This Week

Experiment 3: Forces in Equilibrium

WileyPLUS Assignment 3 is available

Chapters 6 & 7

Due Wednesday, November 11 at 11 pm

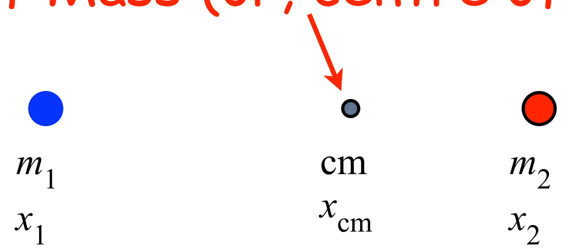
Next Week

Tutorial and Test 3

Chapters 6 & 7

See web page for tutorial questions

Centre of Mass (or, centre of gravity)



The centre of mass of the two objects is defined as:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(The mean position weighted by the masses)

If the masses are moving, the centre of mass moves too:

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

Motion of cm: $$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

The speed of the centre of mass is:

$$v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{p_{tot}}{m_1 + m_2} \qquad (p_{tot} = p_1 + p_2)$$

Or, $p_{tot} = (m_1 + m_2) v_{cm}$

If zero net force acts on the masses, the total momentum is constant and the speed of the centre of mass is constant also, **even after a collision**.

In two or three dimensions:

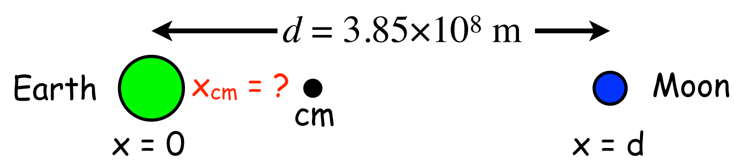
$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \text{ velocity of centre of mass}$$

$$\vec{p}_{tot} = (m_1 + m_2) \vec{v}_{cm}$$

7.-/41: The earth and the moon are separated by a centre-to-centre distance of 3.85×10^8 m. If:

$M_{earth} = 5.98 \times 10^{24}$ kg, $M_{moon} = 7.35 \times 10^{22}$ kg,

how far does the centre of mass lie from the centre of the earth?



Measuring distances from the centre of the earth:

$$x_{cm} = \frac{M_{earth} x_{earth} + M_{moon} x_{moon}}{M_{earth} + M_{moon}} = \frac{M_{earth} \times 0 + M_{moon} \times d}{M_{earth} + M_{moon}}$$

$$= \frac{7.35 \times 10^{22} \times 3.85 \times 10^8}{5.98 \times 10^{24} + 7.35 \times 10^{22}} = 4,675,000 \text{ m}$$

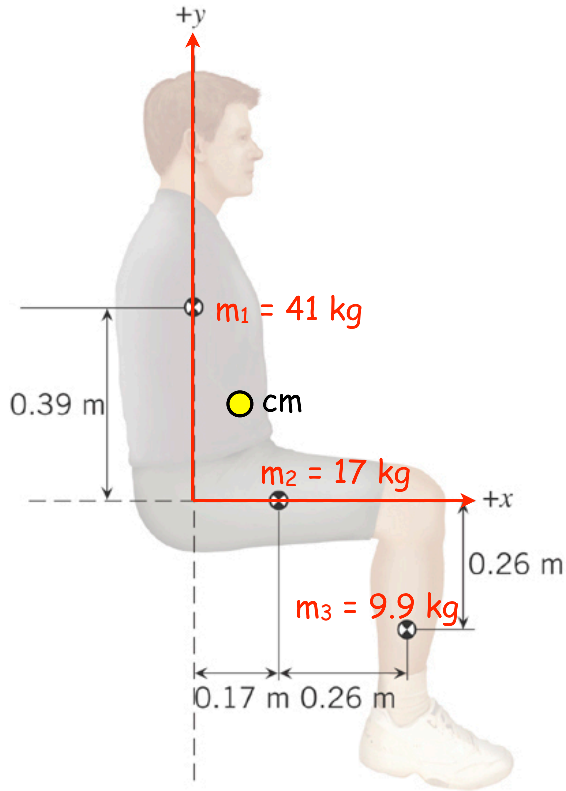
Note, the radius of the earth is 6,380 km, so the cm is inside the earth.

