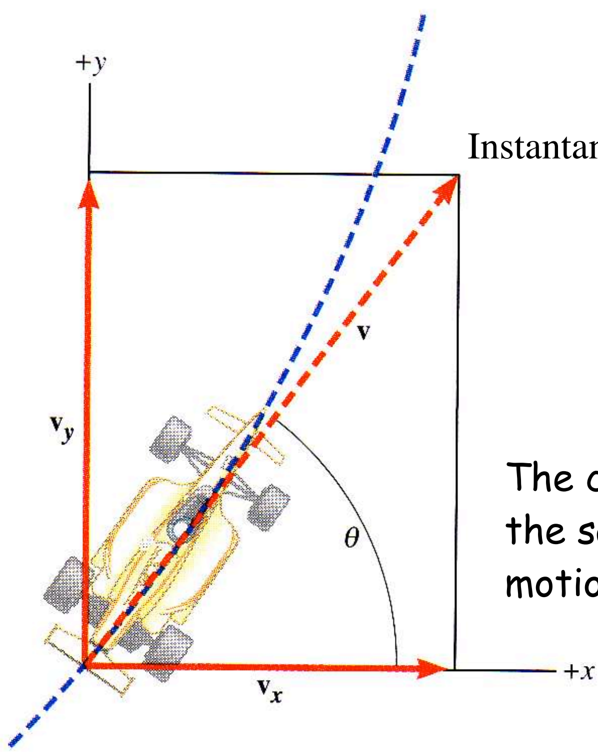
What can be said about the average speeds and velocities for the two parts of the trip?

a) The average speeds are the same, and the average velocities are the same.

b) The average speeds are the same, but the average velocities are different.

c) The average speeds are different, but the average velocities are the same.

Vectors can be resolved into components



$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

The components separately follow the same laws of motion as if the motion were in a straight line

Equations of motion in two dimensions

Same as before, only with subscripts for each direction of motion

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

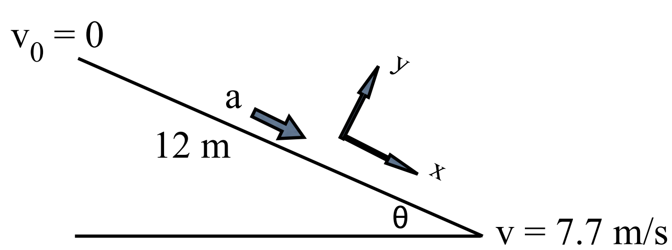
$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \qquad y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t \qquad y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

For projectile motion in absence of air resistance: $a_x = 0$, $a_y = -g$
(y-axis pointing upward)

3.9/7: A skateboarder rolls down a 12 m ramp, reaching a speed of 7.7 m/s at the bottom. What is her average acceleration?

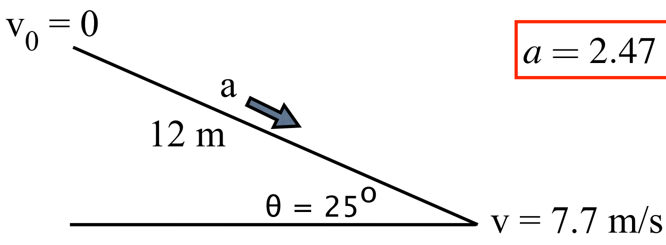


Tilt the x-axis to point down the slope

$$v^2 = v_0^2 + 2ax$$

$$7.7^2 = 0 + 2a \times (12\text{ m}) \quad \rightarrow \underline{a = 2.47\text{ m/s}^2}$$

If $\theta = 25^{o}$, what is the acceleration parallel to the horizontal?



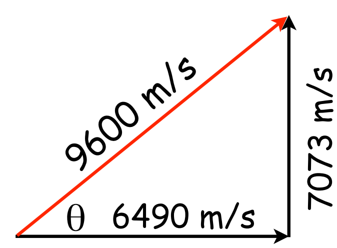
$$\boxed{a = 2.47\ \mathrm{m/s^2}}$$

Acceleration parallel to horizontal $= a\cos\theta = 2.47\cos 25^{\circ} = 2.24\ \mathrm{m/s^2}$

A spacecraft is travelling with a velocity of v_{0x} = 5480 m/s along the +x direction. Two engines are fired for 842 seconds.

