Next Week
Experiment 2:
Measurement of g by Free Fall

WileyPLUS Assignment 2

Due Monday, October 19 at 11:00 pm

Chapters 4 & 5

PHYS 1020 Midterm

Thursday, October 22

7 - 9 pm

Chapters 1 - 5

20 multiple choice questions

Things Centripetal

or, flying off in all directions

Acceleration toward centre of a circular path of radius r:

$$a_c = v^2/r$$

Force needed to maintain the centripetal acceleration = centripetal force:

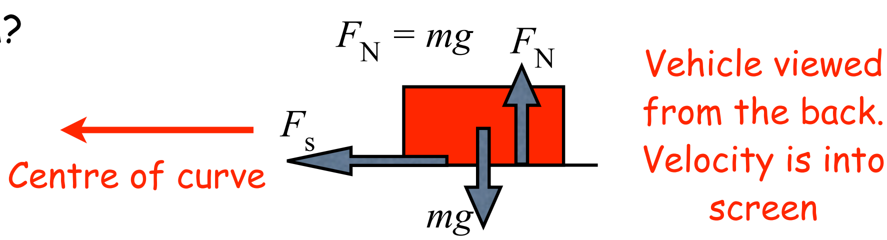
$$F_c = ma_c = mv^2/r$$

Force provided by tension in a string, friction, horizontal component of airplane's lift, gravity...

NB **centrifugal** force is the "force" you feel toward the outside of a curve when going around a corner. It's not really a force, but a consequence of Newton's first law that says things travel at constant velocity (so, in a straight line) unless a force is applied.

Clicker Question

Other things being equal, would it be easier to drive at high speed (no skidding) around an unbanked horizontal curve on the moon than to drive around the same curve on the earth?



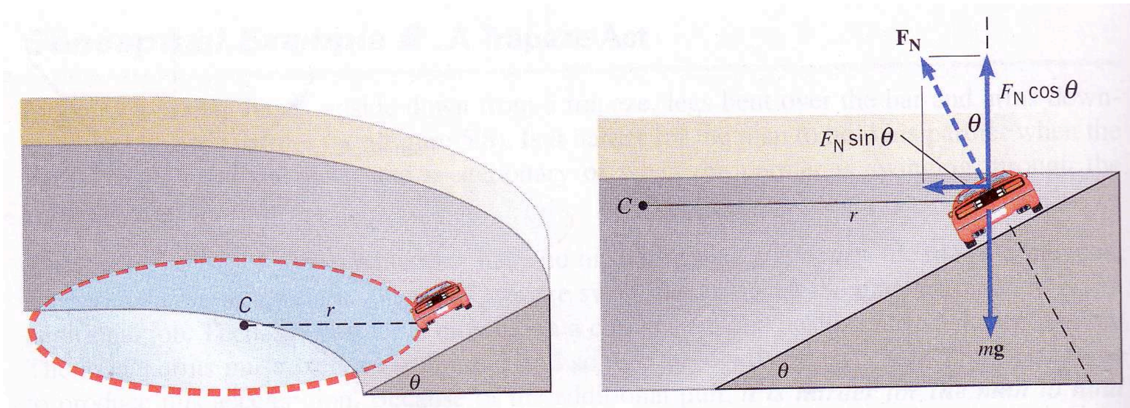
A) Yes, easier on the moon, B) No, more difficult on moon

C) Just the same D) Who knows?

The centripetal acceleration is provided by friction.
The friction force is proportional to the normal force.
The normal force is equal to the weight of the car.
On the moon, the acceleration due to gravity, g_{moon}, is ≈ g/6.

B) More difficult on the moon.

Driving around in circles – banked road



No friction!!

As for plane but with lift force replaced by normal force:

