WileyPLUS Assignment 3

Chapters 6 & 7

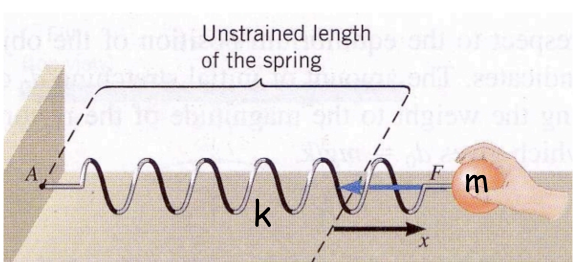
Due Wednesday, November 11 at 11 pm

Next Week

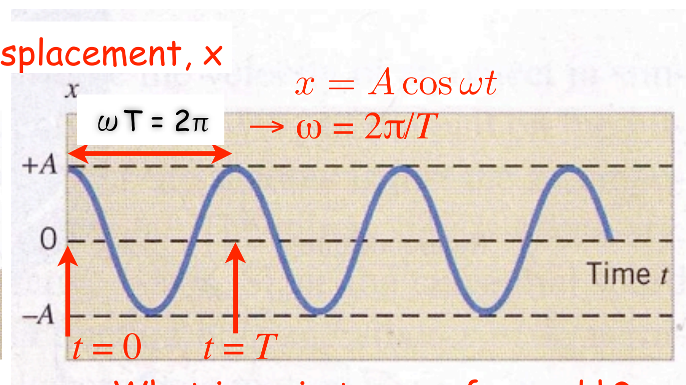
No labs of tutorials

Remembrance Day holiday on Wednesday
(no classes)

Mass on a spring

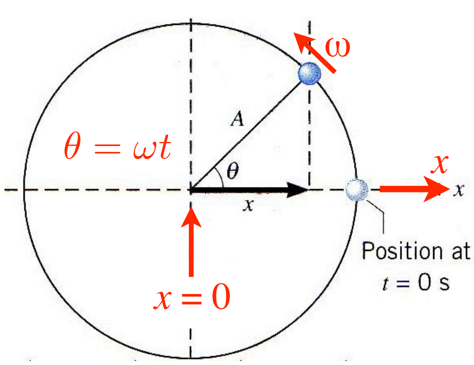


Displacement, x

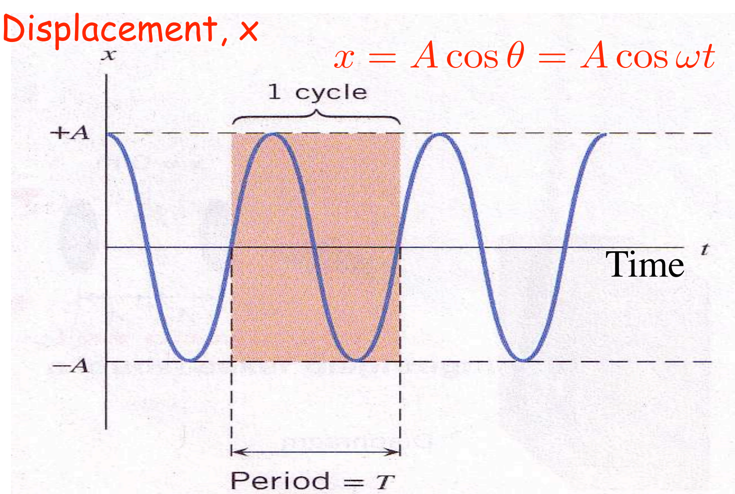


What is ω in terms of m and k?