Engine one: a_x = 1.20 m/s^2
Engine two: a_y = 8.40 m/s^2

Find final v_x, v_y.



Acceleration in x direction: $v_x = v_{0x} + a_x t \ = 5480 + 1.2 \times 842 = 6490$ m/s

Acceleration in y direction: $v_y = v_{0y} + a_y t \ = 0 + 8.4 \times 842 = 7073$ m/s

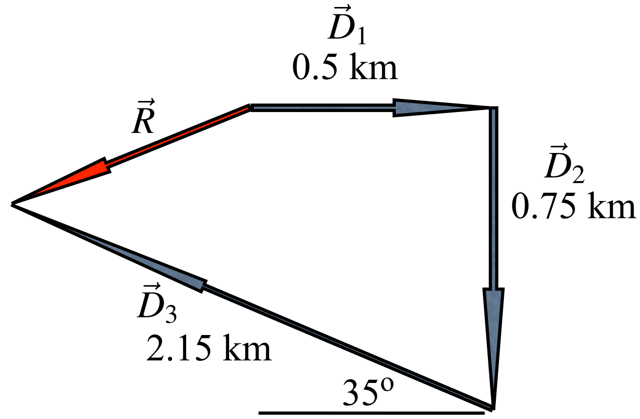
Final speed:

$$\boxed{v = \sqrt{v_x^2 + v_y^2} = \sqrt{6490^2 + 7073^2} = 9600\ \mathrm{m/s}} \quad \tan\theta = \frac{7073}{6490} \rightarrow \quad \theta = 47.5^{\circ}$$

3.11/73: A person walks 0.5 km east, 0.75 km south and 2.15 km at 35^0 north of west in 2.5 h.

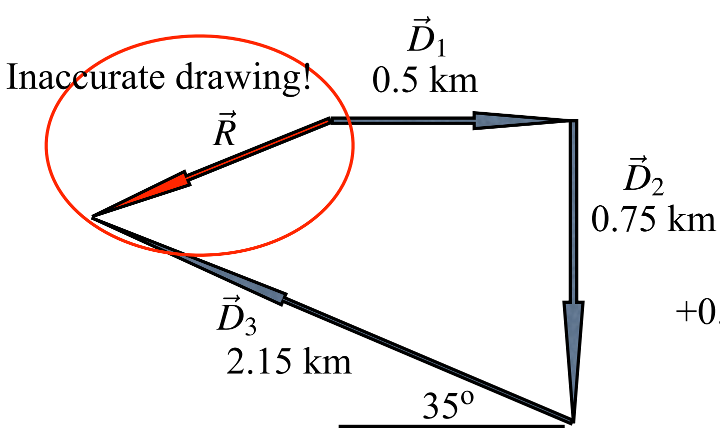
Find the displacement from the starting point and average velocity.

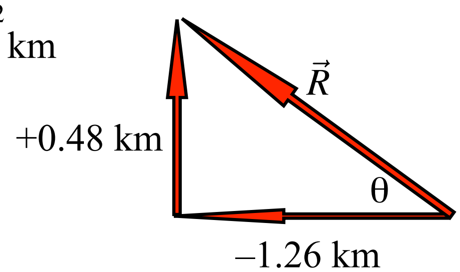
$\vec{R} = ?$



$$\vec{R} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 \qquad \rightarrow R_x = D_{1x} + D_{2x} + D_{3x}$$

$$R_y = D_{1y} + D_{2y} + D_{3y}$$





$$R_x = D_{1x} + D_{2x} + D_{3x} = 0.5 + 0 - 2.15\cos 35^\circ = -1.26 \text{ km}$$

$$R_y = D_{1y} + D_{2y} + D_{3y} = 0 - 0.75 + 2.15\sin 35^\circ = +0.48 \text{ km}$$

$$R = \sqrt{R_x^2 + R_y^2} = 1.35 \text{ km}$$

Angle to west direction: $\tan\theta = 0.48/1.26 \quad \rightarrow \boxed{\theta = 20.9^\circ \text{ north of west}}$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

