WileyPLUS Assignment 3 is available

Chapters 6 & 7

Due Wednesday, November 11 at 11 pm

This Week

Tutorial and Test 3

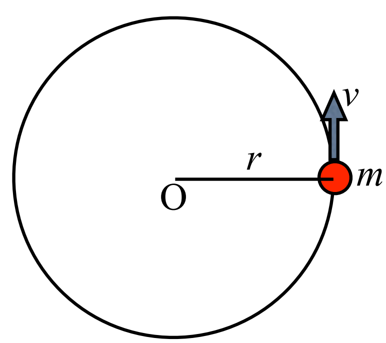
Chapters 6 & 7

Cutnell edition 8	**Cutnell edition 7**
Ch 6:34	Ch 6:31
Ch 6:60	Ch 6:52
Ch 7:13	Ch 7:13
Ch 7:35	Ch 7:31
ch 7:53	Ch 7:17

Angular Momentum

The angular momentum of the mass about O is:

$L = mvr = mr^2\omega$, as $v = r\omega$



Define **$I = mr^2$ = "moment of inertia"** of the mass about centre of circle, then,

angular momentum, $L = I\omega$, and, by applying $F = ma$ to the mass:

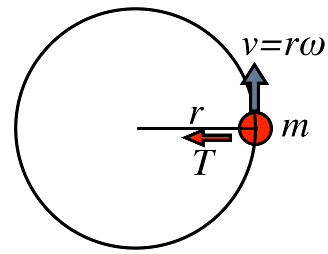
$$\tau = \frac{\Delta L}{\Delta t} = I\frac{\Delta\omega}{\Delta t} \qquad \text{compare } F = ma = m\frac{\Delta v}{\Delta t}$$

Angular momentum is conserved if the net torque acting on an object is zero.

9.60: A small 0.5 kg object moves on a frictionless horizontal table in a circular path of radius 1 m. The angular speed is 6.28 rad/s.

The string is shortened to make the radius of the circle smaller without changing the angular momentum.

If the string breaks when its tension is 105 N, what is the radius of the smallest possible circle in which the object can move?



The tension in the string is:

$T = mv^2/r = 105$ N when string breaks.

What are v and r when T reaches this value?

- use conservation of angular momentum to relate v to r

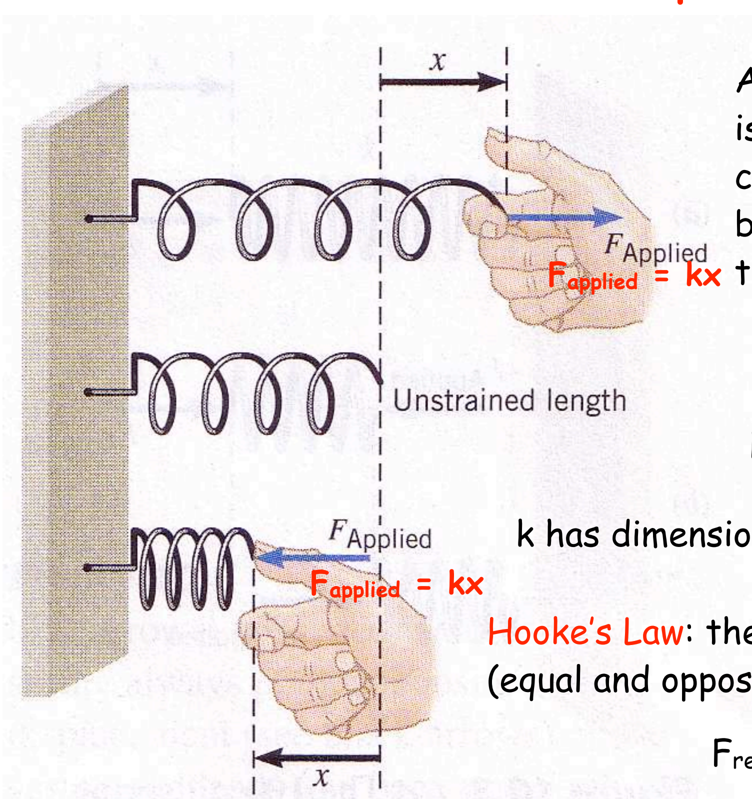
Summary

- Torque, calculated two ways
- First and second conditions of equilibrium
  (1) forces add to zero
  (2) torques add to zero about any point
- Centre of gravity (centre of mass)
  – the point at which the weight of an object may be considered to act
  – torques can be calculated by taking the whole mass to be concentrated at the centre of gravity
- Angular momentum is conserved if the net torque is zero

Chapter 10
Simple Harmonic Motion and Elasticity

- Hooke's Law, motion of a mass on a spring, simple harmonic motion
- Elastic potential energy – the return of the conservation of mechanical energy
- The pendulum and simple harmonic motion
- Read about: 10.5, 10.6, not covered in class
  (damped harmonic motion
  driven harmonic motion
  resonance)
- Forget about: 10.7, 10.8, Elastic deformation, stress, strain...