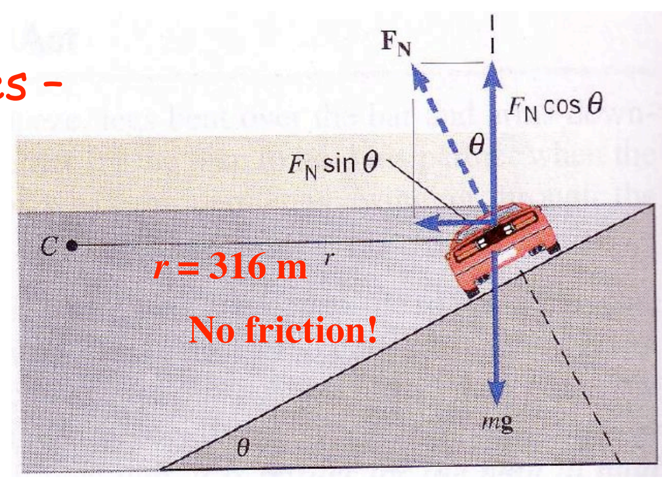
$$F_N \sin\theta = \frac{mv^2}{r}$$

$$F_N \cos\theta = mg$$

$$\rightarrow \tan\theta = \frac{mv^2}{r} \times \frac{1}{mg} = \frac{v^2}{rg}$$

$\rightarrow$ best angle of banking (same as for plane)

Driving around in circles – banked curve



If you drive slowly, you slide **down** the slope.

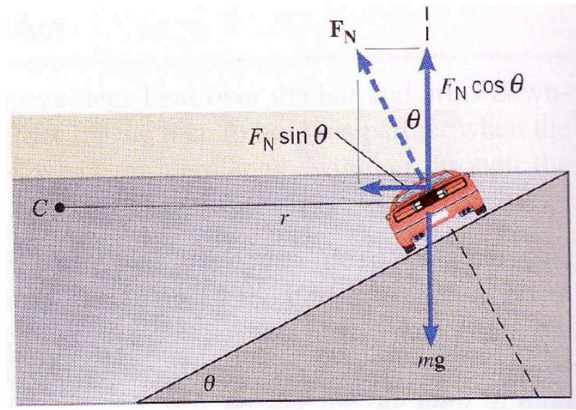
If you drive fast, you skid **up** the slope.

If θ = 31^0 and r = 316 m, and there is no friction, what is the best speed to drive around the banked curve?

$$\tan\theta = \frac{v^2}{rg}, \text{ so } v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{316 \times 9.8 \tan 31^\circ} = 43.1 \text{ m/s} \; = 155 \text{ km/h}$$

5.26/20: Two banked curves have the same radius. Curve A is banked at 13^0, curve B at 19^0. A car can travel around curve A without relying on friction at a speed of 18 m/s. At what speed can this car travel around curve B without relying on friction?



From previous page: $v = \sqrt{rg\tan\theta}$

That is, $v \propto \sqrt{\tan\theta}$

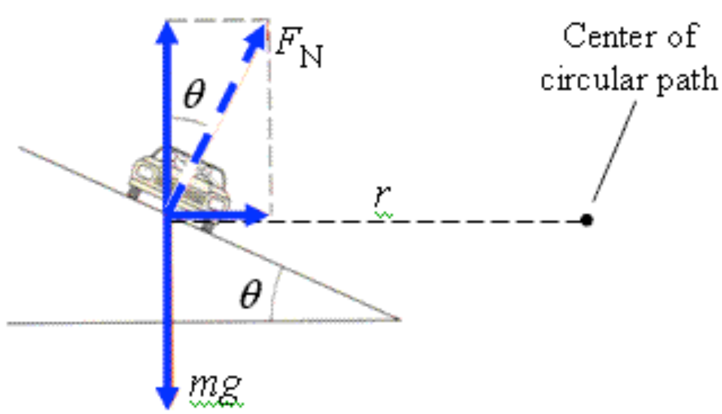
So $\frac{v_A}{v_B} = \sqrt{\frac{\tan\theta_A}{\tan\theta_B}} = \sqrt{\frac{\tan 13^\circ}{\tan 19^\circ}} = 0.819$

Therefore, $v_B = v_A/0.819 = 22$ m/s

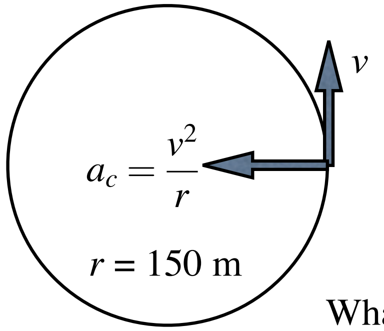
Clicker Question: Focus on Concepts, Question 9

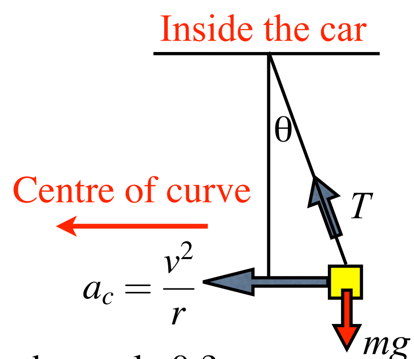
A car is rounding a circular, frictionless curve of radius r on a banked turn. As the drawing indicates, there are two forces acting on the car, its weight mg and the normal force F_N exerted on it by the road. Which force, or force component, provides the centripetal force that keeps the car moving on the circular path?