Circular motion



Displacement, x



Simple Harmonic Motion

Motion is simple harmonic when the restoring force is proportional to the displacement:

$F = -kx.$

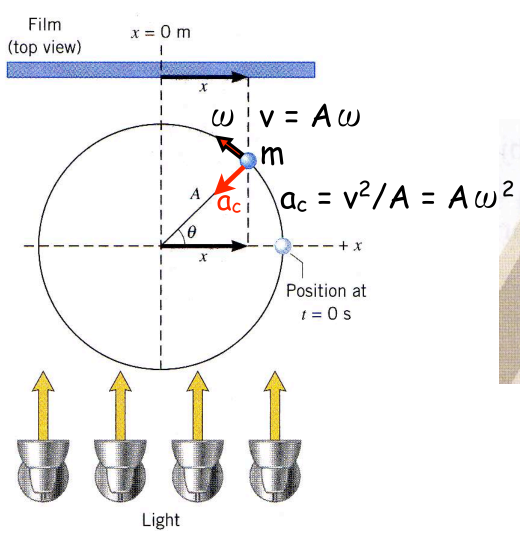
A guide to this motion is provided by a mass moving at constant speed around a circular path:

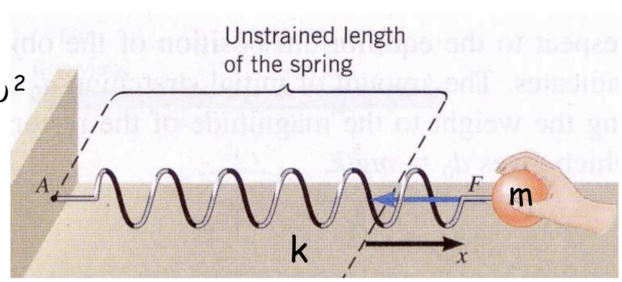
- the force toward $x = 0$ is proportional to x

→ the motion in x is simple harmonic and the period is known

→ gives a relation between the period and the restoring force

Simple Harmonic Motion





Mass on a rotating disk

$F_x = -m\omega^2 x$

Mass on a spring

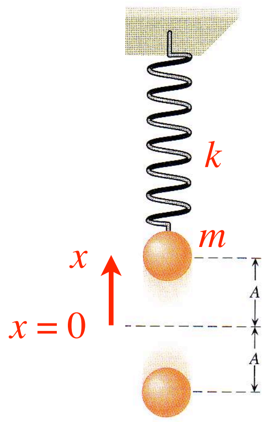
$F_x = -kx$

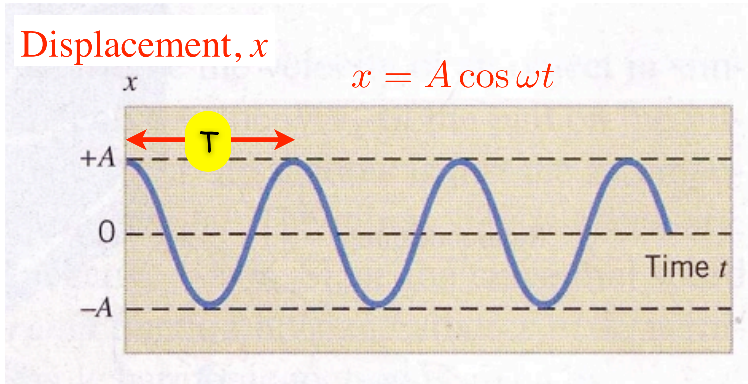
Simple harmonic motion in both cases - restoring force ∝ displacement

COMPARE: if $m\omega^2 = k$, then motions in x are **exactly the same,**

so $\omega^2 = k/m$ for the mass on the spring, and $x = A\cos(\omega t)$

Simple Harmonic Motion





- SHM results when the restoring force is proportional to displacement:

F = -kx

- The resulting motion is:

$$x = A\cos\omega t, \quad (\text{or } x = A\sin\omega t)$$

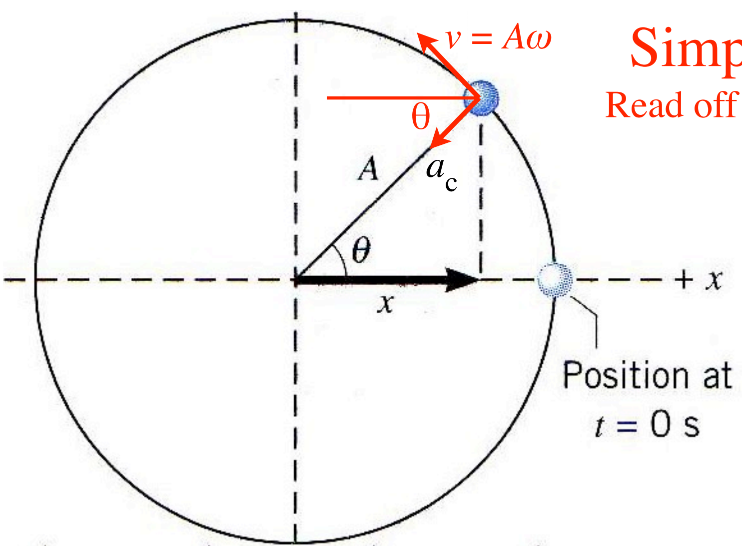
$$\omega = \sqrt{\frac{k}{m}} \text{ and } T = 2\pi\sqrt{\frac{m}{k}}$$

rad/s → ω, Hz → f, s → T:

ω = 2πf = 2π/T

Simple Harmonic Motion

Read off motion from the reference circle



$$\theta = \omega t$$
$$x = A\cos\omega t$$
$$v_x = -A\omega\sin\omega t$$
$$a_x = -A\omega^2\cos\omega t = -\omega^2 x$$