The Ideal Spring



An equal amount of force is required to stretch or compress an ideal spring by the same amount – up to a point.

$$F_{applied} = k\,x$$

k is the spring constant

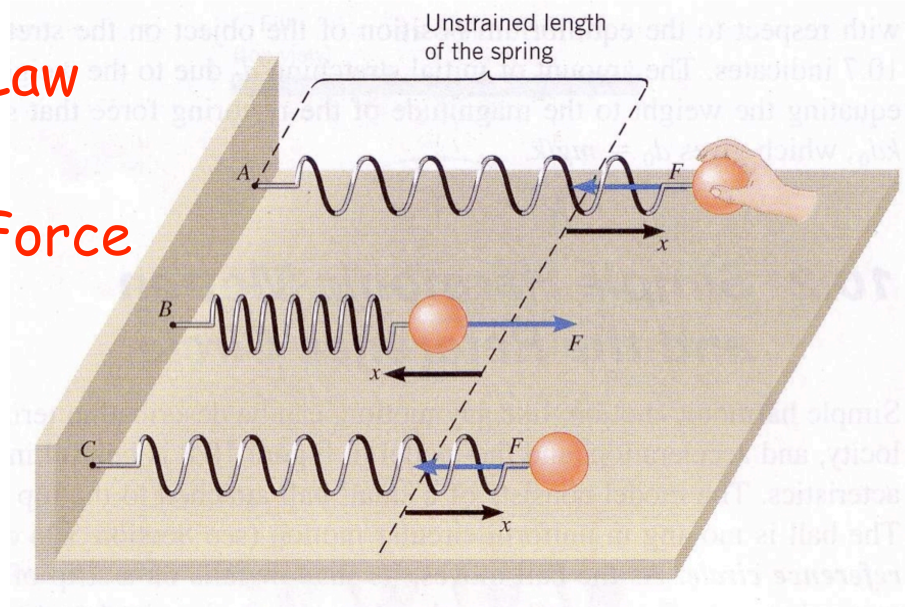
k has dimensions of force/length, N/m

Hooke's Law: the restoring force is –kx (equal and opposite to the applied force)

$F_{restoring}$ = -kx

Hooke's Law

Restoring Force

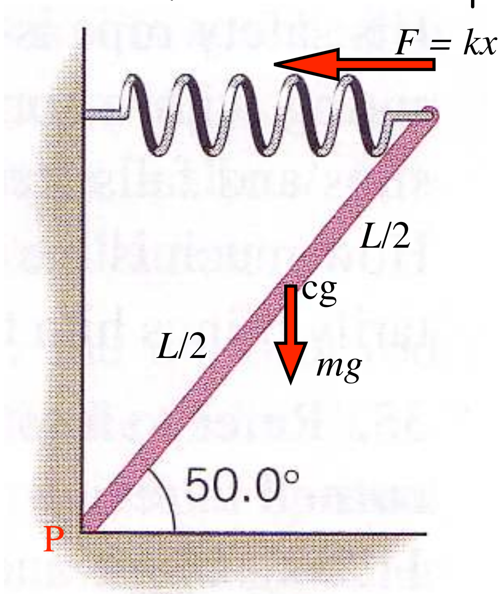


The **restoring force** is the force the spring exerts when stretched or compressed - tries to move spring back to equilibrium state.

$$\text{Restoring force}, F = -F_{applied} = -kx$$

$$\boxed{F = -kx, \text{ Hooke's Law}}$$

10.-/8: A 10.1 kg uniform board is held in place by a spring. The spring constant is k = 176 N/m.
How much has the the spring stretched at equilibrium?



The length of the plank is L

Torques about P at the floor:

$$-mg(L/2)\cos 50^\circ + FL\sin 50^\circ = 0$$

$$F = \frac{mg}{2\tan 50^\circ} = kx$$

$$\text{So}, x = \frac{mg}{2k\tan 50^\circ} = \frac{10.1 \times 9.8}{2 \times 176\tan 50^\circ}$$

$$\boxed{x = 0.24 \text{ m}}$$

Is a short spring easier to compress?

Imagine the long spring as two half-length springs joined together.

The applied force compresses each half spring by x/2.

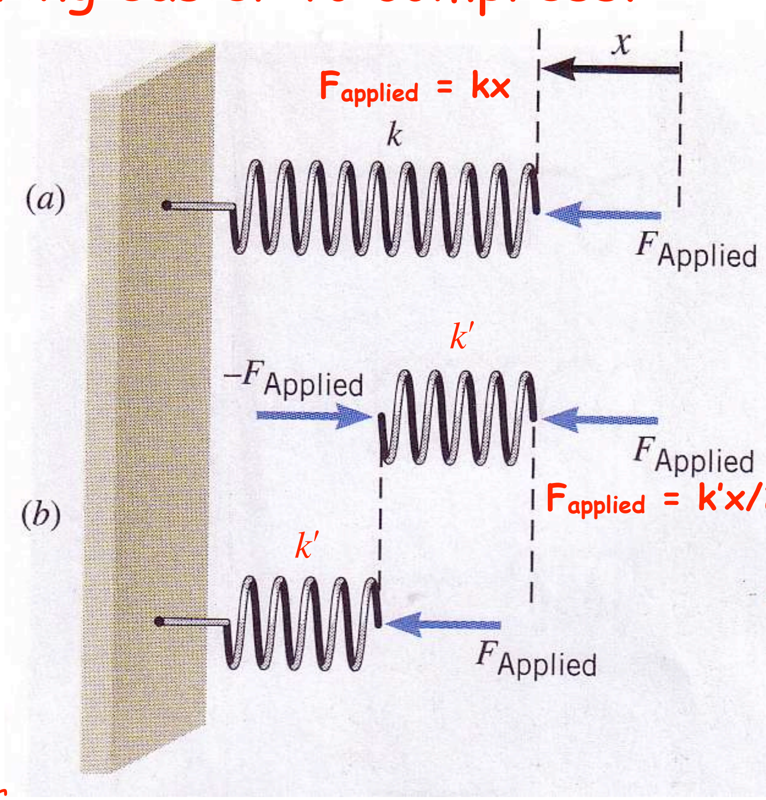
The spring constant for the short springs is given by:

$F_{applied} = k' \times (x/2)$ and

$F_{applied} = kx$ (full spring)

So $k'x/2 = kx$, and $k' = 2k$

The shorter spring is stiffer



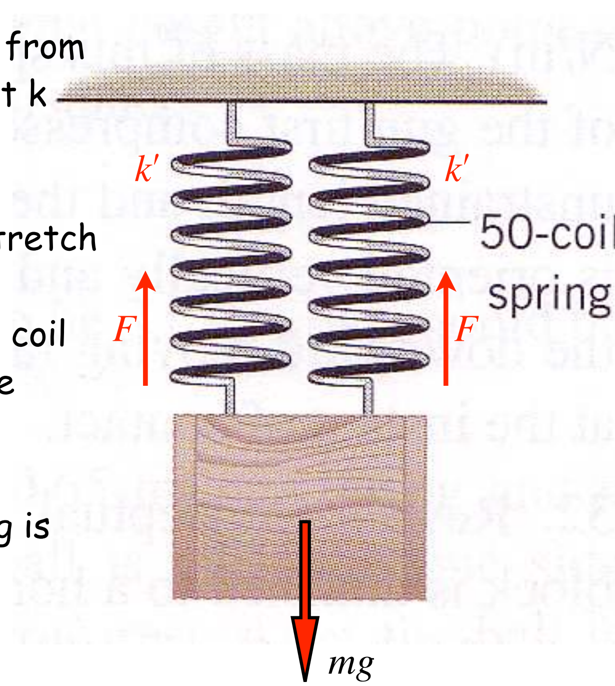
10.13/12: A mass, m, is suspended from a 100 coil spring of spring constant k and its equilibrium position noted:

$x_0 = mg/k = 0.16$ m = equilibrium stretch

The spring is then cut into two 50 coil springs as shown. How much do the springs stretch?

The spring constant of each spring is

$k' = 2k$



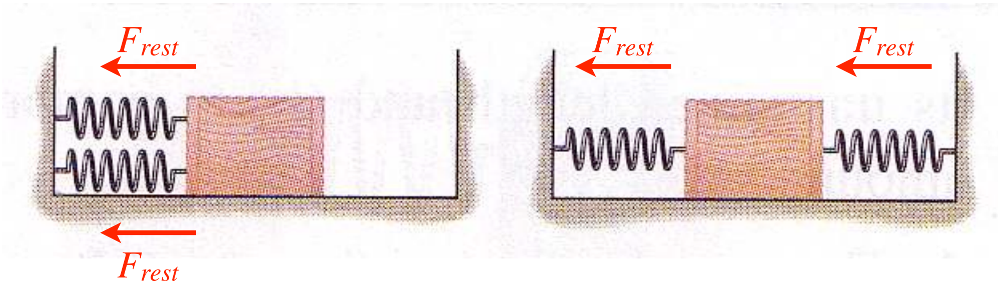
There are two of these springs, so the total restoring force is

$F_{tot} = 2 \times (k'x) = 2 \times (2kx) = mg$, so $x = mg/(4k) = x_0/4 = 0.04$ m

Clickers!

The springs are identical and initially unstrained.

The boxes are pulled to the right by the same distance and released. Which box feels the greater restoring force from the springs?

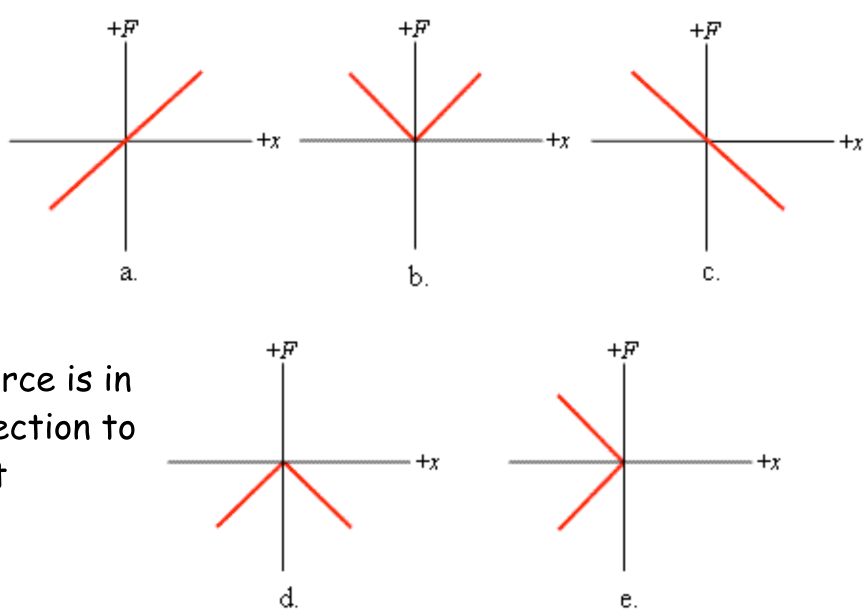


A) box on left feels the greater force
B) box on right feels the greater force
C) the boxes feel the same force