A) The component $F_N \cos\theta$ of the normal force.
B) The component $F_N \sin\theta$ of the normal force.
C) Both the normal force, F_N, and the weight, mg, of the car.
D) The normal force, F_N.
E) The weight, mg, of the car.



5.21/18: A car travels at 28 m/s around a curve of radius 150 m. A mass is suspended from a string from inside the roof.





What is the angle θ ?

Force toward centre of circular path due to tension in the string:

$$\frac{mv^2}{r} = T\sin\theta \qquad \rightarrow \tan\theta = \frac{v^2}{rg}$$

Forces in the vertical direction: $mg = T\cos\theta$

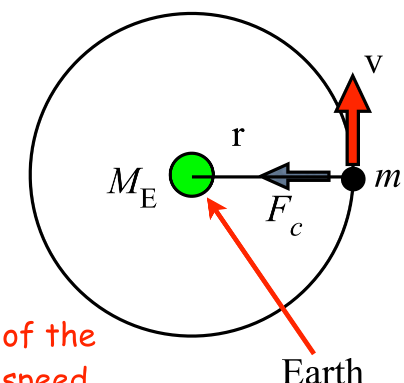
$$\tan\theta = \frac{28^2}{150g} = 0.5333$$

$$\theta = 28.1^\circ$$

Orbiting the Earth

"The secret to flying is to throw yourself at the earth and miss."
Hitch Hiker's Guide to the Galaxy

The centripetal force on the satellite is provided by the gravitational force from the earth.



$$F_c = \frac{mv^2}{r} = \frac{GM_E m}{r^2}$$

$$\text{So } v = \sqrt{\frac{GM_E}{r}}$$

the smaller the radius of the orbit, the greater the speed

Synchronous orbit: period = 24 hours

- satellite stays above same part of the earth (above the equator)
- used by communications satellites
- what is the radius of the orbit?

Synchronous Orbit - what is its radius?

The period of an orbit is: $T = \dfrac{\text{circumference of orbit}}{\text{speed of satellite}} = \dfrac{2\pi r}{v}$

What is v?

From previous page, $v = \sqrt{\dfrac{GM_E}{r}}$

So $T = 2\pi r \times \sqrt{\dfrac{r}{GM_E}} = 2\pi \times r^{3/2}\sqrt{\dfrac{1}{GM_E}}$

$$r^3 = GM_E\left(\frac{T}{2\pi}\right)^2$$

(Kepler's 3rd law of planetary motion: $T^2 \propto r^3$)

With $T = 24 \times 3600$ s, $r = 4.23 \times 10^7$ m $= 42,300$ km from centre of earth

The speed of the satellite is: $v = \dfrac{2\pi r}{T} = 3070$ m/s $= 11,000$ km/h

5.-/32: The earth orbits the sun once per year at a distance of 1.5×10^{11} m.

Venus orbits the sun at a distance of 1.08×10^{11} m.

What is the length of the year on Venus?

Kepler: $T^2 \propto R^3$,

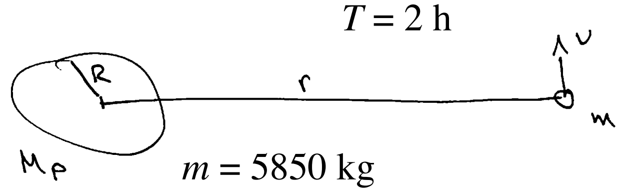
so, $\left(\dfrac{T_V}{T_E}\right)^2 = \left(\dfrac{R_V}{R_E}\right)^3 = \left(\dfrac{1.08}{1.5}\right)^3 = 0.373$

$$T_V = 0.611 T_E$$

The length of the year on Venus is 0.611 Earth years.

5.-/33: A satellite has a mass of 5850 kg and is in a circular orbit 4.1×10^5 m above the surface of a planet. The period of the orbit is 2 hours. The radius of the planet is 4.15×10^6 m. What is the weight of the satellite when it is at rest on the planet's surface?

$T = 2$ h



What is g at $r = R$?
Need the mass M_p of the planet.