7.64/44/-: The figure shows the centres of mass of:

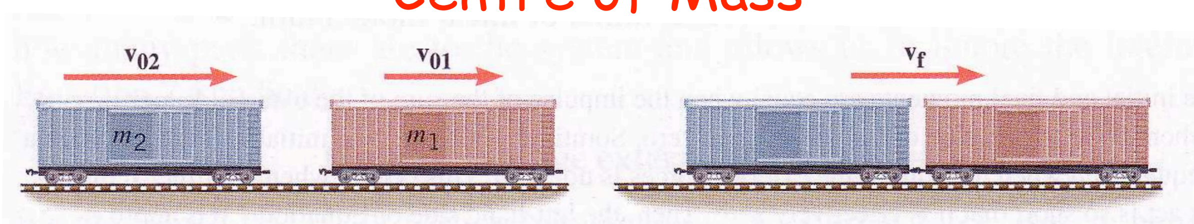
1) torso, neck and head, mass m_1 = 41 kg,
2) upper legs, mass m_2 = 17 kg,
3) lower legs and feet, mass m_3 = 9.9 kg.

Find the position of the centre of mass of the seated person.

(NB minor appendages such as arms and hands have been left out for simplicity).



Centre of Mass



(a) Before (b) After

Velocity of centre of mass before collision:

$$v_{cm} = \frac{m_1 v_{01} + m_2 v_{02}}{m_1 + m_2}$$

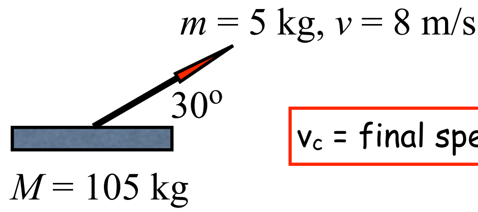
Velocity of centre of mass after collision:

$$v'_{cm} = v_f$$ (the railcars are moving as a unit)

Conservation of momentum:

$$m_1 v_{01} + m_2 v_{02} = (m_1 + m_2) v_f \rightarrow v'_{cm} = v_{cm}$$ (centre of mass continues moving at same speed)

7.-/54: A person stands in a stationary canoe and throws a 5 kg stone at 8 m/s at 30^0 above the horizontal. What is the recoil velocity of the canoe?



v_c = final speed of canoe

Conservation of momentum: canoe initially at rest, so momentum = 0
Or, centre of mass is at rest and remains so.

Summary

Momentum

$$\vec{p} = m\vec{v}$$

Impulse and Momentum – Newton's 2nd law in another form

$$\text{Impulse: } \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$$

Conservation of Momentum

$$\text{In the absence of applied forces: } \vec{p}_1 + \vec{p}_2 = \vec{p}_1{}' + \vec{p}_2{}'$$

Centre of Mass

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

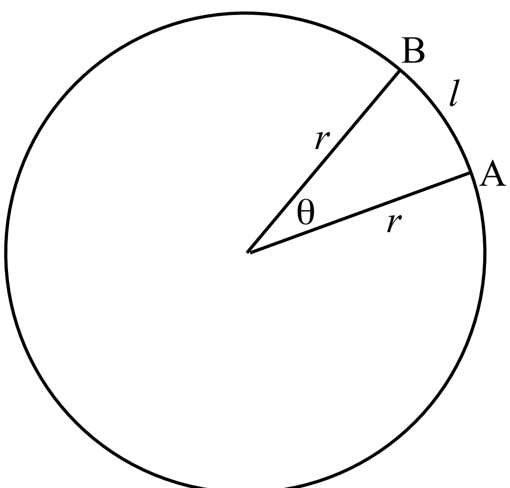
continues to move at constant velocity if no applied forces

Chapter 8, Rotational Kinematics

Sections 1 – 3 only

- Rotational motion and angular displacement
- Angular velocity and angular acceleration
- Equations of rotational kinematics

Angular Displacement



The length of the arc of the circle of radius r from A to B is:

$$l = r\theta$$

where θ is in radians.

