WileyPLUS Assignment 3

Chapters 6 & 7

Due Wednesday, November 11 at 11 pm

This Week

No labs of tutorials

Remembrance Day holiday on Wednesday
(no classes)

Mechanical Energy

Mechanical energy, conserved in the absence of nonconservative (applied and friction) forces:

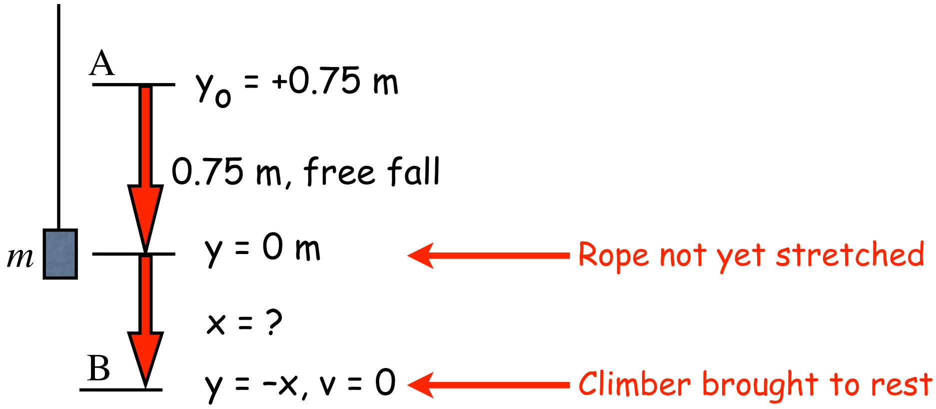
$$E = KE + PE_{grav} + PE_{elastic}$$
$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

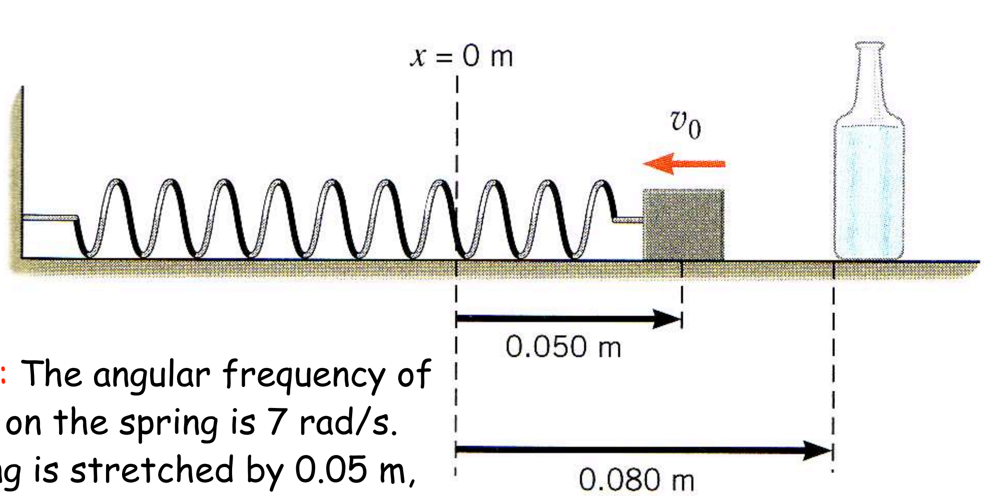
In the presence of nonconservative forces:

$$W_{nc} = \Delta E = \Delta KE + \Delta PE_{grav} + \Delta PE_{elastic}$$

(Work-Energy Theorem)

10.92/84: An 86 kg climber is scaling the vertical wall of a mountain. His safety rope when stretched acts like a spring with spring constant k = 1200 N/m. He falls 0.75 m before the rope becomes taut. How much does the rope stretch when it breaks his fall and momentarily brings him to rest?





10.39/34: The angular frequency of the mass on the spring is 7 rad/s. The spring is stretched by 0.05 m, as shown, and the block is thrown to the left.

Find the minimum speed v_0 so that the bottle gets hit (ignore width of block).

