Q1) Radius of electron orbit: $r = \frac{n^2 a_0}{Z}$

$$r = \frac{16 a_0}{3} = 16\frac{(0.0529\ nM)}{3} = 0.2821\ nM$$

$$= 2.821 \times 10^{-10} m.$$

$$F_{centripetal} = F_{coulomb}$$

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

$$v^2 = \frac{kZe^2}{mr} = \frac{(8.99\times10^9 \frac{Nm^2}{C^2})\, 3\, (1.60\times10^{-19} C)^2}{(9.11\times10^{-31} kg)(2.821\times10^{-10} m)}$$

$$V = 1.639 \times 10^6 \frac{m}{s}$$

$$T = \frac{C}{V} = \frac{2\pi r}{V} = \frac{2\pi(2.821\times10^{-10})}{(1.639\times10^6)} = 1.082\times10^{-15} s.$$

$$N = \frac{10^{-8}}{T} = 9.246\times10^6 \text{ revolutions}$$

/3?

0.986 E-

2.) Can we confine an electron inside an atomic nucleus?
Suppose the scale of the nucleus is $\sim 10^{-15}$ m.
Use the uncertainty principle to calculate a momentum uncertainty and then find the corresponding kinetic energy.

$$\Delta p \Delta x \geq \frac{\hbar}{2} \qquad \Delta p \geq \frac{\hbar}{2\Delta x} = \frac{6.63\times10^{-34}}{4\pi \times 10^{-15}\,\text{m}}$$

$$= 5.28\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{(5.28\times10^{-20})^2}{2(9.11\times10^{-31})}\ \frac{\text{kg}\cdot\text{m/s}}{\text{kg}} = 1.53\times10^{-9}\,\text{J}$$

$$= 9.57\times10^{9}\,\text{eV}.$$

$$m_e c^2 = 8.19\times10^{-14}\,\text{J} = 5.11\times10^{5}\,\text{eV}$$

$$(\Delta E)^2 = (\Delta p)^2 c^2 + m_e^2 c^4$$

$$(\Delta E)^2 = \left[(5.28\times10^{-20})(3\times10^{8})\right]^2 + \left(9.1\times10^{-31}\cdot(3\times10^{8})^2\right)^2$$

$$(\Delta E)^2 = 2.51\times10^{-22}\,\text{J}$$

$$\Delta E = 1.58\times10^{-11}\,\text{J}$$

$$= 9.9\,\text{MeV} \qquad 99\,\text{MeV}$$

Ionization energy $13.6\,\text{eV} \ll \Delta E$

electron cannot be confined to nucleus!

Q3) $\frac{p^2}{2m} = E$ $\qquad$ $d = 5\,nM$ atomic spacing

$$p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2mc^2 E}}$$

$$\lambda = \frac{1240}{\sqrt{2 \cdot (3.727 \times 10^9 eV) \cdot (7.7 \times 10^6 eV)}}$$

$$= 5.176 \times 10^{-6}\, nM.$$

b.) $\lambda = \frac{hc}{\sqrt{2mc^2 E}}$

$$\lambda^2 = \frac{(hc)^2}{2mc^2 E}$$

$$E = \frac{(hc)^2}{2mc^2 \lambda^2} = \frac{(1240)^2}{2(3.727 \times 10^9)(5^2)} = 8.251 \times 10^{-6}\, eV$$

c.) It is not surprising since $\lambda \ll d$
and E is <u>tiny</u> to get $\lambda \sim d$.