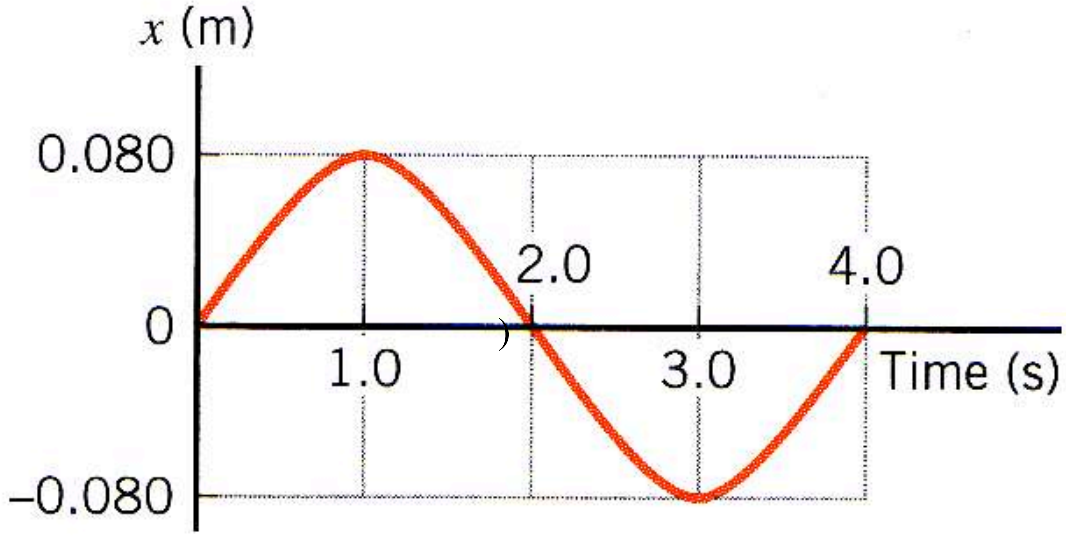
Also by differentiating

$$\text{Maximum } v_x = \pm A\omega \text{ when } x = 0$$

$$\text{Maximum } a_x = \mp A\omega^2 \text{ when } x = \pm A$$

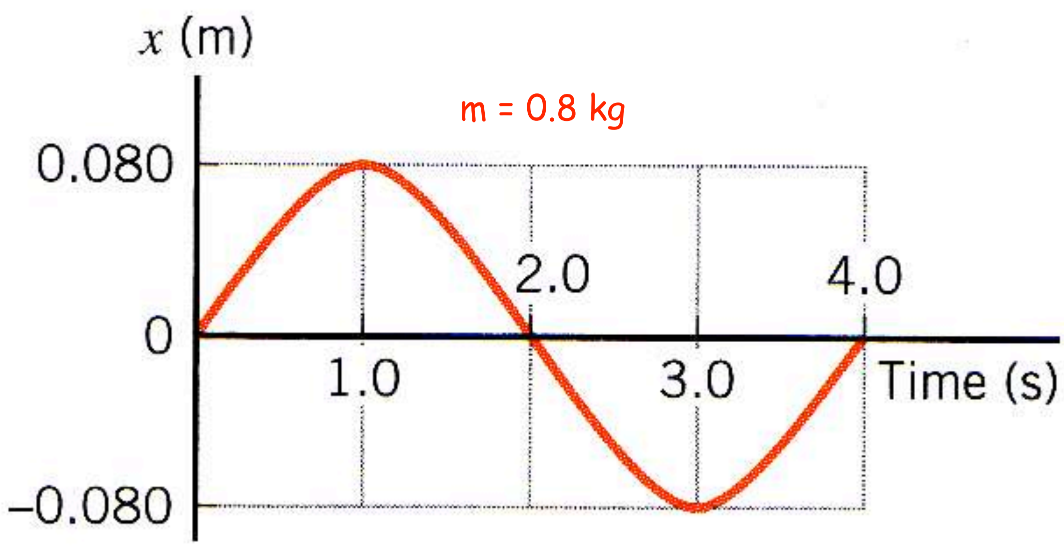
$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

10.19/18: A 0.8 kg mass is attached to a spring.



a) Find the amplitude, A, of the motion.

A = maximum displacement from the equilibrium position = 0.08 m

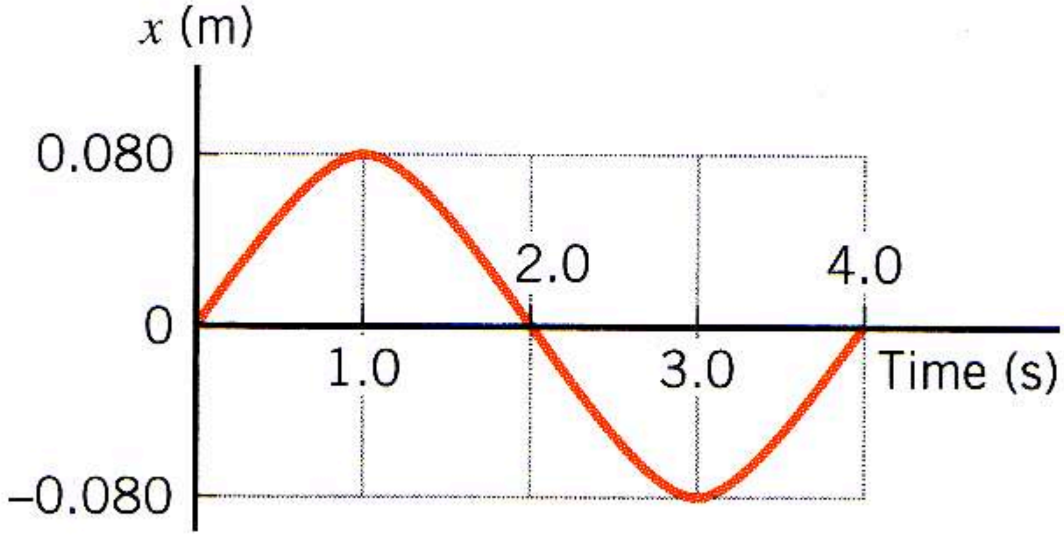


b) Find the angular frequency, ω

$\omega = 2\pi/T$ and T = 4 s, so $\omega = 2\pi/4 = 1.57$ rad/s

c) Find the spring constant, k.

$\omega^2 = k/m$, so $k = m\omega^2 = (0.8 \text{ kg})(1.57 \text{ rad/s})^2 = 1.97$ N/m



d) Find the speed of the object at t = 1 s.
v = 0 (has reached maximum x and has come momentarily to rest)

e) Find the magnitude of the acceleration at t = 1 s.
$a = -\omega^2 x = -(1.57 \text{ rad/s})^2 \times (0.08 \text{ m}) = -0.20 \text{ m/s}^2$

(Check: $a = F/m = -kx/m = -1.97 \times 0.08/0.8 = -0.20 \text{ m/s}^2$)

Clickers!

A steel ball is dropped onto a concrete floor. Over and over again, it rebounds to its original height.