$m = 5850$ kg
$R = 4.15 \times 10^6$ m
$r = 4.1 \times 10^5 + R = 4.56 \times 10^6$ m

The speed of the satellite in orbit is $v = 2\pi r/T$

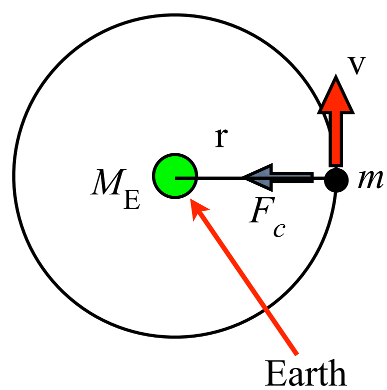
$$\text{Centripetal force, } F_c = \boxed{\frac{mv^2}{r} = \frac{GmM_p}{r^2}} \rightarrow GM_p = v^2 r = \left[\frac{2\pi}{T}\right]^2 r^3$$

$$\text{Then, weight on planet's surface, } mg_p = \frac{GmM_p}{R^2} = \frac{m}{R^2}\left[\frac{2\pi}{T}\right]^2 r^3$$

$$\text{Weight} = \frac{5850}{(4.15 \times 10^6)^2}\left[\frac{2\pi}{2 \times 3600}\right]^2 \times (4.56 \times 10^6)^3 = 2.45 \times 10^4 \text{ N}$$

($g_p = 4.2$ m/s^2)

Free Fall, Weightlessness



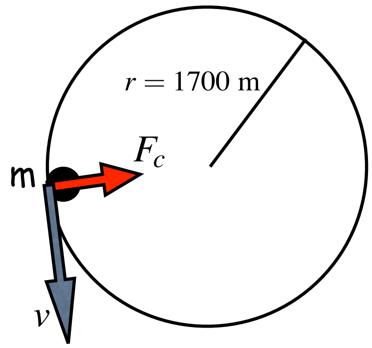
An orbiting satellite is in free fall – there's nothing to hold it up.

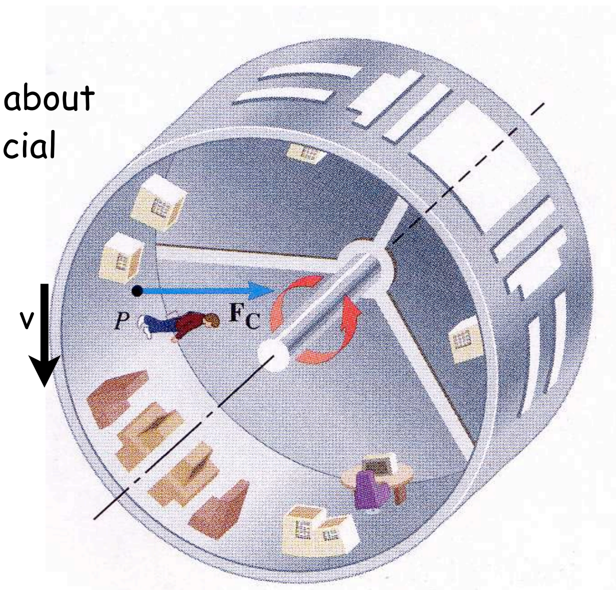
Everything in the satellite is accelerated toward the centre of the earth at the same rate.

An object exerts no force on the bathroom scales as the scales are also being accelerated toward the centre of the earth at the same rate as the object.

Artificial Gravity

A space station is rotating about its axis to provide an artificial gravity.



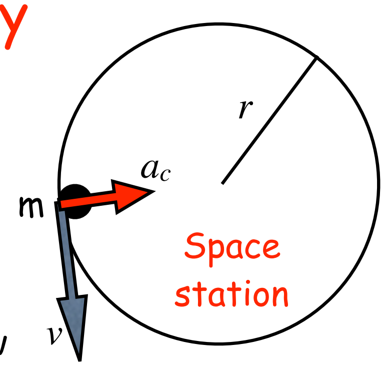


$F_c = \frac{mv^2}{r}$ - make this equal to the person's weight on earth, mg

$$\frac{mv^2}{r} = mg \quad \rightarrow v = \sqrt{rg} = \sqrt{1700 \times 9.8} = 129 \text{ m/s}$$ ($2\pi r/v = 83$ seconds per revolution)

Artificial Gravity

5.50/28: Problems of motion sickness start to appear in a rotating environment when the rotation rate is greater than 2 revolutions per minute.