C) the boxes feel the same force

Clicker Question: Focus on Concepts, Question 2

Which one of the following graphs correctly represents the restoring force F of an ideal spring as a function of the displacement x of the spring from its unstrained length?



c) $F = -kx$,
The restoring force is in the opposite direction to the displacement

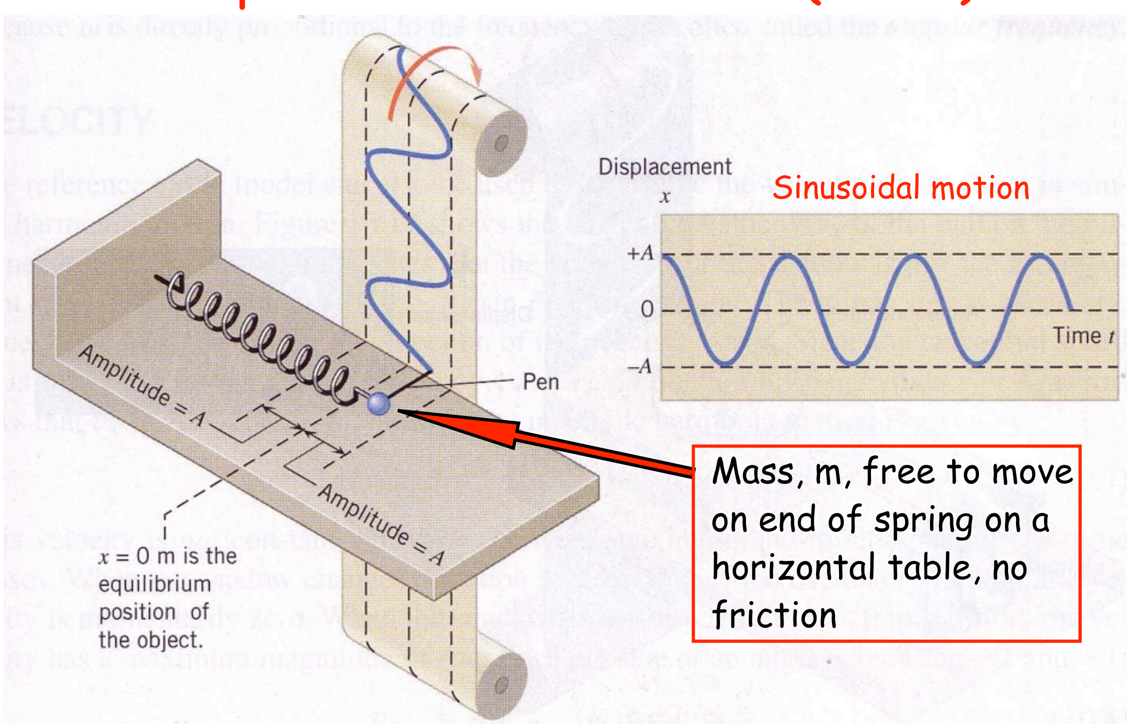
Useful for experiment 4:

10.11/11: A small ball is attached to one end of a spring that has an unstrained length of 0.2 m. The spring is held by the other end, and the ball is whirled around in a horizontal circle at a speed of 3 m/s. The spring remains nearly parallel to the ground and is observed to stretch by 0.01 m. By how much would the spring stretch if it were attached to the ceiling and the ball allowed to hang straight down, motionless?

Find the spring constant from the amount the spring stretches when the ball is whirled around in a circle.

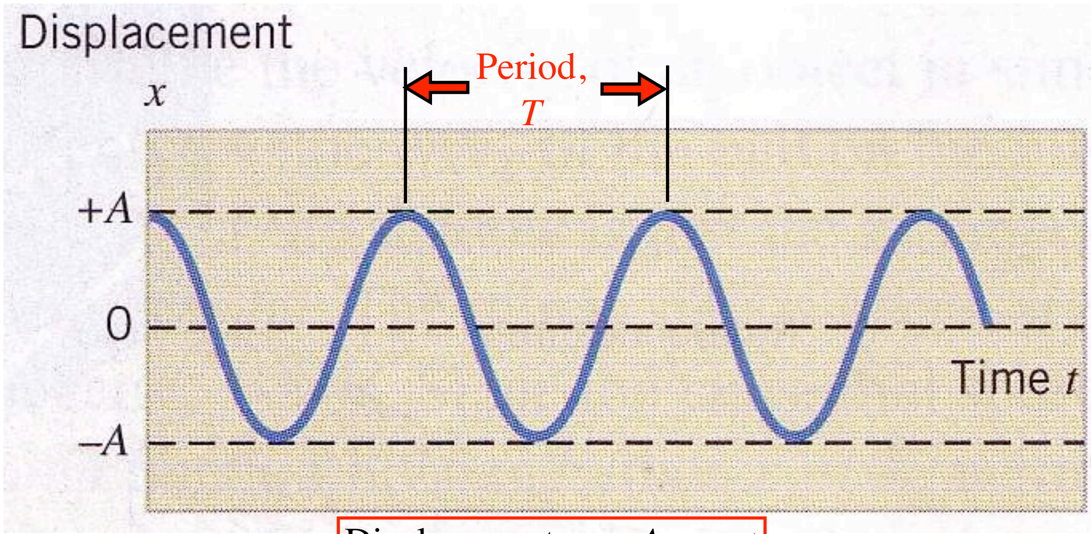
→ Find the stretch when the ball is suspended.

