This Week

Experiment 3: Forces in Equilibrium

PHYS 1020 Final Exam

Friday, December 18

1:30 - 4:30

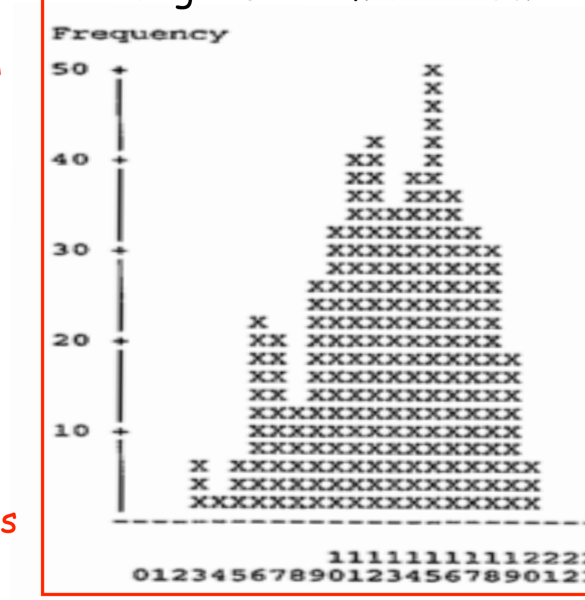
30 multiple choice questions

Formula sheet provided

If you wrote paper B (yellow) and entered answers in the left columns (paper A), let me know!

Midterm

Average: 64%: marks on web



Impulse and Momentum

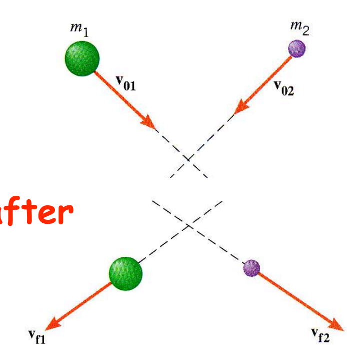
Impulse = FΔt = Δp = change in momentum

Momentum is constant if there is zero net applied force

Alternative statement of Newton's second law: $\vec{F} = \dfrac{\Delta \vec{p}}{\Delta t}$

For a system of 2 or more colliding objects, momentum is conserved if there is zero net applied force:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1{}' + \vec{p}_2{}'$$



Total momentum before = total momentum after

Clickers!

A canoe with two people aboard is coasting with an initial momentum of +110 kg.m/s. Then, one of the people (person 1) dives off the back of the canoe. During this time, the net average external force acting on the system of canoe and two people is zero.

The table lists four possibilities for the final momentum of person 1 and the final momentum of the canoe with person 2, immediately after person 1 leaves the canoe. Only one possibility could be correct. Which is it?

	Person 1	Person 2 and canoe
A)	-60 kg.m/s	+170 kg.m/s
B)	-30 kg.m/s	+110 kg.m/s
C)	-40 kg.m/s	-70 kg.m/s
D)	+80 kg.m/s	-30 kg.m/s

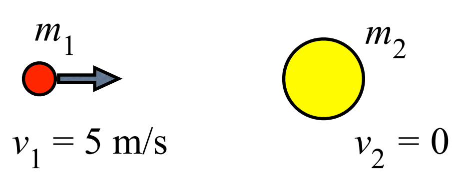
Momentum is conserved!

A) Total momentum = 110 kg.m/s

• **Elastic collision**: the total kinetic energy after collision is equal to the total before collision.

• **Inelastic collision**: the total kinetic energy is not conserved. If objects stick together after collision, the collision is "perfectly inelastic" – there is no bounce.

Example: A ball of mass m_1 = 0.25 kg makes a perfectly elastic collision with a ball of mass m_2 = 0.8 kg.



Initial momentum = $m_1 v_1 + 0$

Momentum after impact = $m_1 v_{f1} + m_2 v_{f2}$

$\boxed{\text{Momentum is conserved: } m_1 v_1 = m_1 v_{f1} + m_2 v_{f2}}$

$$\boxed{\text{Momentum conserved: } m_1 v_1 = m_1 v_{f1} + m_2 v_{f2}}$$

So, $v_{f2} = \dfrac{m_1 v_1 - m_1 v_{f1}}{m_2}$

substitute for v_{f2}

The collision is elastic, so:

KE: $$\frac{m_1 v_1^2}{2} + 0 = \frac{m_1 v_{f1}^2}{2} + \frac{m_2 v_{f2}^2}{2}$$

Solution, after some algebra, is:

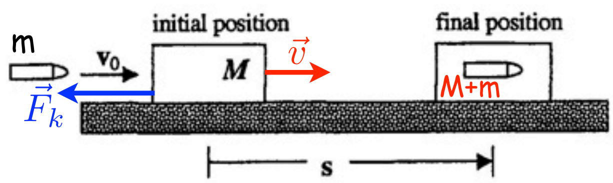
$$v_{f1} = v_1 \left[\frac{m_1 - m_2}{m_1 + m_2}\right]$$

$$v_{f2} = v_1 \left[\frac{2m_1}{m_1 + m_2}\right]$$

If $m_1 = m_2$, then $v_{f1} = 0$, $v_{f2} = v_1$

If $m_1 = 0.25$ kg, $m_2 = 0.8$ kg, then $v_{f1} = -2.62$ m/s, $v_{f2} = 2.38$ m/s

Dec 2003 Final, Q26: A bullet of mass m is fired with speed v_0 into a block of wood of mass M. The bullet comes to rest in the block. The block with the bullet inside slides along a horizontal surface with coefficient of kinetic friction μ_k. How far does the block slide before it comes to rest?



For the impact -

bullet

Conservation of momentum: $mv_0 = (m + M)v$ right after impact

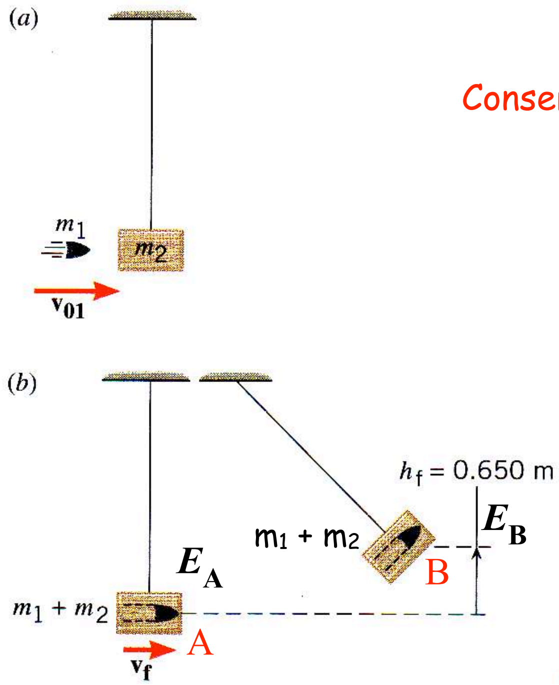
So, speed of block + bullet immediately after impact is: $v = \dfrac{mv_0}{m+M}$

Work-energy theorem, block coming to rest: $W_{nc} = F_k s = \Delta KE + \Delta PE$

That is: $-\mu_k (m+M) g \times s = -(m+M)v^2/2 + 0$

So, $$s = \frac{v^2/2}{\mu_k g} = \frac{1}{2\mu_k g}\left[\frac{mv_0}{m+M}\right]^2$$

Ballistic pendulum
Measure the speed of a bullet



Conservation of momentum for initial impact:

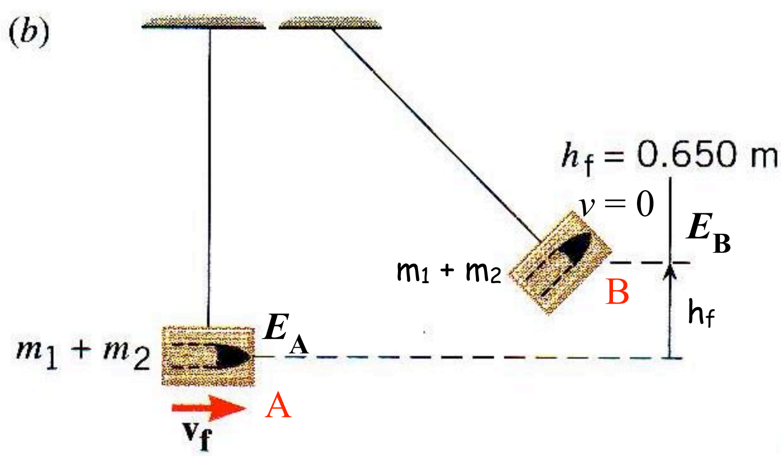
$$\boxed{m_1 v_{01} = (m_1 + m_2) v_f}$$

The collision is inelastic – the bullet drills into the block, generating a lot of heat.

BUT, after the collision, the block + bullet swing up to a highest point and conserve mechanical energy.