Q4) $\Delta E = 20.0 - 13.6 = 6.4\, eV$

$$6.4\, eV = \frac{1}{2} m v^2 .$$

$$V = \sqrt{\frac{2 \times 6.4}{m}}$$

$$v = \left(\frac{2 \times (6.4) \times 1.6 \times 10^{-19}\, J/eV}{9.11 \times 10^{-31}\, kg} \right)^{1/2}$$

$$V = \sqrt{2.248 \times 10^{12}}\ m/s$$

$$V = 1.49 \times 10^{6}\ m/s .$$

Q5) $E_n = -\dfrac{13.6}{n^2}\ \text{eV} \qquad n = 1, 2, 3 \ldots$

for a transition from $n=1$ to $n=4$ we have

$$\Delta E = E_4 - E_1 = -13.6\left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$

$$= 12.75\,\text{eV}. \qquad 2$$

b.) Different ways the atom can de-excite:

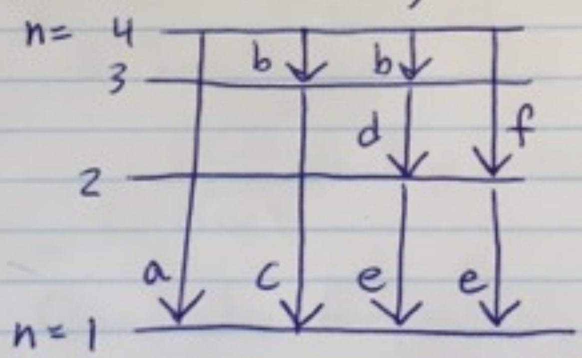
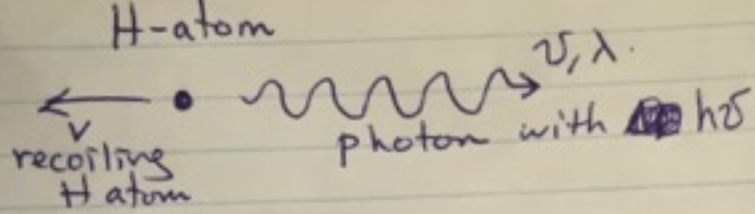


Table of energies:

a:	12.75 eV	
b:	0.6319 eV	6
c:	12.089 eV	
d:	1.889 eV	
e:	10.200 eV	
f:	2.550 eV	

c.)



Conservation of energy: $\Delta E = \frac{1}{2}mv^2 + h\nu$

Conservation of momentum: $p = mv = \frac{h}{\lambda} = \frac{h\nu}{c}$

From momentum:

$$\frac{p^2}{2m} = \frac{h^2\nu^2}{2mc^2}$$

So we have

$$\frac{h^2\nu^2}{2mc^2} + h\nu - \Delta E = 0$$

Quadratic formula:

$$\nu = \frac{-h \pm \sqrt{h^2 + \frac{2h^2\Delta E}{mc^2}}}{\left(h^2/mc^2\right)}$$

$$h\nu = \left(-1 \pm \sqrt{1 + \frac{2\Delta E}{mc^2}}\right)mc^2$$

Use the positive solution!

$$h\nu = \left(-1 + \sqrt{1 + \frac{2\Delta E}{mc^2}}\right)mc^2$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{x^2}{8} + \dots$$

$$h\nu = mc^2\left(-1 + 1 + \frac{1}{2}\left(\frac{2\Delta E}{mc^2}\right) - \frac{1}{8}\left(\frac{2\Delta E}{mc^2}\right)^2 + \cdots\right)$$

$$h\nu = \Delta E - \frac{1}{2}\frac{(\Delta E)^2}{mc^2} + \ldots$$

Energy of recoiling atom

$$K = \Delta E - h\nu$$

$$= \frac{1}{2}\left(\frac{(12.75\,eV)^2}{9.38\times10^{8}}\right)$$

$$= 8.128\times10^{-8}\,eV = 1.3\times10^{-26}\,J$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times1.3\times10^{-26}\,J}{1.67\times10^{-27}\,kg}}$$

$$v = \overset{4.07}{\cancel{3.94}}\ m/s.$$

→ If only the 1st order term in the binomial expansion is kept $h\nu = \Delta E$ and the atom recoil vanishes. $(v=0)$

subtract 2/10 marks for this bad assumption!