Simple Harmonic Motion (SHM)



Pull the mass a distance A to the right, release, and observe motion...

Simple Harmonic Motion (SHM)



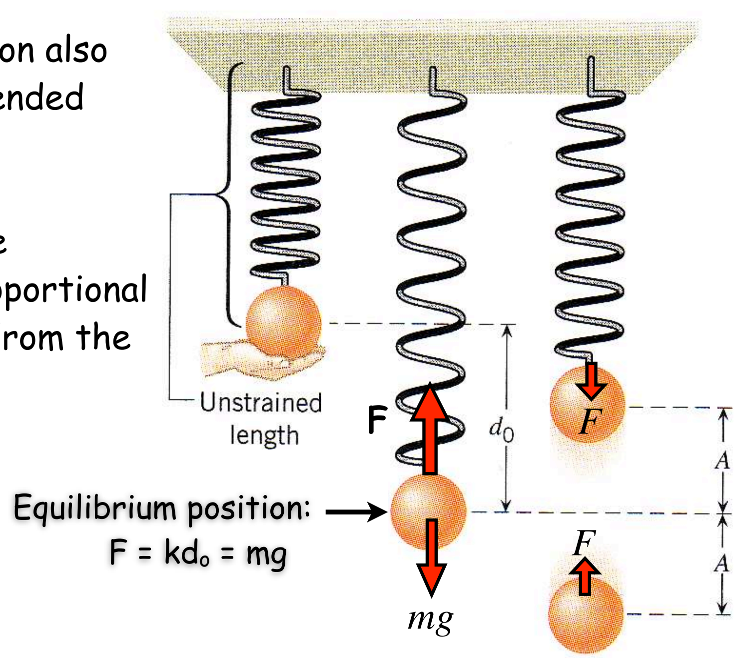
$\boxed{\text{Displacement}, x = A\cos\omega t}$

The time for one cycle is the period, T
So $\omega T = 2\pi$ radians (i.e., 360^0, 1 cycle), and $\omega = 2\pi/T = 2\pi f$,
f = frequency of the SHM, cycles/second or Hertz (Hz)

Simple Harmonic Motion (SHM)

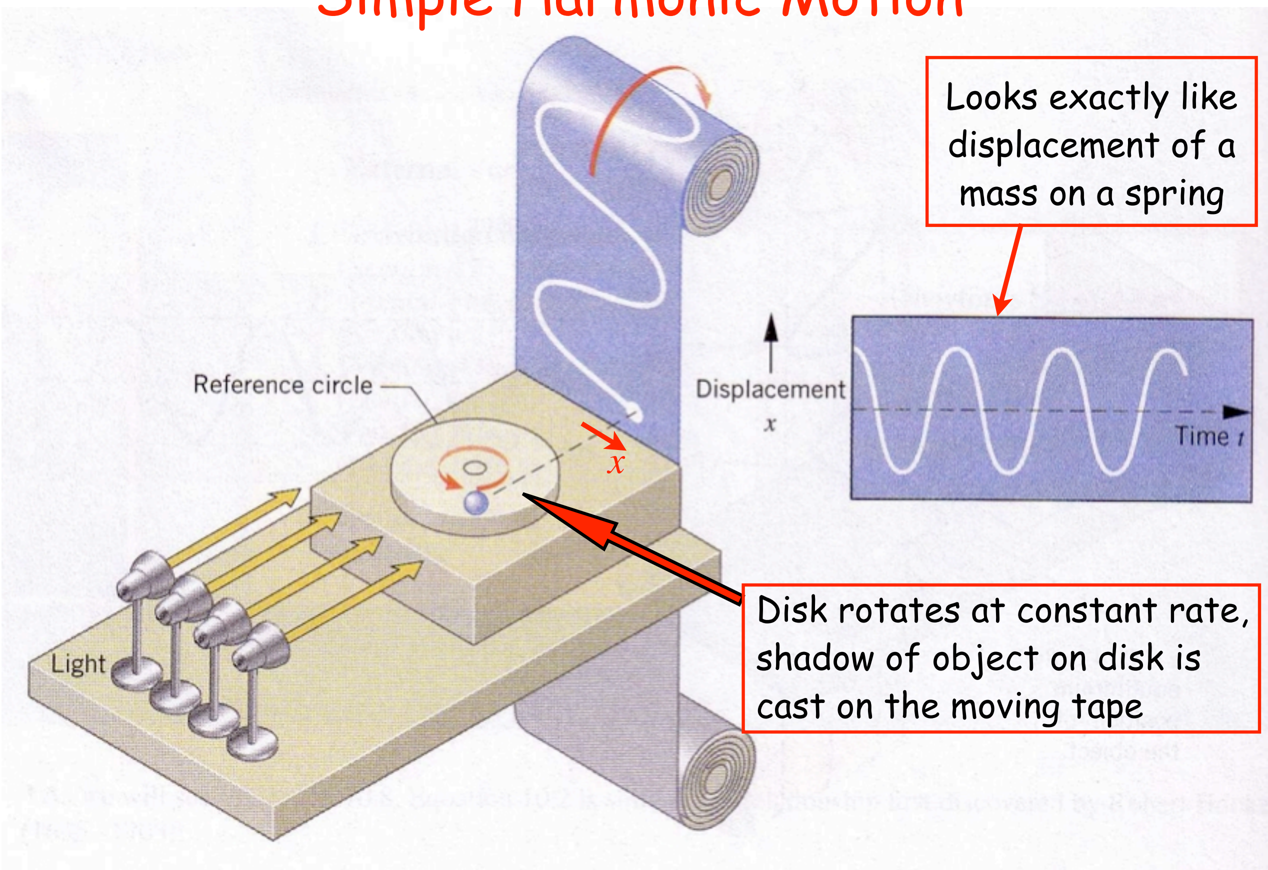
Simple harmonic motion also seen for a mass suspended from a spring.