Conservation of mechanical energy:

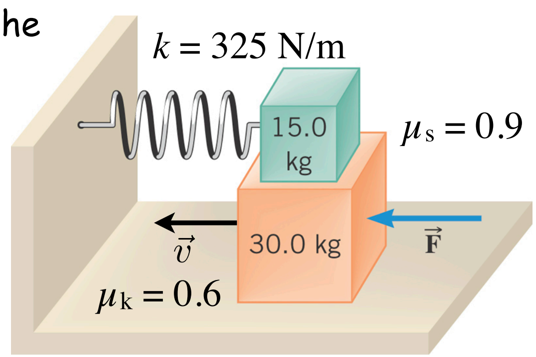
$PE_0 + KE_0 = PE + KE$ ← As block touches bottle

10:6/85: A spring lies on a horizontal table, and the left end of the spring is attached to a wall. The other end is connected to a box. The box is pulled to the right, stretching the spring.

The mass of the box is 0.90 kg, and the spring has a spring constant of 71 N/m. The coefficient of static friction between the box and the table on which it rests is μ_s = 0.67.

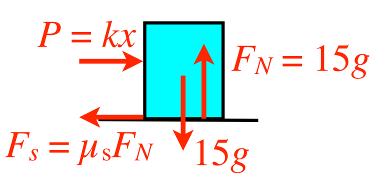
How far can the spring be stretched from its unstrained position without the box moving when it is released?

10.14/13: A horizontal force F is applied to the lower block in such a way that the blocks move at constant speed. At the point where the upper block begins to slip, determine a) the amount by which the spring is compressed and b) the magnitude of the force, F.



There is no acceleration, so the net force on each block is zero.

a) Forces on the upper block:



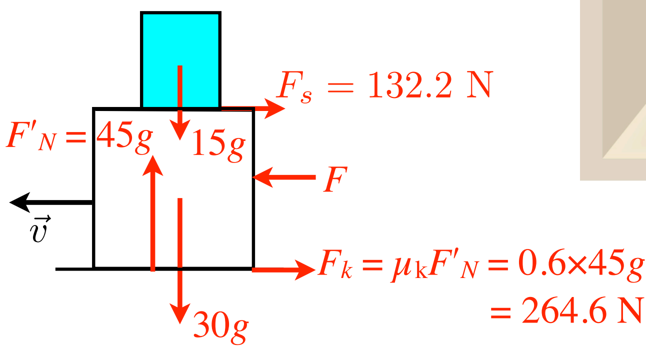
Block slips when P = $(F_s)_{max} = \mu_s F_N$

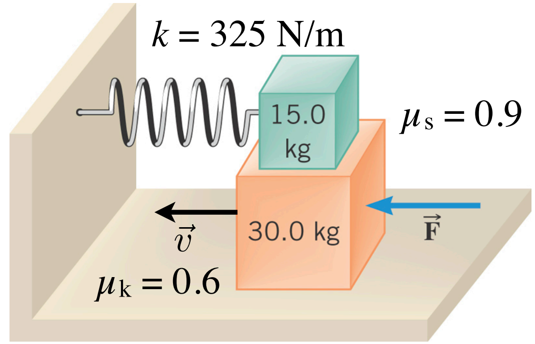
That is: kx = $\mu_s \times 15g$

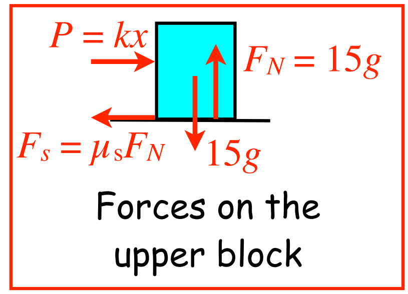
$$\rightarrow x = \frac{15\mu_s g}{k} = \frac{15 \times 0.9g}{325} = 0.407 \text{ m}$$

$$\text{and } F_s = 15\mu_s g = 132.2 \text{ N}$$

b) Forces on the lower block:



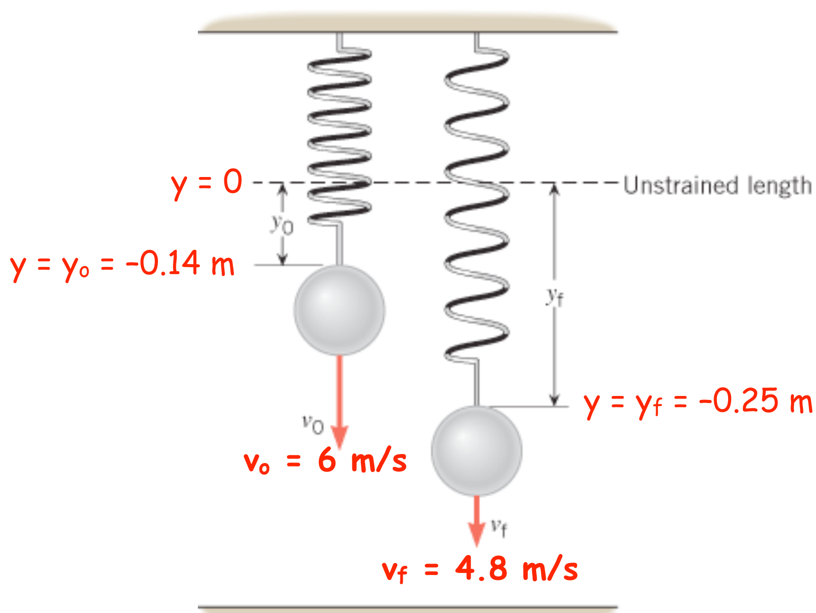




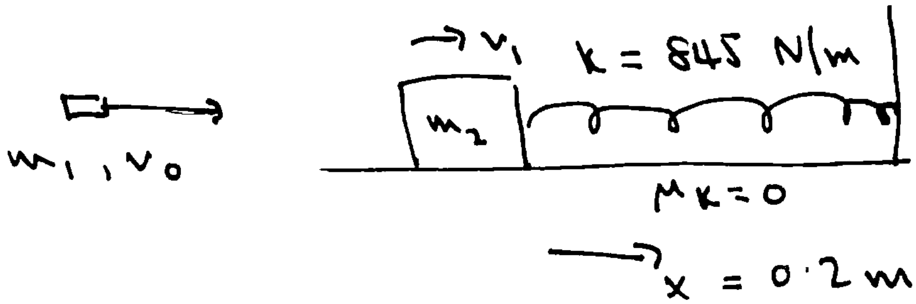
Acceleration = 0, so net force on block = 0

$F = F_s + F_k = 132.2 + 264.6 = 396.8$ N

10:38/- A 0.49-kg metal sphere oscillates at the end of a vertical spring. As the spring stretches from 0.14 to 0.25 m (relative to its unstrained length), the speed of the sphere decreases from 6.0 to 4.8 m/s. What is the spring constant of the spring?