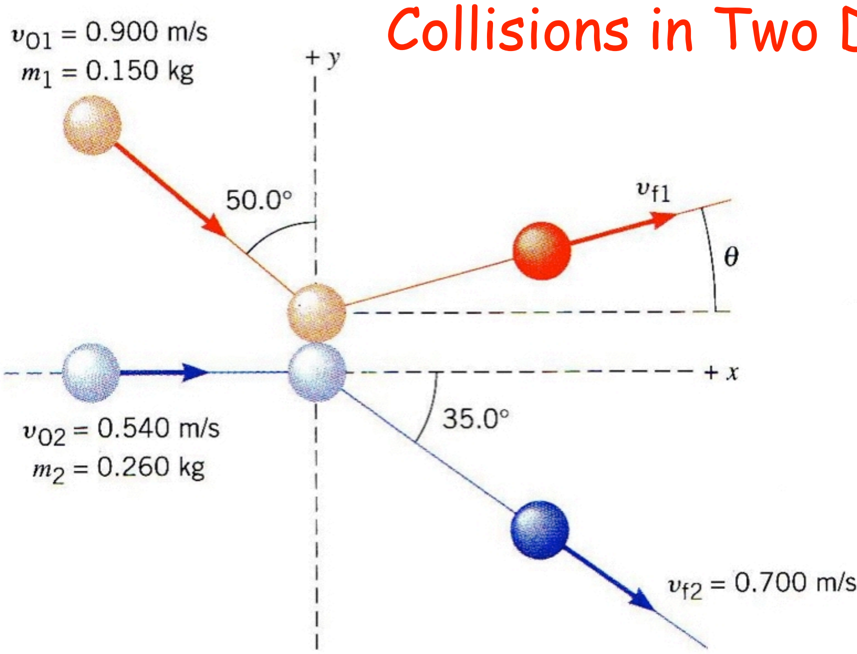
$$E_A(\textbf{after impact}) = E_B$$

$$\boxed{m_1 v_{01} = (m_1 + m_2) v_f \;\rightarrow\; v_f = \frac{m_1 v_{01}}{(m_1 + m_2)}}$$