Is the motion simple harmonic motion?

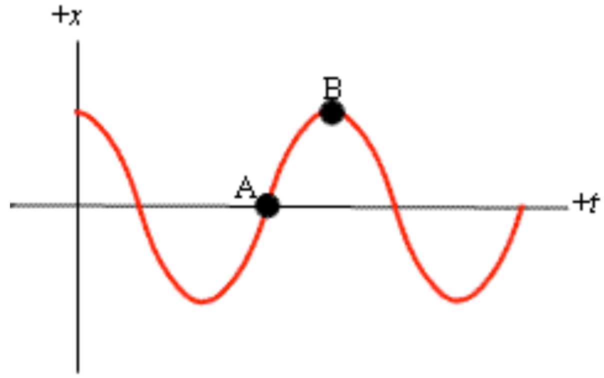
A) Yes, the motion is simple harmonic, as the motion is periodic, repeating itself after a fixed time.

B) No, the motion is not simple harmonic.

C) I'd have to watch the movie.

B) The motion is not simple harmonic

Focus on Concepts, Question 7



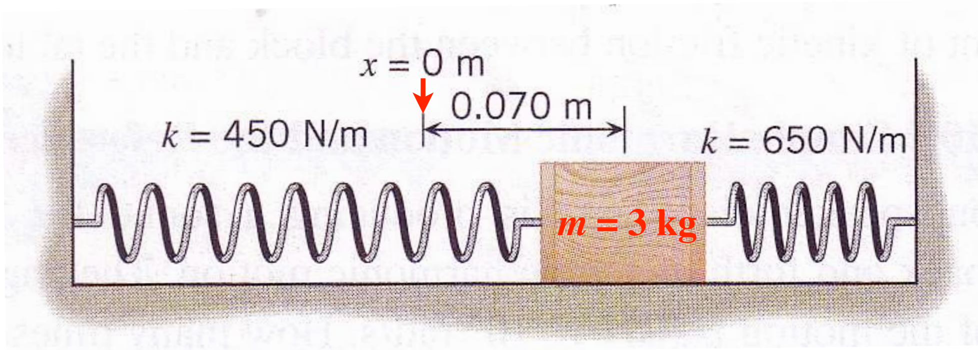
The drawing shows a graph of displacement x versus time t for simple harmonic motion of an object on a horizontal spring.

Which one of the following answers correctly gives the **magnitude** v of the velocity and the **magnitude** a of the acceleration at points A and B in the graph?

A) v_A = 0 m/s, a_A = maximum, v_B = maximum, a_B = 0 m/s^2
B) v_A = maximum, a_A = maximum, v_B = 0 m/s, a_B = maximum
C) v_A = maximum, a_A = maximum, v_B = 0 m/s, a_B = 0 m/s^2
D) v_A = maximum, a_A = 0 m/s^2, v_B = 0 m/s, a_B = maximum
E) v_A = 0 m/s, a_A = 0 m/s^2, v_B = maximum, a_B = maximum

D) v_A = maximum, a_A = 0 m/s^2, v_B = 0 m/s, a_B = maximum

When a mass m_1 is hung from a vertical spring and set into vertical simple harmonic motion, its frequency is 12 Hz. When another object of mass m_2 is hung on the spring along with m_1, the frequency of motion is 4 Hz. Find m_2/m_1.



A 3 kg block is placed between two horizontal springs. The springs are neither strained nor compressed when the block is at x = 0. The block is displaced to x = 0.07 m and is released.

Find the speed of the block when it passes back through x = 0 and the angular frequency of the system.