Find the minimum radius of the station to allow an artificial gravity of one gee (a_c = 9.8 m/s^2) while avoiding motion sickness.

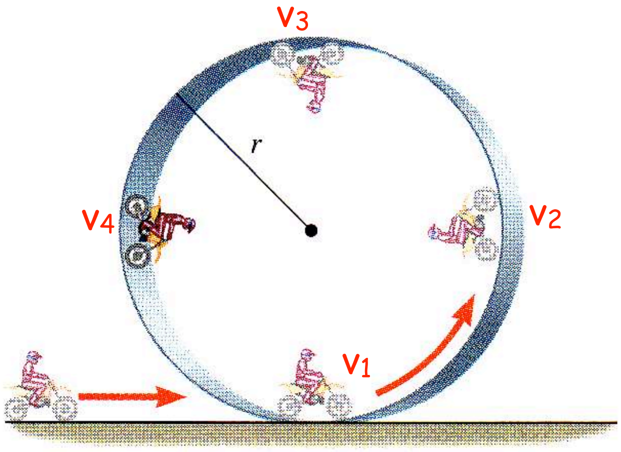
From previous slide, get artificial gravity a_c = g when: $v = \sqrt{rg}$

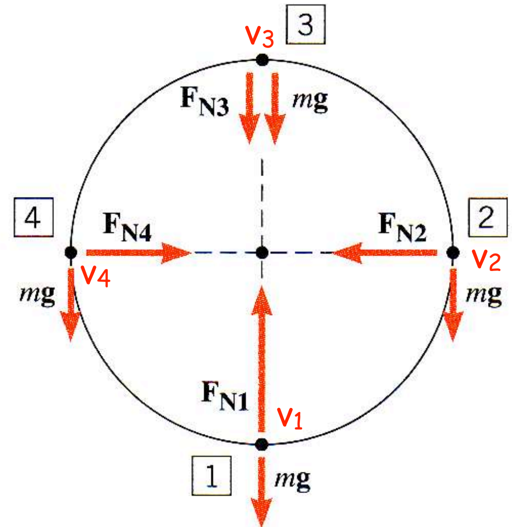
$$\text{Period of rotation, } \boxed{T} = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg}} \boxed{= 2\pi\sqrt{\frac{r}{g}}}$$

$$\text{So, } r = \left[\frac{T}{2\pi}\right]^2 g = 223 \text{ m} \qquad (\text{for } T = 30 \text{ s})$$

The minimum radius of the space station is 223 m

Vertical Circular Motion



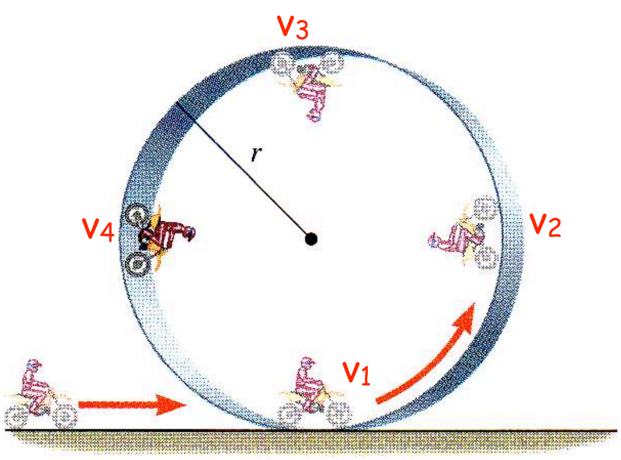


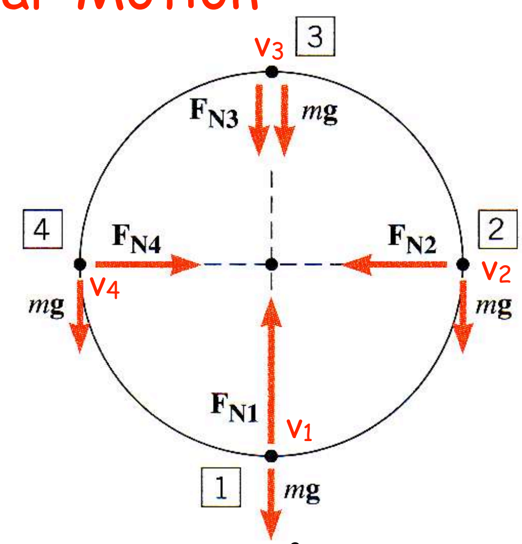
At (1): Net force toward centre of circle $= F_{N1} - mg = \dfrac{mv_1^2}{r}$