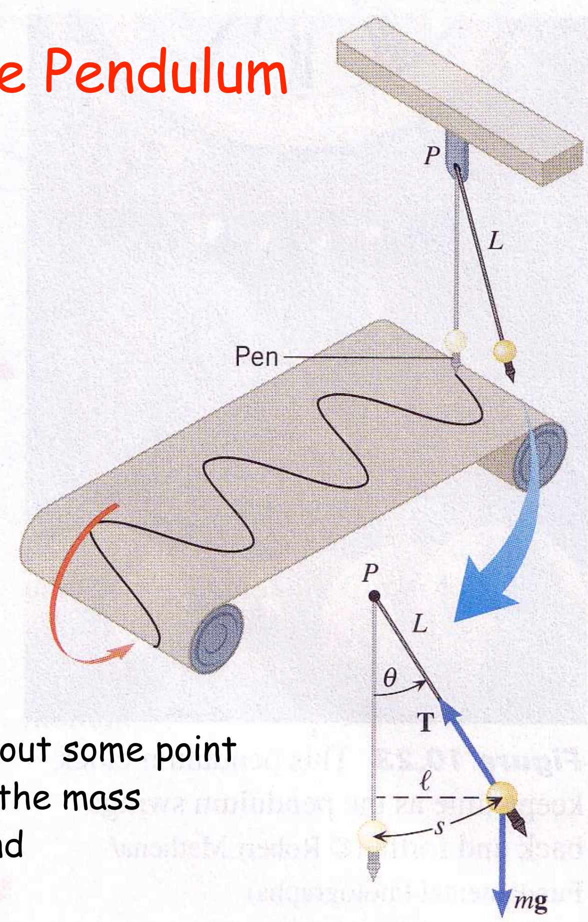
10.-/88: A 0.01 kg bullet is fired horizontally into a 2.5 kg wooden block attached to one end of a massless horizontal spring, k = 845 N/m. The other end of the spring is fixed in place. The spring is initially unstrained. The bullet stops in the block while the spring is compressed, causing the bullet+block to oscillate with an amplitude of 0.2 m. What is the initial speed of the bullet?



The Simple Pendulum

Simple Pendulum:

- a mass on the end of a string
- executes SHM for small displacements



"Physical Pendulum"
(not covered)
– an extended mass pivoting about some point
– example, a solid bar in which the mass is not concentrated at one end

Simple Pendulum

The restoring force along the arc s along which the mass moves is:

$$F = -mg\sin\theta \simeq -mg\theta \text{ for small angles}$$

and $\theta = \frac{s}{L}$ radians

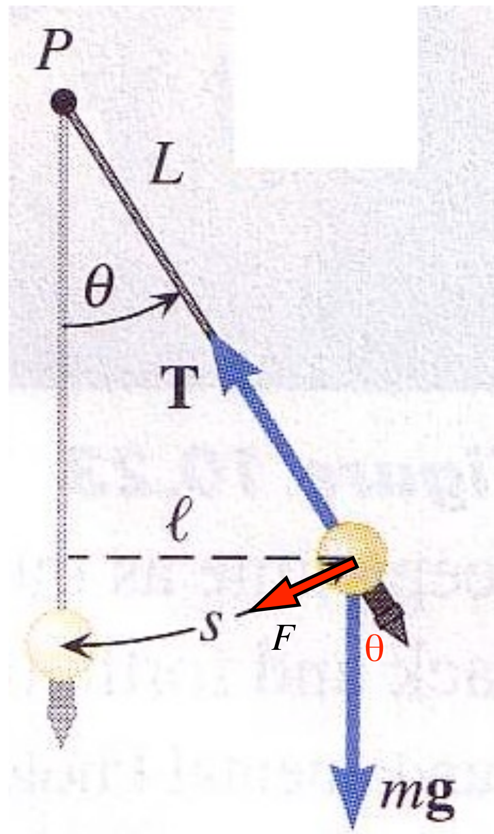
Force pulls mass back to $\theta = 0$

$$\text{So, } F = -\left(\frac{mg}{L}\right)s$$

This is of the same form as for a mass on a spring:

F = -kx, with s taking the place of x and with an effective spring constant:

k = mg/L



Effective spring constant, $k = mg/L$

Then, the angular frequency for the motion is:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}}$$

