PHYS 2380 Assignment 1.

① $u(\lambda)\,d\lambda = u(\nu)\,d\nu$

$$u(\lambda) = u(\nu)\frac{d\nu}{d\lambda}$$

$$\nu = \frac{c}{\lambda} \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

At this point we have to think about what this means.

The function $u(\lambda)$ is a distribution of measured values - like a histogram. It is positive definite and cannot be negative. We can also think of the - sign as an artifact of expressing this in terms of λ instead of ν, which are inverse to one another. So changing from λ to ν introduces the - sign because the rate at which ν changes wrt λ is decreasing as ~~ν~~ increases. However this should <u>not</u> affect the $u(\lambda)$ distribution except to shift it - ie the $u(\lambda)$ and $u(\nu)$ are both positive definite. So take the absolute value. $\left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2}$

Then $u(\nu) = \dfrac{8\pi h \nu^3}{c^3(e^{h\nu/k_BT}-1)} \longrightarrow \dfrac{8\pi h}{\lambda^3(e^{hc/\lambda k_BT}-1)}$

$$u(\lambda) = \frac{8\pi h}{\lambda^3(e^{hc/\lambda k_BT}-1)}\,\frac{c}{\lambda^2} \quad \text{or} \quad \boxed{u(\lambda) = \frac{8\pi hc}{\lambda^5}\,\frac{1}{(e^{hc/\lambda k_BT}-1)}}$$

(2.) $T = 2.72548\ K$

$$\lambda_m = \frac{\text{const}}{T} = \frac{2.898 \times 10^{-3}\ mK}{2.72548\ K} = 1.06330 \times 10^{-3}\ m$$

$$= 1.06330\ mm.$$

b.) $\frac{c}{\lambda_m} = \nu_m = 2.82141 \times 10^{11}\ Hz.$

c.) $R = \sigma T^4 = 3.12864 \times 10^{-6} \frac{W}{m^2}$, $\sigma = 5.670 \times 10^{-8} \frac{W}{m^2 K^4}$

↖ Stefan-Boltzmann Law

$$A = 4\pi R_{\oplus}^2 = 4\pi(6.38 \times 10^6)^2 = 5.12 \times 10^{14}\ m^2.$$

$$P = RA = 1.60 \times 10^9\ W$$

③ Power radiated by the human body

$T = 37°C \rightarrow 310K$

$R_{emitted} = \sigma T^4 = 524 \frac{W}{m^2}$.

b.) $T_e = 27°C \rightarrow 300K$

$R_{absorbed} = \sigma T_e^4 = 459 \frac{W}{m^2}$

$R_{net} = R_{emitted} - R_{abs}$

$= 64.7 \frac{W}{m^2}$

c.) $A = 1.73 m^2$

$P = 64.7 \frac{W}{m^2} \cdot 1.73 m^2 = 111.98 W$

$111.98 \frac{J}{s} \cdot \frac{1}{4180} \frac{Cal}{J} = 0.0268 \frac{Cal}{s}$

$0.0268 \frac{Cal}{s} \; \frac{60 s}{min} \; \frac{60 min}{hr} \; \frac{24 hr}{day}$

$= 2302 \frac{Cal}{day}$

$= 2.30 \times 10^3 \frac{cal}{day}$.

4.) Start from Maxwell-Boltzmann

$$f(E) = A e^{-E/k_bT}$$

$$\int_0^\infty f(E)\,dE = 1.$$

$$A\int_0^\infty e^{-E/k_bT}\,dE = 1 \qquad \text{set } x = \frac{E}{k_bT}, \quad dx = \frac{dE}{k_bT}$$

$$A k_bT \int_0^\infty e^{-x}\,dx = A\,k_bT\left(-e^{-x}\Big|_0^\infty\right) = A k_bT = 1$$

$$\boxed{A = \frac{1}{k_bT}}.$$

b.) $$\bar{E} = \frac{\int_0^\infty E f(E)\,dE}{\int_0^\infty f(E)\,dE} = \frac{1}{k_bT}\int_0^\infty E e^{-E/k_bT}\,dE$$

$$x = \frac{E}{k_bT} \qquad E = k_bT x, \quad dE = k_bT\,dx$$

$$\bar{E} = k_bT\int_0^\infty x e^{-x}\,dx$$

Integration by parts

$u = x$

$dv = e^{-x}dx$

$v = -e^{-x}$

$$uv - \int v\,du = -xe^{-x} + \int e^{-x}dx$$

$$= -xe^{-x} - e^{-x}\Big|_0^\infty$$

$$= 1.$$

So $\boxed{\bar{E} = k_bT}$.

5.) $$\bar{E} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/k_bT}}{\sum_{n=0}^{\infty} e^{-h\nu/k_bT}} = h\nu\left(\frac{\sum_{n=0}^{\infty} ne^{-nh\nu/k_bT}}{\sum_{n=0}^{\infty} e^{-h\nu/k_bT}}\right)$$

Get this in the right form:

$$\alpha = \frac{h\nu}{k_bT}$$

$$\bar{E} = \alpha k_bT\left(\frac{-d}{d\alpha}\left[\ln\left(\frac{1}{1-e^{-\alpha}}\right)\right]\right)$$

$$\bar{E} = -\alpha k_bT\frac{d}{d\alpha}\left[\ln\left(\frac{1}{1-e^{-\alpha}}\right)\right]$$

b.) Big derivative!

$$\frac{d}{d\alpha}\ln\left[(1-e^{-\alpha})^{-1}\right] = \frac{1}{(1-e^{-\alpha})}\left[-(1-e^{-\alpha})^{-2}\right]e^{-\alpha}$$

$$= \frac{-e^{-\alpha}}{1-e^{-\alpha}} = \frac{-e^{-\alpha}}{\left(\frac{e^{+\alpha}}{e^{+\alpha}}-\frac{1}{e^{+\alpha}}\right)}$$

$$= -\frac{e^{-\alpha}}{\frac{1}{e^{\alpha}}(e^{\alpha}-1)} = -\frac{1}{e^{\alpha}-1}$$

so

$$\bar{E} = \frac{\alpha k_bT}{(e^{\alpha}-1)} = \frac{h\nu}{(e^{h\nu/k_bT}-1)}$$