$$F_{N1} = mg + \frac{mv_1^2}{r} \qquad \text{(greater than the weight)}$$

At (2): Force toward centre of circle $= F_{N2} = \dfrac{mv_2^2}{r}$

Vertical Circular Motion





At (3): Net force toward centre of circle $= F_{N3} + mg = \dfrac{mv_3^2}{r}$

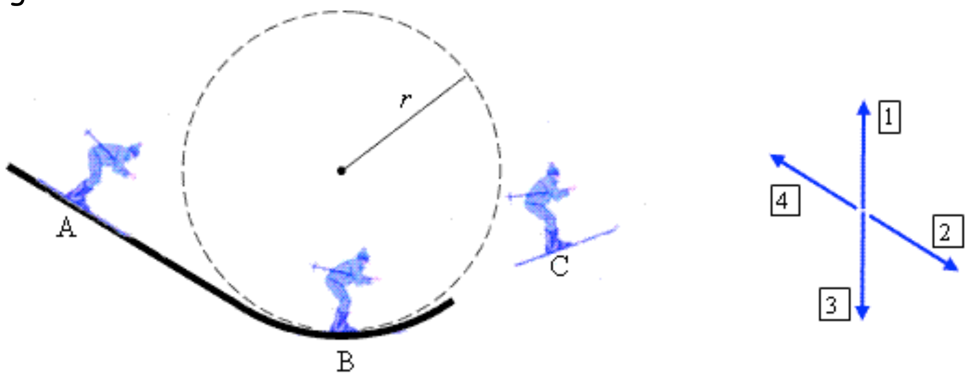
$$F_{N3} = \frac{mv_3^2}{r} - mg \qquad \text{Falls off if } F_{N3} \leq 0, \text{ i.e. } v_3 \leq \sqrt{rg}$$

At (4): as for (2)

Clicker Question: Focus on Concepts, Question 14

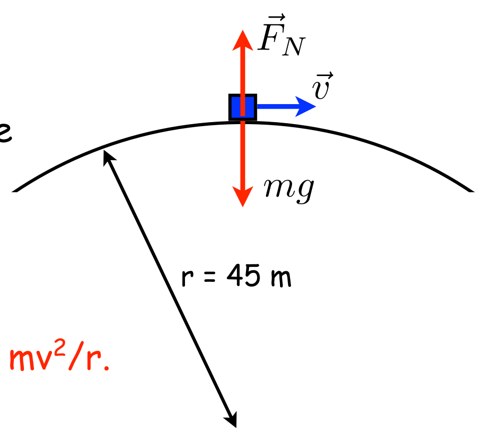
The drawing shows an extreme skier at three locations on a ski run: (A) a straight section, (B) a circular section, and (C) an airborne phase in which the skier is in free-fall. At the right of the drawing are four possible directions for the net force that acts on the skier. What is the direction of the net force at A, B, and C? Assume that there is no friction or air resistance at any point along the ski run.

A) 2, 3, 2
B) 2, 1, 1
C) 4, 3, 2
D) 3, 1, 3
E) 2, 1, 3



E) 2, 1, 3

5.46/40: A motorcycle is travelling up one side of a hill and down the other side. The crest is a circular arc with a radius of 45 m. Determine the maximum speed that the motorcycle can have while moving over the crest without losing contact with the road.



The net downward force on the bike at the crest of the hill allows the motorbike to remain in contact with the ground. Then $F_N > 0$.

That is:
net downward force = centripetal force, mv^2/r.

$$mg - F_N = \frac{mv^2}{r}$$

If F_N drops to zero, bike loses contact with ground.
Set F_N = 0.
Then, $v^2 = gr = 9.8 \times 45$, and v = 21 m/s (76 km/h)

Chapter 5: Uniform Circular Motion

- Period of circular motion: $T = 2\pi r/v$
- Centripetal acceleration: $a_c = v^2/r$
- Centripetal force: $F_c = ma_c = mv^2/r$
- For motion in a horizontal circle,
  - equilibrium in the vertical direction, vertical forces cancel
  - use Newton's second law to relate net horizontal force to the centripetal acceleration
- For motion in a vertical circle,
  - net force toward centre of circle = mv^2/r
- Artificial gravity,
  - spin the spacecraft, "g" = $a_c = v^2/r$