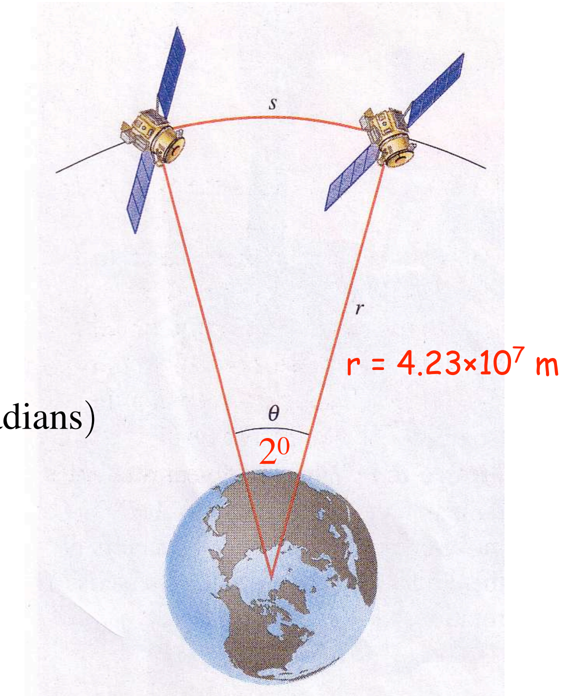
$$\theta\ (\text{radians}) = \frac{\text{arc length}}{\text{radius}} = \frac{l}{r}$$

One complete revolution corresponds to:

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians} \equiv 360^\circ$$

π radians = 180^0 **Radians = (degrees) x π/180**

Two geostationary satellites are at an altitude of 4.23×10^7 m. Their angular separation θ is 2^0.

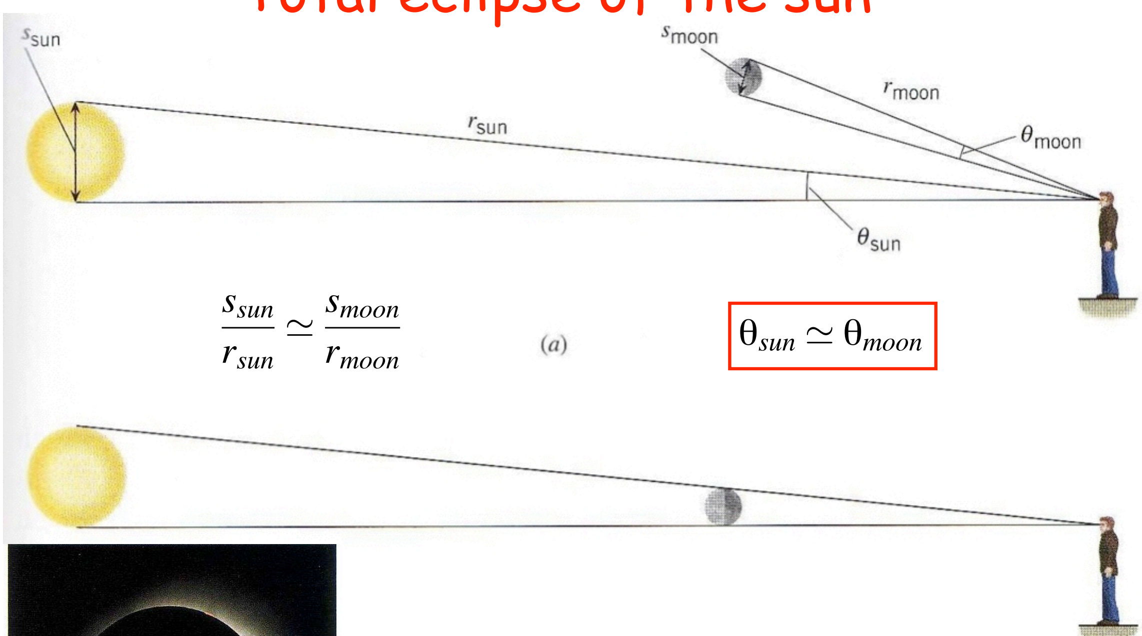


Find the arc length s.

$$s = r\,\theta = (4.23\times10^7\ \text{m})\times(\overset{2^o}{\downarrow}\!2\pi/180\ \text{radians})$$

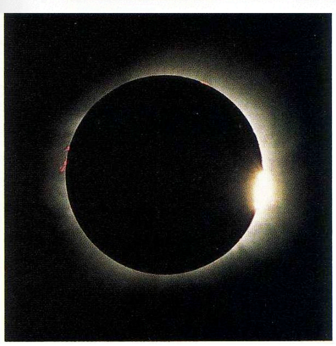
$= 1.48\times10^6$ m

Total eclipse of the sun



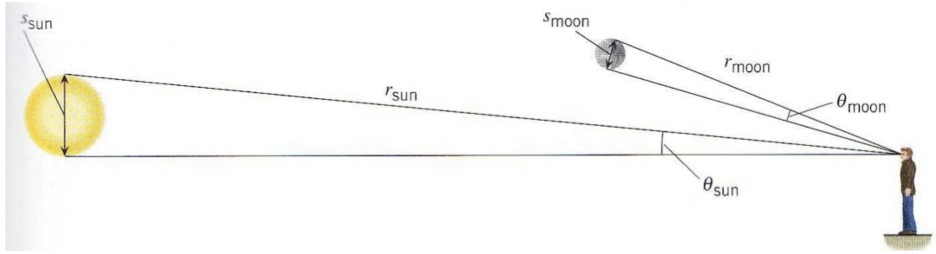
$$\frac{s_{sun}}{r_{sun}} \simeq \frac{s_{moon}}{r_{moon}}$$