$\boxed{E_A = E_B}$

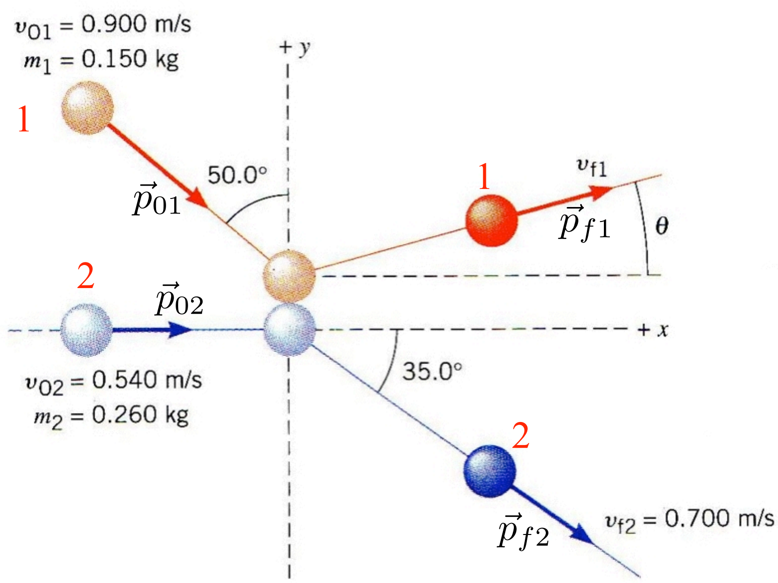
Collisions in Two Dimensions



Conservation of momentum: $\vec{p}_0 = \vec{p}_f$

x and y components: $p_{0x} = p_{fx}$

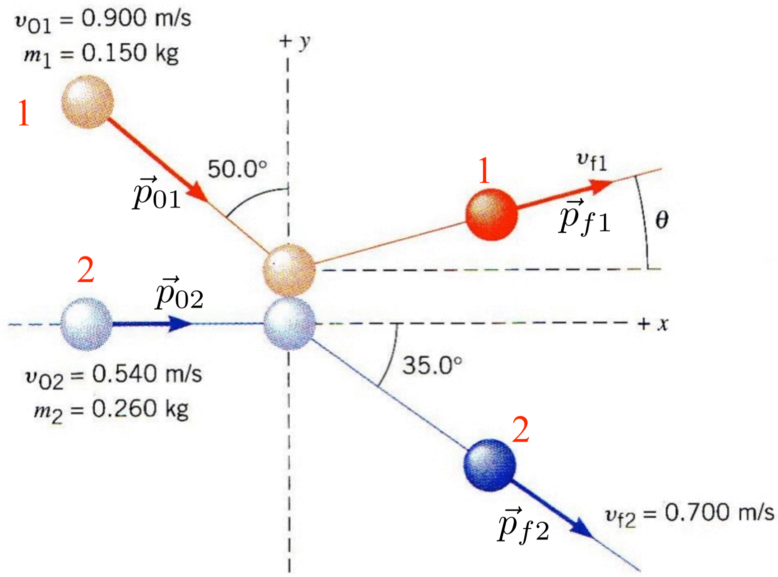
$$p_{0y} = p_{fy}$$



x: $p_{01} \sin 50^\circ + p_{02} = p_{f1x} + p_{2f} \cos 35^\circ$

That is, $p_{f1x} = 0.9 \times 0.15 \sin 50^\circ + 0.26 \times 0.54 - 0.26 \times 0.7 \cos 35^\circ$

$$\boxed{p_{f1x} = 0.0947 \text{ kg.m/s}}$$

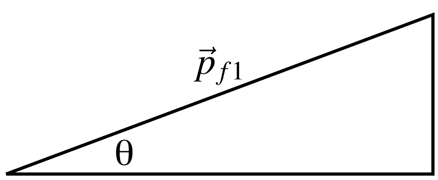


y: $-p_{01}\cos 50^\circ + 0 = p_{f1y} - p_{f2}\sin 35^\circ$

So, $p_{f1y} = p_{f2}\sin 35^\circ - p_{01}\cos 50^\circ$

$$= 0.26 \times 0.7 \sin 35 - 0.15 \times 0.9 \cos 50^\circ$$

$$\boxed{p_{f1y} = 0.0176 \text{ kg.m/s}}$$



$p_{f1y} = 0.0176$ kg.m/s

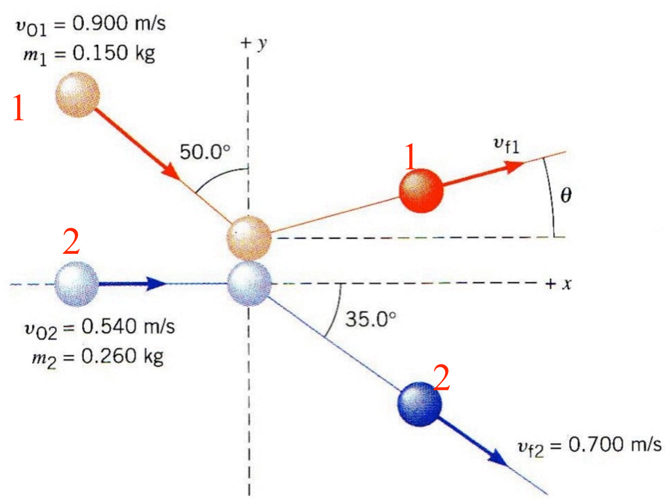
$p_{f1x} = 0.0947$ kg.m/s

$$p_{f1} = \sqrt{0.0947^2 + 0.0176^2} = 0.0963 \text{ kg.m/s}$$

$$v_{f1} = p_{f1}/m_1 = 0.64 \text{ m/s}$$

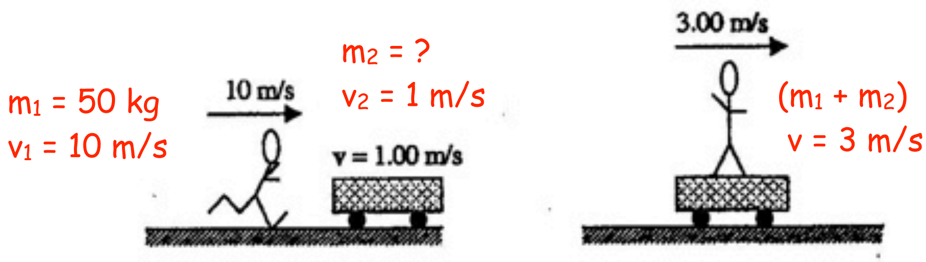
$$\tan\theta = \frac{0.0176}{0.0947} = 0.1859$$

$$\theta = 10.5^\circ$$



Dec 2003 Final, Q25. A boy who weighs 50 kg runs at a speed of 10 m/s and jumps onto a cart, as shown. What is the mass of the cart?