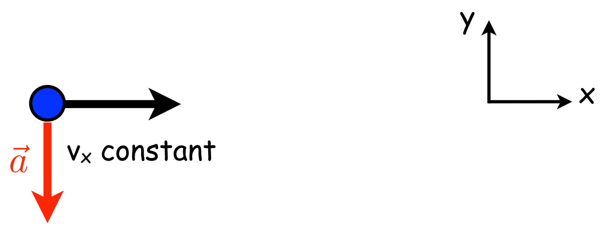
$$= \frac{1.35 \text{ km at } 20.9^\circ \text{ north of west}}{2.5 \text{ h}}$$

$$= 0.54 \text{ km/h at } 20.9^\circ \text{ north of west}$$

Projectile Motion

- Consider motion in x and y separately
- Ignore air resistance $\rightarrow$ velocity in x-direction is constant
- Write down positions in x and y as a function of time
- Remember that the projectile travels up and down (y) in the same time that it is travelling sideways (x)

Projectile Motion



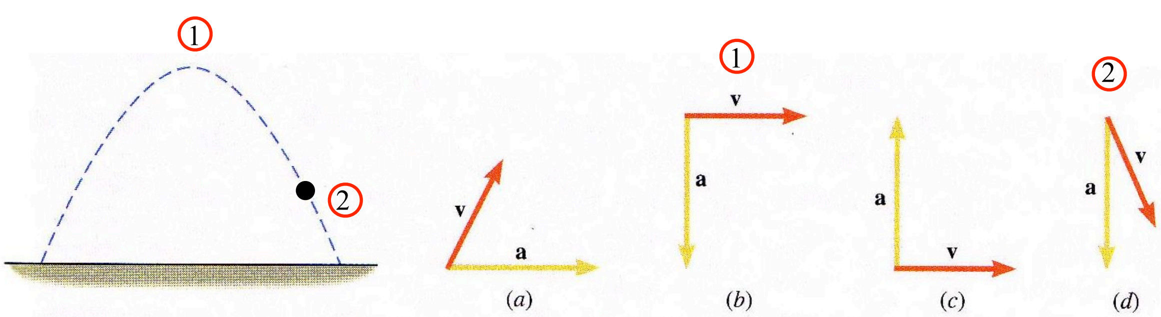
In absence of air resistance: no forces act in x-direction, so v_x, the speed in x-direction is constant throughout the path.

Speed changes in y-direction because of gravity and $a_y = -g$.

Clicker Question

The projectile has velocity $\vec{v}$ and acceleration $\vec{a}$

There is no air resistance



Which of (a), (b), (c) and (d) could **not** represent the directions of the vectors at any point of the trajectory?

A: (a)
B: (b)
C: (c)
D: (d)
E: (a) and (c)

Clicker Question: Focus on Concepts, Question 2

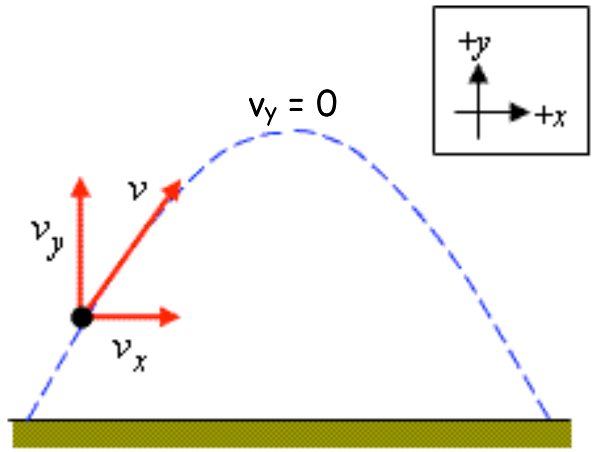
At a certain point along the path in projectile motion, the projectile has a velocity **v** whose scalar components are v_x = +30 m/s and v_y = +40 m/s. As the projectile moves along the path, what would be its minimum speed?