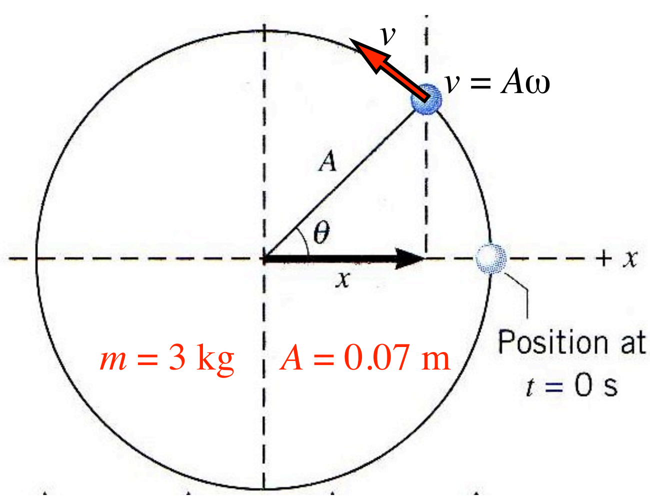
Question: What is the effective spring constant? That is, what is the restoring force when the block is displaced unit distance?

When the block is moved x to the right, the restoring force is:

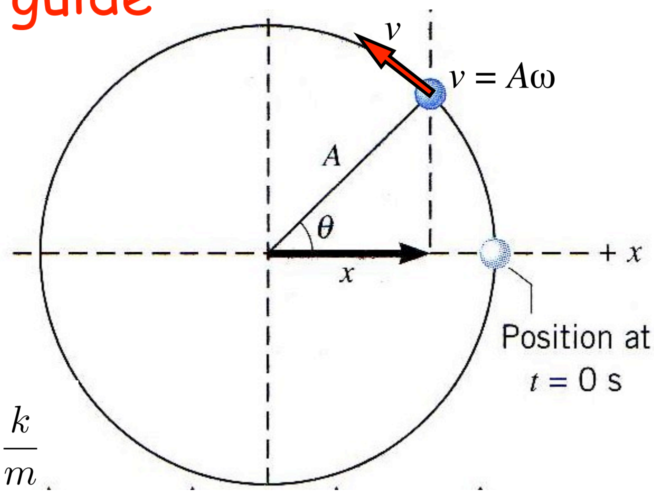
$F = -(450 + 650)x = -1100x$ N.

$F = -1100x$ N

The effective spring constant is 1100 N/m.



Motion of mass on spring using the reference circle as a guide



$$\omega = \sqrt{\frac{k}{m}}$$

At $x = 0$, $v_x = -A\omega$

Considering the motion in x:

$$\text{KE} = \frac{mv_x^2}{2} = \frac{mA^2\omega^2}{2} = \frac{mA^2}{2}\frac{k}{m}$$

$$\text{KE} = \frac{kA^2}{2}$$

Conservation of mechanical energy: when the spring is stretched to $x = A$, it has PE = $kA^2/2$, which is converted entirely to KE when $x = 0$.

Energy and Simple Harmonic Motion

Elastic Potential Energy:

Energy is stored in a spring when it is stretched or compressed. The potential energy is released when the spring is released.

The restoring force exerted by the spring when stretched by x is:

$F = -kx$

The work done by the restoring force when the spring is stretched from x_0 to x_f is:

$$W_{elastic} = F_{average} \times (x_f - x_0) = \frac{-k(x_f + x_0)}{2} \times (x_f - x_0)$$

$$W_{elastic} = \frac{-k(x_f + x_0)}{2} \times (x_f - x_0)$$

$$W_{elastic} = \frac{1}{2}kx_0^2 - \frac{1}{2}kx_f^2$$ = work done **by the restoring force**

Initial elastic PE

Final elastic PE

$$PE_{elastic} = \frac{1}{2}kx^2$$

The total mechanical energy is now:

$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

Mechanical Energy

Elastic Potential Energy:

$$PE_{elastic} = \frac{1}{2}kx^2$$

Mechanical Energy:

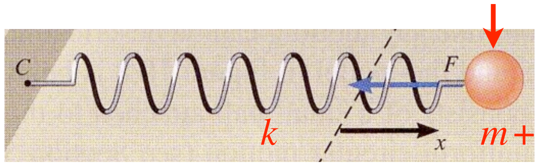
$$\begin{aligned} E &= KE + PE_{grav} + PE_{elastic} \\ &= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2 \end{aligned}$$

Work-Energy Theoreom: the work done by nonconservative (applied and friction) forces:

$$W_{nc} = \Delta E = \Delta KE + \Delta PE_{grav} + \Delta PE_{elastic}$$

A block is attached to a horizontal spring and slides back and forth in simple harmonic motion on a frictionless horizontal surface. A second identical block is suddenly attached to the first block at the moment the block reaches its greatest displacement and is at rest.