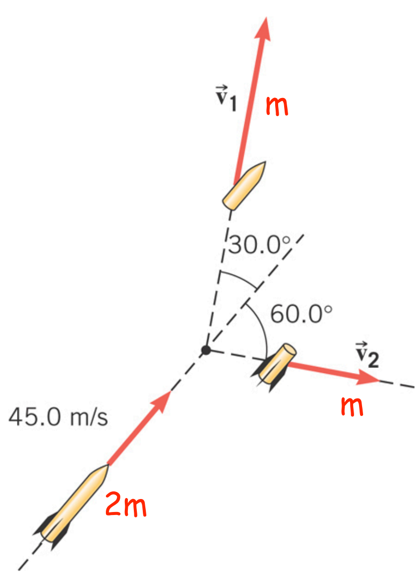
(a) 175 kg (b) 350 kg (c) 117 kg (d) 88 kg (e) 163 kg



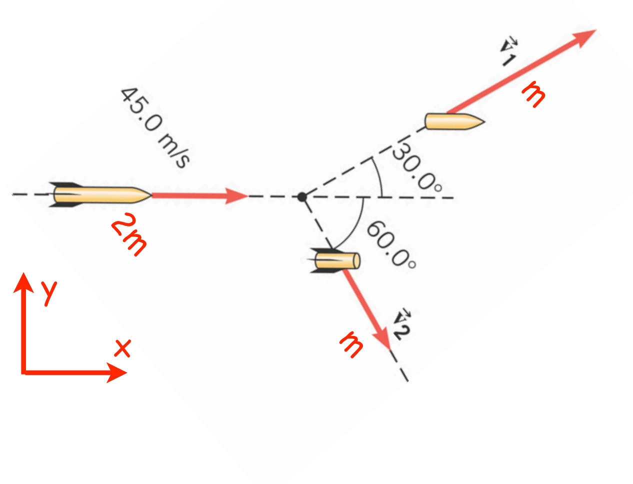
Conservation of momentum: $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

Solve for m_2: $m_2 = \dfrac{m_1(v_1 - v)}{v - v_2} = \dfrac{50(10-3)}{3-1} = 175\ \text{kg}$

7.22/19: A fireworks rocket breaks into two pieces of equal mass. Find the velocities of the pieces.

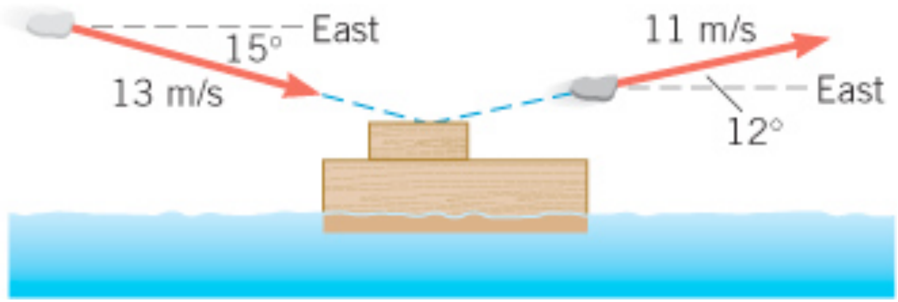


Rotate:

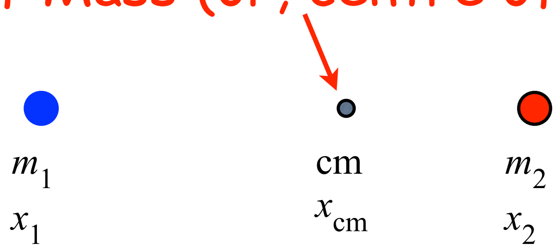


Prob. 7.40/-

A girl is skipping stones across a lake. One of the stones accidentally ricochets off a toy boat that is initially at rest in the water (see the drawing). The 0.072-kg stone strikes the boat at a velocity of 13 m/s, 15° below due east, and ricochets off at a velocity of 11 m/s, 12° above due east. After being struck by the stone, the boat's velocity is 2.1 m/s, due east. What is the mass of the boat? Assume the water offers no resistance to the boat's motion.



Centre of Mass (or, centre of gravity)



The centre of mass of the two objects is defined as:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(The mean position weighted by the masses)

If the masses are moving, the centre of mass moves too:

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

Motion of cm: $$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

The speed of the centre of mass is:

$$v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{p_{tot}}{m_1 + m_2} \qquad (p_{tot} = p_1 + p_2)$$

Or, $p_{tot} = (m_1 + m_2) v_{cm}$

If zero net force acts on the masses, the total momentum is constant and the speed of the centre of mass is constant also, **even after a collision**.

In two or three dimensions:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \text{ velocity of centre of mass}$$

$$\vec{p}_{tot} = (m_1 + m_2) \vec{v}_{cm}$$