A) 40 m/s

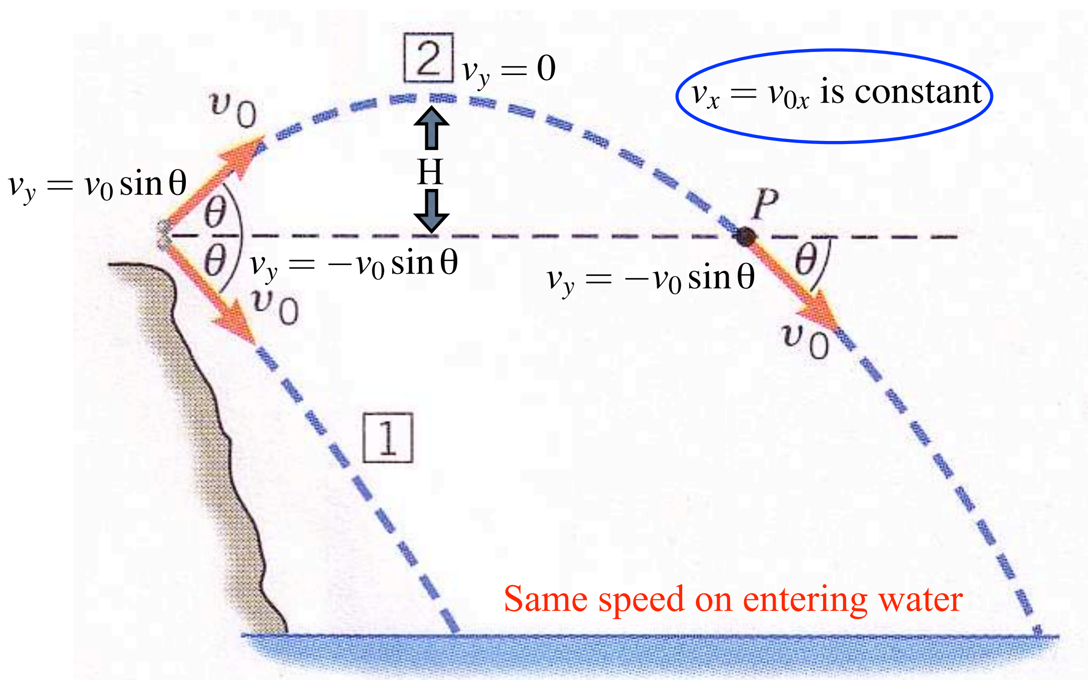
B) 0 m/s

C) $\sqrt{30^2 + 40^2} = 50 \text{ m/s}$

D) 30 m/s



v_x = constant = 30 m/s, v_y varies



Stones 1 and 2 are thrown with the same speed, v_o, but at angles θ above and below the horizontal. Which hits the water with the greater speed?

Stone 2 at P has the same velocity as stone 1 at the start → same speed when they hit the water

Clickers!

A rifle, at a height H above the ground, fires a bullet parallel to the ground.

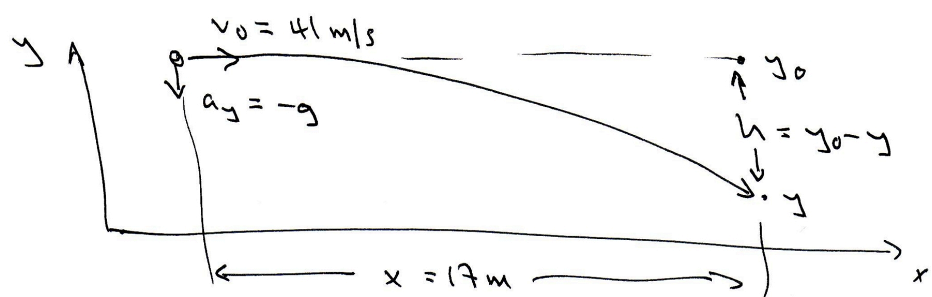
At the same instant and at the same height, a second bullet is dropped from rest.

In the absence of air resistance, which bullet strikes the ground first?

A) The bullet that is dropped strikes the ground first
B) The bullet fired from the rifle strikes the ground first
C) The bullets strike the ground at the same time
D) Impossible to say without knowing the speed of the bullet

Projectile Motion

3.30/24: A ball is thrown horizontally at 41 m/s. How much does it drop while travelling a horizontal distance of 17 m?