SHM occurs when the restoring force is proportional to the displacement from the equilibrium position



SHM is about x = d_o with amplitude A.

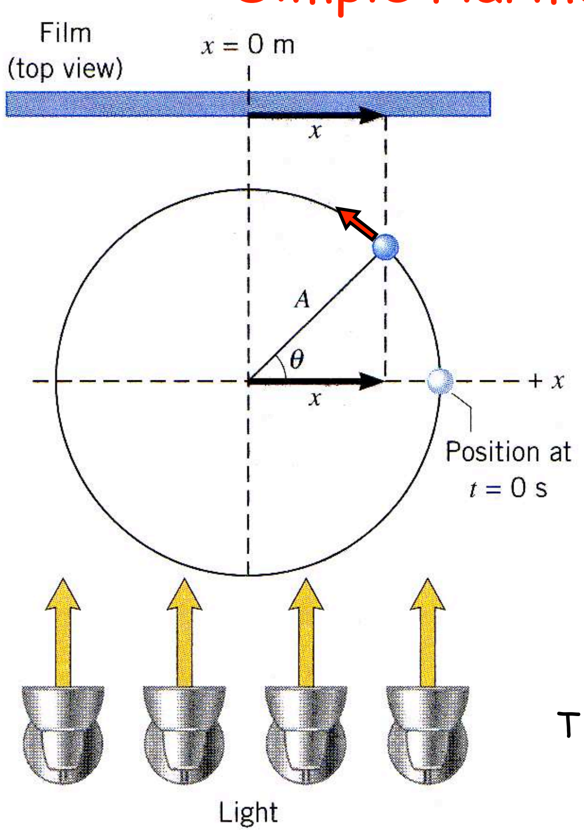
Simple Harmonic Motion



Looks exactly like displacement of a mass on a spring

Disk rotates at constant rate, shadow of object on disk is cast on the moving tape

Simple Harmonic Motion



Shadow appears on screen at

$x = A\cos\theta = A\cos\omega t$

when rotation is at constant angular velocity ω rad/s.