$$\boxed{\theta_{sun} \simeq \theta_{moon}}$$



The sun and moon have almost exactly the same angular size as seen from the earth even though the moon is much smaller than the sun.

8.18/68



Moon: diameter = 3.48×10^6 m,
distance from earth = 3.85×10^8 m

Sun: diameter = 1.39×10^9 m,
distance from earth = 1.50×10^{11} m

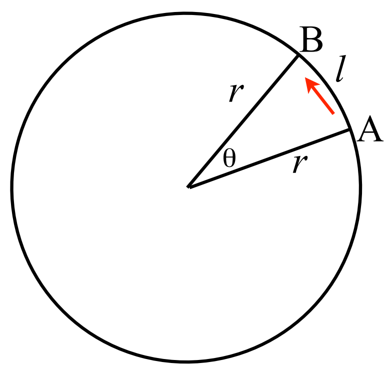
a) Angular size of moon = $(3.48 \times 10^6 \text{ m})/(3.85 \times 10^8 \text{ m})$ = 0.00904 rad

Angular size of sun = $(1.39 \times 10^9 \text{ m})/(1.50 \times 10^{11} \text{ m})$ = 0.00927 rad

b) The moon has slightly smaller angular size than the sun.

c) Apparent area of moon = $(0.00904/0.00927)^2$ × apparent area of sun
= 95.1% of apparent area of sun.

Angular velocity – motion around a circular path



Angular velocity is:

$$\bar{\omega} = \frac{\theta_B - \theta_A}{t_B - t_A} \text{ rad/s}$$ (average)

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ rad/s}$$ (instantaneous)

The instantaneous angular velocity is measured over a vanishingly small time interval.

If the motion from A to B takes time t, then

$$v = \frac{l}{t} = \frac{r\theta}{t} = r\omega$$

That is, $v = r\omega$

Angular acceleration

Angular acceleration is the rate of change of angular velocity:

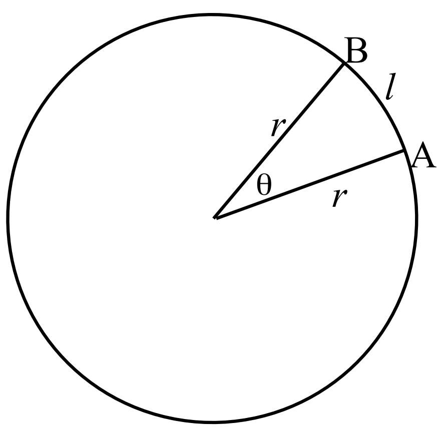
Average angular acceleration $\bar{\alpha} = \dfrac{\omega - \omega_0}{t - t_0} = \dfrac{\Delta\omega}{\Delta t}$

Instantaneous angular acceleration $\alpha = \lim_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t}$

Compare with linear acceleration:

$$\bar{a} = \frac{v - v_0}{t - t_0} = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$



Angular Displacement

The length of the arc of the circle of radius r from A to B is:

$l = r\theta$, where θ is in radians

π radians = 180^0

Angular Velocity

$\bar{\omega} = \dfrac{\theta_B - \theta_A}{t_B - t_A}$ rad/s (average)

$\omega = \dfrac{\Delta\theta}{\Delta t}$ rad/s (instantaneous)

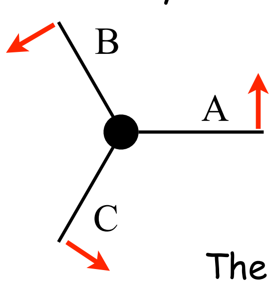
And, $v = r\omega$

Equations for rotational motion

Analogous to the equations for linear motion.

$\omega = \omega_0 + \alpha t$		$v = v_0 + at$
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	$\omega \Leftrightarrow v$	$x = \frac{1}{2}(v_0 + v)t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$\alpha \Leftrightarrow a$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$\theta \Leftrightarrow x$	$v^2 = v_0^2 + 2ax$

8.17/12: A stroboscope is a light that flashes on and off at a constant rate to illuminate a rotating object. If the flashing rate is adjusted properly, the rotating object appears to be stationary.