Motion in x direction:

$$v_x = v_0 = 41 \text{ m/s (constant in absence of air resistance)}$$

Time to travel 17 m in x direction: $t = \dfrac{17 \text{ m}}{41 \text{ m/s}} = 0.4146 \text{ s}$

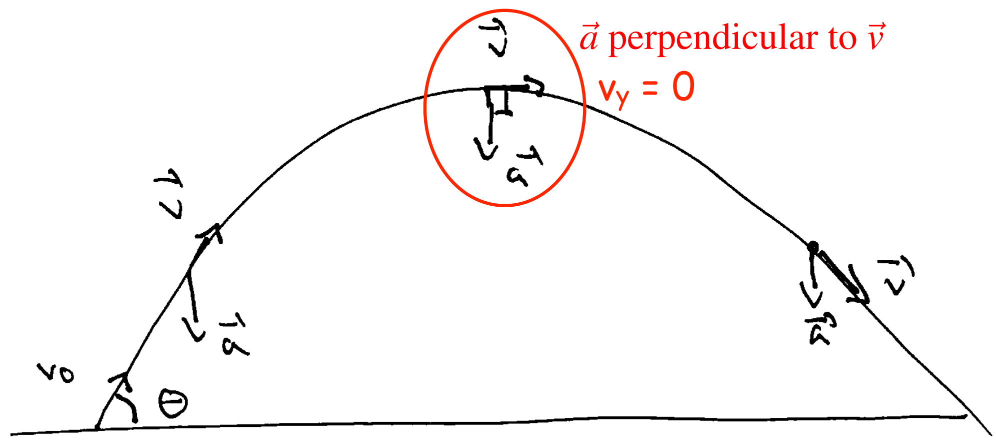
Motion in y: $y = y_0 + v_{0y}t - \dfrac{1}{2}gt^2$

Ball drops by: $h = y_0 - y = 0 + \dfrac{1}{2}g \times 0.4146^2 = 0.84 \text{ m}$

An object is thrown up in the air at an angle θ that is less than 90^0.

a) Is there a point where the acceleration and velocity are perpendicular?

b) Is there any point where velocity and acceleration are parallel?

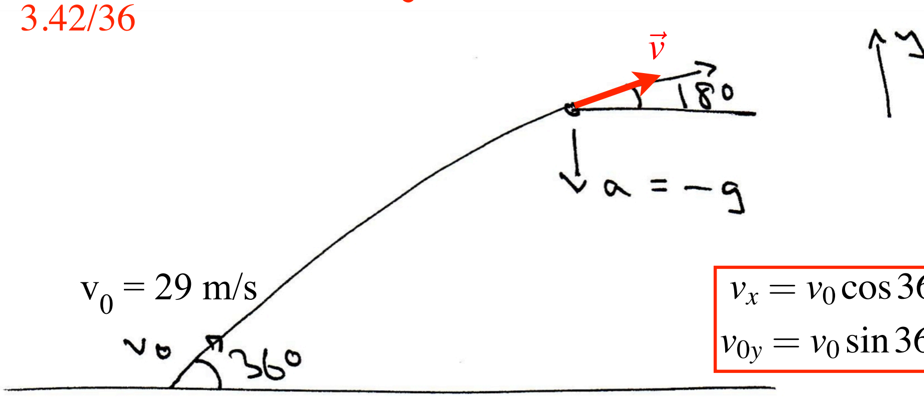


Acceleration is always downward – gravity always pulls downward

Velocity is always tangent to the trajectory

Projectile Motion

3.42/36



$$v_x = v_0 \cos 36^\circ \text{ (constant)}$$
$$v_{0y} = v_0 \sin 36^\circ$$

A projectile is launched with initial speed v_0 = 29 m/s at 36^0 to the horizontal. When does the path make an angle of 18^0 to the horizontal?

The angle to the horizontal is given by: $\tan\theta = \dfrac{v_y}{v_x}$

$\boxed{\text{So need to find when: } v_y = v_x \tan 18^\circ}$

$\boxed{\text{So need to find when: } v_y = v_x \tan 18^\circ}$

$v_0 = 29$ m/s

$$\text{So, } v_y = \underbrace{v_0 \cos 36^\circ}_{v_x} \tan 18^\circ = 0.2629 v_0$$

Famous equation for v_y:

$$v_y = v_{0y} - gt$$

$$\text{Therefore, } t = \frac{v_{0y} - v_y}{g} = \frac{v_0 \sin 36^\circ - 0.2629 v_0}{g}$$

$$\boxed{t = (29 \text{ m/s}) \times \frac{(\sin 36^\circ - 0.2629)}{9.8} = 0.961 \text{ s}}$$