Explain how a) the amplitude, b) the frequency, c) the maximum speed of the oscillation change.



a) Amplitude unchanged

b) $\omega = \sqrt{\frac{k}{m}} \rightarrow \sqrt{\frac{k}{2m}}$ frequency reduced by factor of $\sqrt{2}$

c) Mechanical energy: at $x = A$: KE = 0, PE = $kA^2/2$ – unchanged

At $x = 0$: $KE_{max} = kA^2/2 = 2mv_{max}^2/2$ is unchanged

As the mass is doubled, v_{max} is reduced by factor of $\sqrt{2}$

Mechanical Energy

An object of mass 0.2 kg is oscillating on a spring on a horizontal frictionless table. The spring constant is k = 545 N/m.

The spring is stretched to x_0 = 4.5 cm, then released from rest.

Find the speed of the mass when (a) x_f = 2.25 cm, (b) x_f = 0 cm.

An archer pulls the bowstring back 0.47 m. The bow and string act like a spring with spring constant k = 425 N/m.

What is the elastic potential energy of the drawn bow?

$$E = \frac{1}{2}kx^2 = \frac{1}{2} \times 425 \times 0.47^2 = 46.9 \text{ J}$$

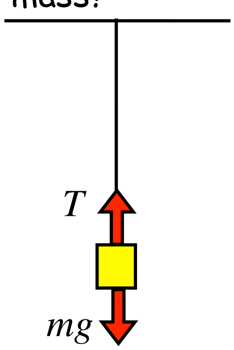
The arrow has a mass m = 0.03 kg. How fast will it travel when it leaves the bow?

$$E = \frac{1}{2}kx^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$46.9 \text{ J} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.03v^2$$

$$v = \sqrt{2 \times 46.9/0.03} = 55.9 \text{ m/s}$$

10.30/28: A 3.2 kg block hangs stationary from the end of a vertical spring attached to the ceiling. The elastic potential energy of the spring/mass system is 1.8 J. What is the elastic potential energy when the 3.2 kg mass is replaced by a 5 kg mass?



At equilibrium, mg = T = kx,

where x is the amount the spring is stretched.

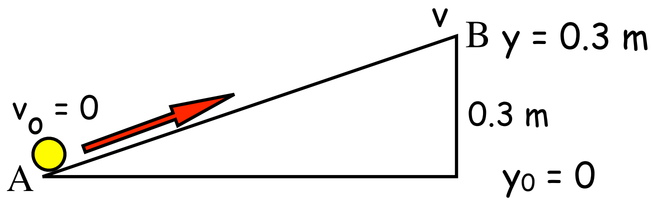
So, x = mg/k.

The elastic potential energy is

$PE_{elastic} = kx^2/2 = k(mg/k)^2/2 = m^2g^2/2k$.

That is, $PE_{elastic} \propto m^2$
So $PE_{elastic} = (5/3.2)^2 \times 1.8 = 4.4$ J

The spring in a pinball machine (k = 675 N/m) is compressed 0.065 m. The ball (m = 0.0585 kg) is at rest against the spring at point A. When the spring is released, the ball slides to point B, which is 0.3 m higher than point A. How fast is the ball moving at B? (no friction)



Conservation of mechanical energy:

At A: $E_A = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}kx_0^2$

$= 0 + 0 + \frac{1}{2} \times 675 \times 0.065^2 = 1.426 \text{ J}$

At B: $E_B = \frac{1}{2}mv^2 + 0.3mg + 0 \;= 1.426 \text{ J} \;\rightarrow v = 6.55 \text{ m/s}$

Mechanical Energy

Mechanical energy, conserved in the absence of nonconservative (applied and friction) forces:

$$E = KE + PE_{grav} + PE_{elastic}$$
$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

In the presence of nonconservative forces:

$$W_{nc} = \Delta E = \Delta KE + \Delta PE_{grav} + \Delta PE_{elastic}$$