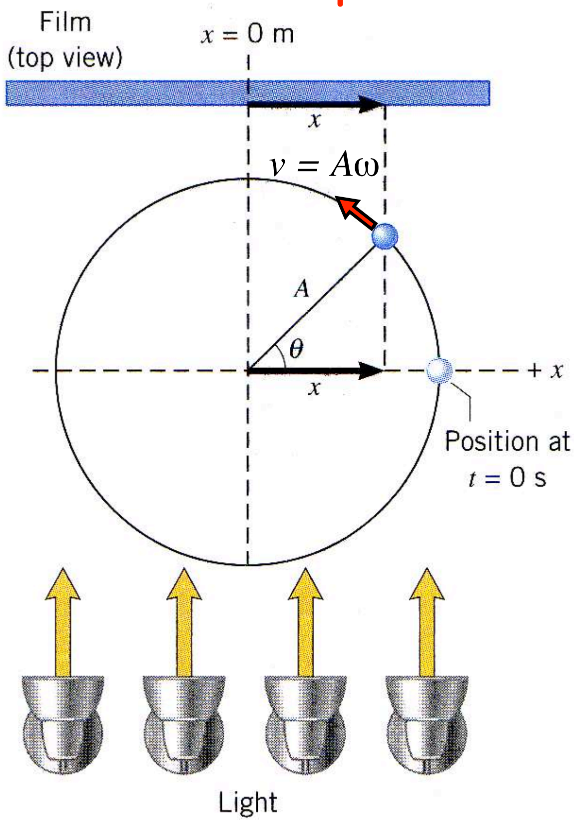
When t = T, the period,

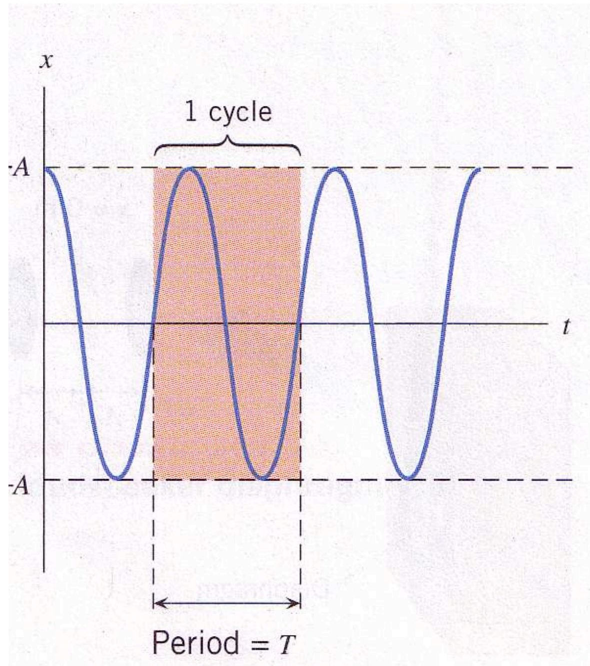
$\theta = \omega T = 2\pi$ radians, and then,

$\omega = 2\pi/T = 2\pi f$

T = period, f = 1/T = frequency (Hz)
$\omega = 2\pi f$

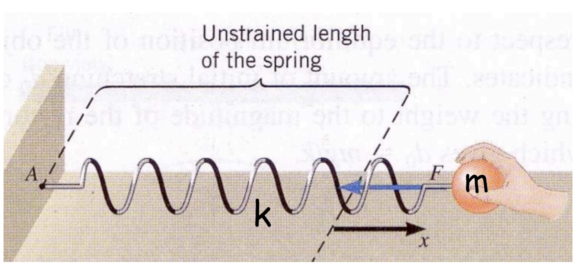
Simple Harmonic Motion



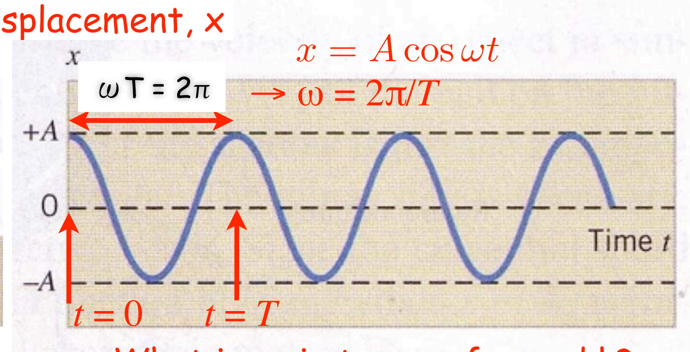


Sinusoidal motion – just like the mass on a spring

Mass on a spring

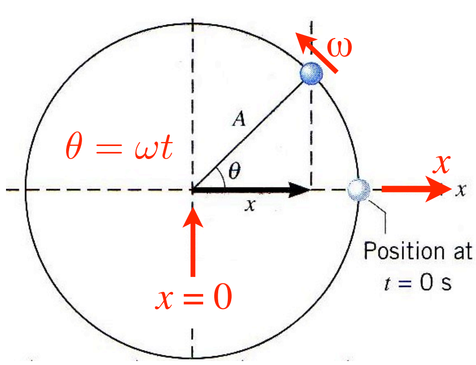


Displacement, x

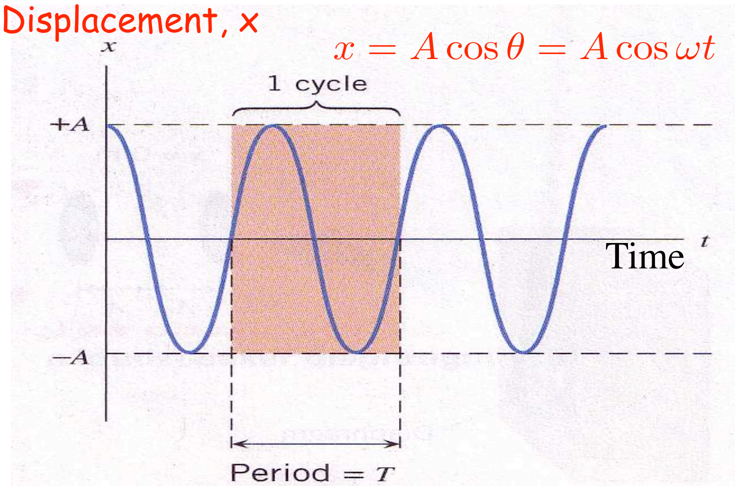


What is ω in terms of m and k?

Circular motion



Displacement, x



Simple Harmonic Motion

- Simple harmonic motion results when the restoring force is proportional to the displacement –

  F = –kx

- For the mass on the spring, how is the period of the harmonic motion related to the spring constant, k?