A three blade propeller is rotating at 16.7 revolutions per second.

What should be the shortest time between flashes to make the fan appear to be stationary?

The propeller appears to be stationary if it rotates from A to B between flashes.

This is 1/3 of a revolution, which takes the propeller a time:

$$t = \frac{1}{3} \times \frac{1}{16.7} = 0.020 \text{ seconds}$$

For the next shortest time, the propeller rotates from A to C in 0.040 s.

The angular speed of the rotor in a centrifuge increases from 420 to 1420 rad/s in 5 s.

a) Through what angle does the rotor move in this time?

Average angular speed = (420 + 1420)/2 = 920 rad/s.

In 5 s, rotor turns through 920 × 5 = 4600 radians.

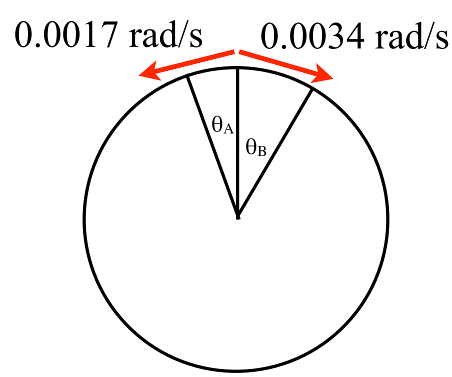
$$\text{or, } \frac{4600}{2\pi} = 732.1 \text{ revolutions}$$

b) What is the angular acceleration?

$$\omega = \omega_0 + \alpha t \qquad \text{(think of } v = v_0 + at\text{)}$$

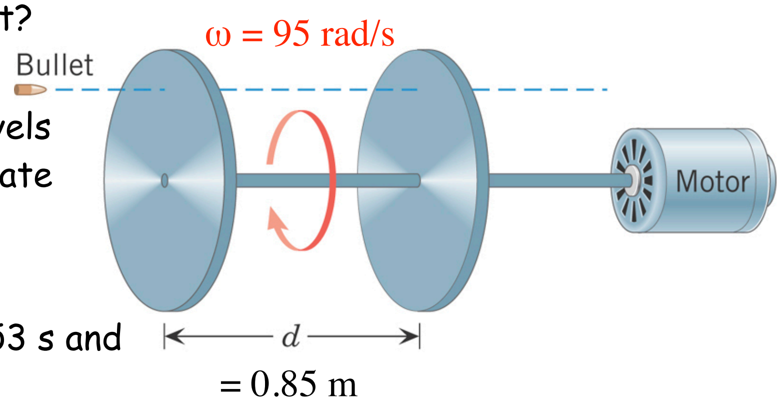
$$\text{So, } \alpha = \frac{(1420 - 420)\ \text{rad/s}}{5\ \text{s}} = 200\ \text{rad/s}^2$$

8.13/10: Two people start at the same place and walk around a circular lake in opposite directions. One has angular speed 0.0017 rad/s, the other 0.0034 rad/s. How long before they meet?



8.16/13: The speed of a bullet can be measured with the apparatus shown. The bullet passes through two disks that are rotating together. The disks rotate as the bullet travels from one disk to the other, so the holes in the disks do not line up.

The angular displacement between the bullet holes is 0.24 rad. What is the speed of the bullet?



In time t, the bullet travels 0.85 m and the disks rotate 0.24 rad at 95 rad/s.

$(\theta = \omega t)$

So, t = 0.24/95 = 0.00253 s and

$(v = d/t)$

v = (0.85 m)/(0.00253 s) = 336 m/s

8.25/20: The wheels of a bicycle have an angular velocity of 20 rad/s. The brakes are applied, bringing the bicycle to a uniform stop. During braking, the angular displacement of the wheels is 15.92 revolutions.

a) How much time does it take to stop?
b) What is the angular acceleration of the wheels?

Equations for rotational motion

Analogous to the equations for linear motion.

$\omega = \omega_0 + \alpha t$		$v = v_0 + at$
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	$\omega \Leftrightarrow v$	$x = \frac{1}{2}(v_0 + v)t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$\alpha \Leftrightarrow a$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$\theta \Leftrightarrow x$	$v^2 = v_0^2 + 2ax$

arc length, $l = r\,\theta$

speed, $v = r\omega$

π radians = 180°