As, $\omega = 2\pi f = 2\pi/T$, the period is

$$T = 2\pi\sqrt{\frac{L}{g}}$$ Period of a simple pendulum

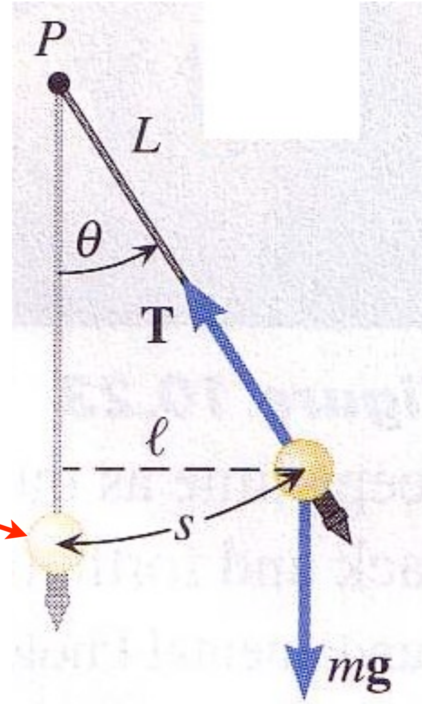
"Physical pendulum": an extended object pivoting about a point

Not covered!

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

I = moment of inertia
L = distance from pivot to centre of gravity

10.48/43: A pendulum consists of a ball on the end of a string 0.65 m long. The ball is pulled to one side through a small angle and released. How long does it take the ball to reach its greatest speed?



Conservation of mechanical energy:

$$PE_o + KE_o = PE + KE, \text{ or}$$

$$PE_o + 0 = 0 + KE$$

At lowest point, PE = 0, so KE = maximum value

Time to reach lowest point is T/4:

$$t = \frac{1}{4} \times 2\pi\sqrt{L/g} = 0.40 \text{ s}$$

10:76/41: The height of a tower is measured by attaching a mass to the lower end of a rope to form a simple pendulum with a length equal to the height of the tower.

If the period of the pendulum is 9.2 s, what is the height of the tower?

$$\text{Period}, T = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{So}, l = g\left[\frac{T}{2\pi}\right]^2 = g\left[\frac{9.2}{2\pi}\right]^2$$

$$l = 21 \text{ m}$$

A pendulum clock acts as a simple pendulum of length 1 m. It keeps accurate time at a location where the acceleration due to gravity is 9.83 m/s^2. What must be the length of the pendulum to keep accurate time if the local acceleration due to gravity is 9.78 m/s^2?

$$\text{Period}, T = 2\pi\sqrt{\frac{L}{g}}$$

For a fixed period, L/g = constant.

$$\text{So}, \frac{L_1}{g_1} = \frac{L_2}{g_2} \text{ to keep time}$$

$$L_2 = L_1 \times \frac{g_2}{g_1} = (1\text{ m}) \times \frac{9.78}{9.83}$$

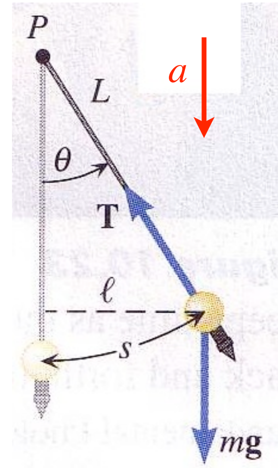
$$L_2 = 0.995\text{ m}$$

Astronauts on a distant planet set up a simple pendulum of length 1.2 m. The pendulum executes simple harmonic motion and makes 100 complete swings in 280 s. What is the acceleration due to gravity on the planet?

$$\text{Period}, T = 2\pi\sqrt{\frac{L}{g}} = \frac{280}{100} = 2.8\text{ s}$$

$$g = L\left[\frac{2\pi}{T}\right]^2 = 1.2\left[\frac{2\pi}{2.8}\right]^2 = 6.0\text{ m/s}^2$$

Clickers!



You have a simple pendulum in an elevator that is **accelerating downward** with acceleration a.

Does the pendulum swing more slowly, more quickly, or at the same rate as it does when the elevator is at rest?

A) The pendulum swings more slowly

B) The pendulum swings more quickly

C) The pendulum swings at unchanged rate

Hint: the tension in a string from which a mass is suspended is m(g - a), as if the acceleration due to gravity has been reduced...

Simple Harmonic Motion

- The restoring force has the form: F = –kx
- The motion is: x = A cos(ωt), or x = A sin(ωt)
- The angular frequency is: $\omega = \sqrt{\frac{k}{m}}$

$$\omega = 2\pi f = 2\pi / T \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

- Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$