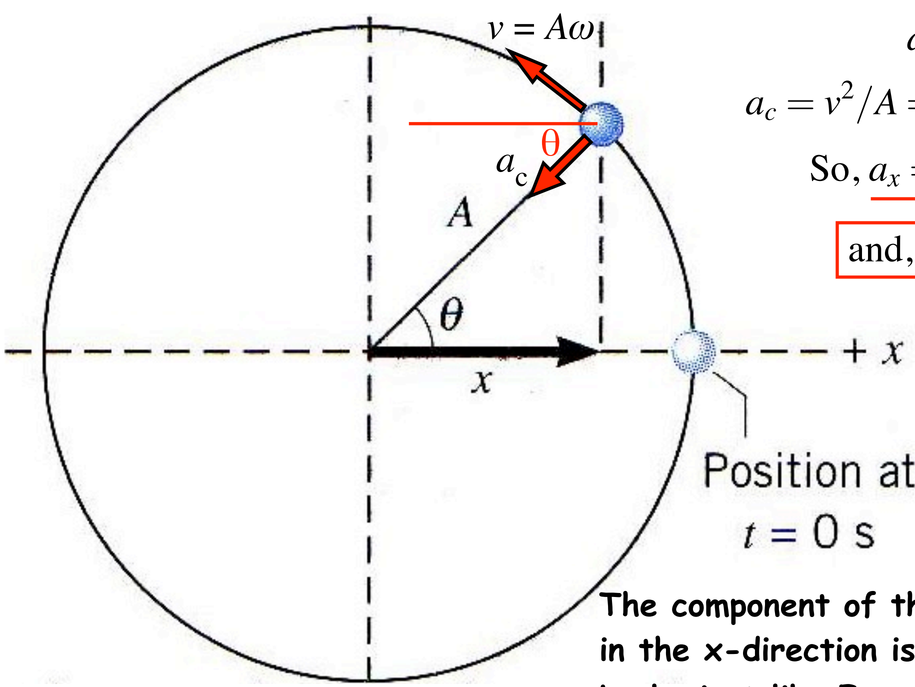
Clue:

- The motion of the mass on the spring looks just like the x component of the motion of the mass on the rotating disk.

Strategy – solve the easier problem:

- Look at the motion in a circle and find out what is the acceleration, a_x, as that can be related to a restoring force and an effective spring constant...

Simple Harmonic Motion
The "Reference Circle"



$$\underline{x = A\cos\theta}$$

$$a_x = -a_c\cos\theta$$

$$a_c = v^2/A = (A\omega)^2/A = A\omega^2$$

$$\text{So, } \underline{a_x = -A\omega^2\cos\theta}$$

$$\boxed{\text{and, } a_x = -\omega^2 x}$$

Restoring force proportional to displacement!

The component of the centripetal force in the x-direction is $F_x = ma_x = -m\omega^2 x$. Looks just like $F_x = -kx$ for the spring with $k = m\omega^2$!

Rotating disk: $a_x = -\omega^2 x$ — acceleration is proportional to displacement, x

Mass on a spring: the restoring force is: $F = -kx = ma_x$

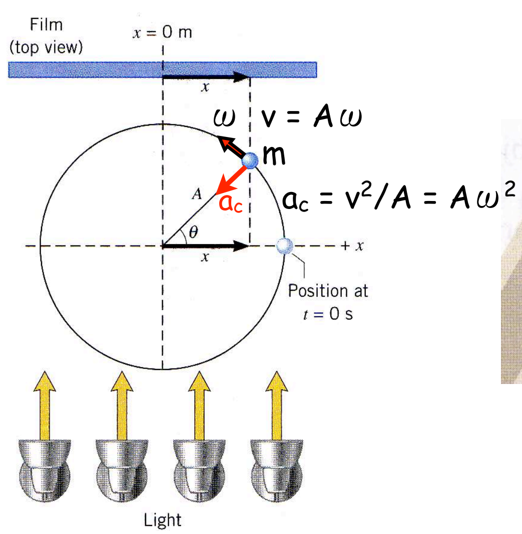
That is, $a_x = -(k/m)x$ — acceleration also proportional to displacement, x

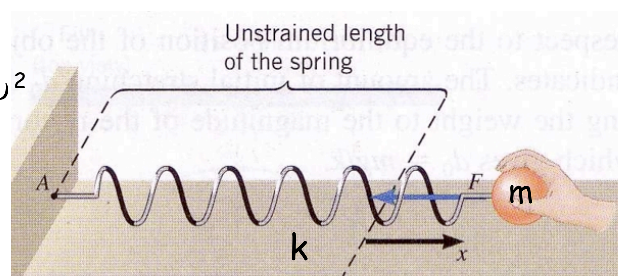
COMPARE: The mass on the spring **moves in the same way** as a mass on a disk that is rotating with angular frequency $\omega = [k/m]^{1/2}$.

Then: $\omega = 2\pi f = \dfrac{2\pi}{T}$

$$\boxed{\text{So, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{for the mass on the spring}}$$

Simple Harmonic Motion





Mass on a rotating disk
$F_x = -m\omega^2 x$

Mass on a spring
$F_x = -kx$

Simple harmonic motion in both cases - restoring force ∝ displacement

COMPARE: if $m\omega^2 = k$, then motions in x are **exactly the same,**

so $\omega^2 = k/m$ for the mass on the spring, and $x = A\cos(\omega t)$