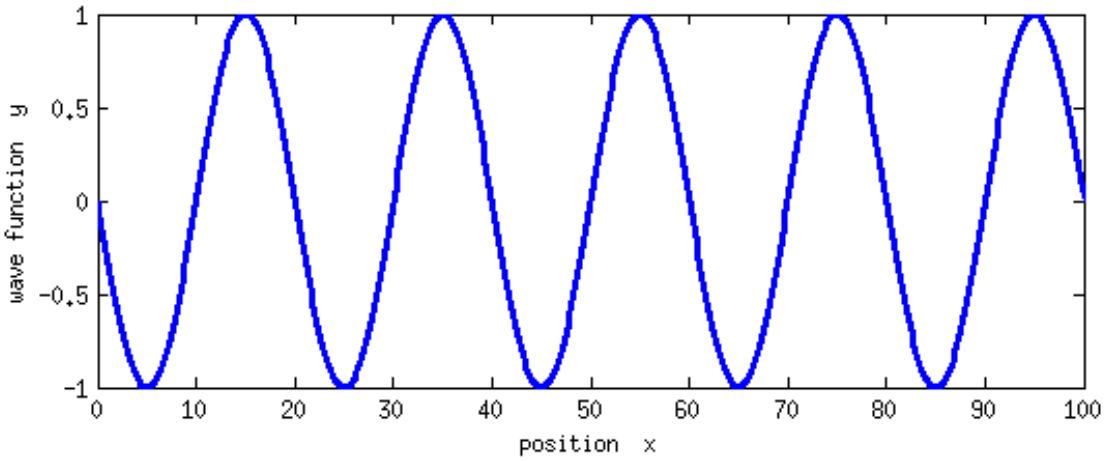
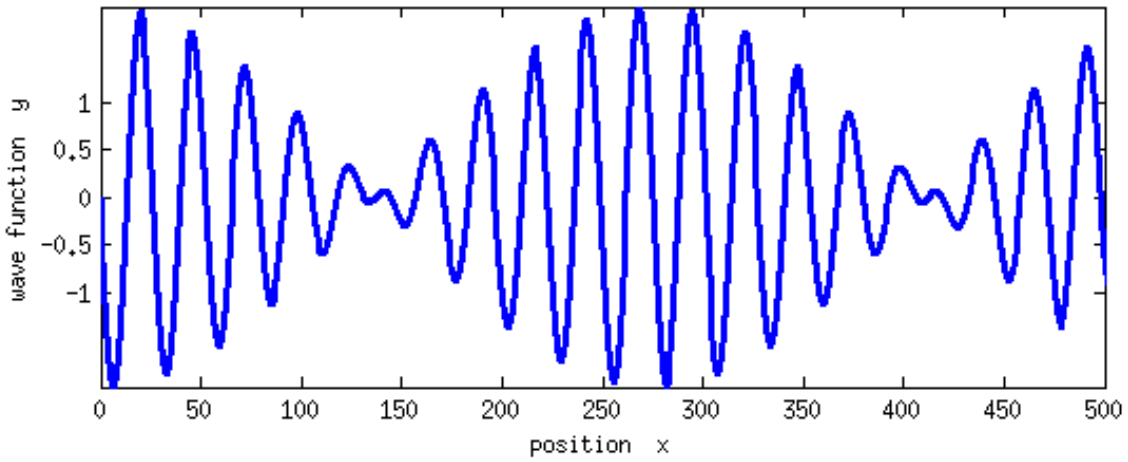
Waves and Fourier Expansion

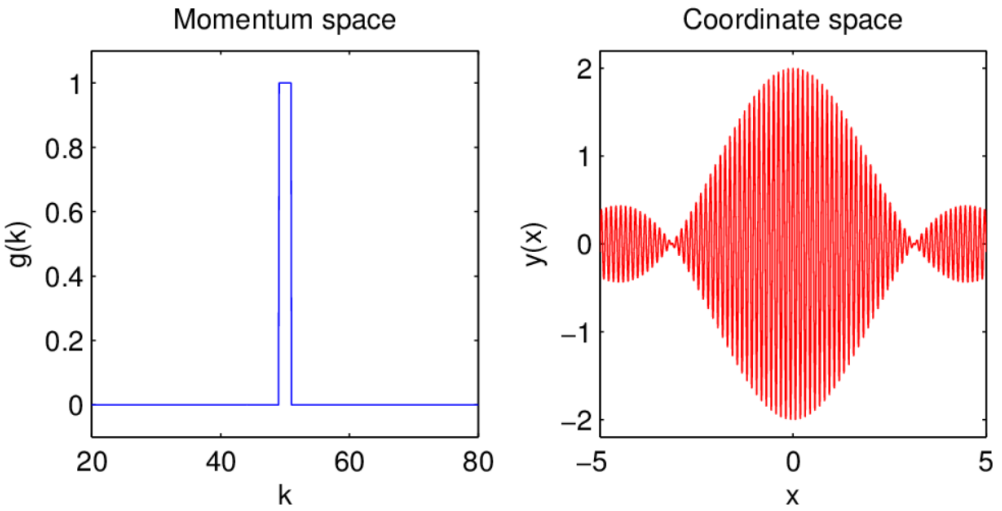
A single travelling sine wave.



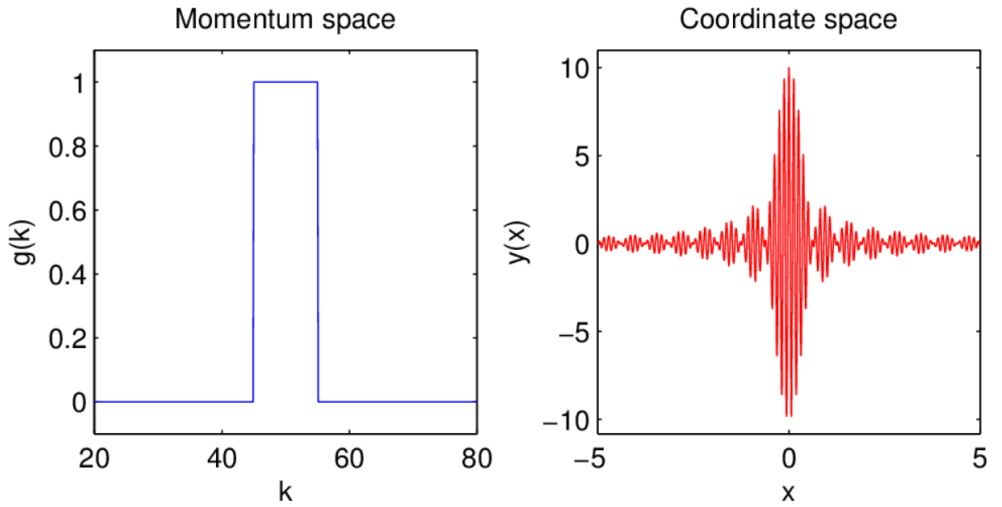
An example of beats by adding two sine waves.



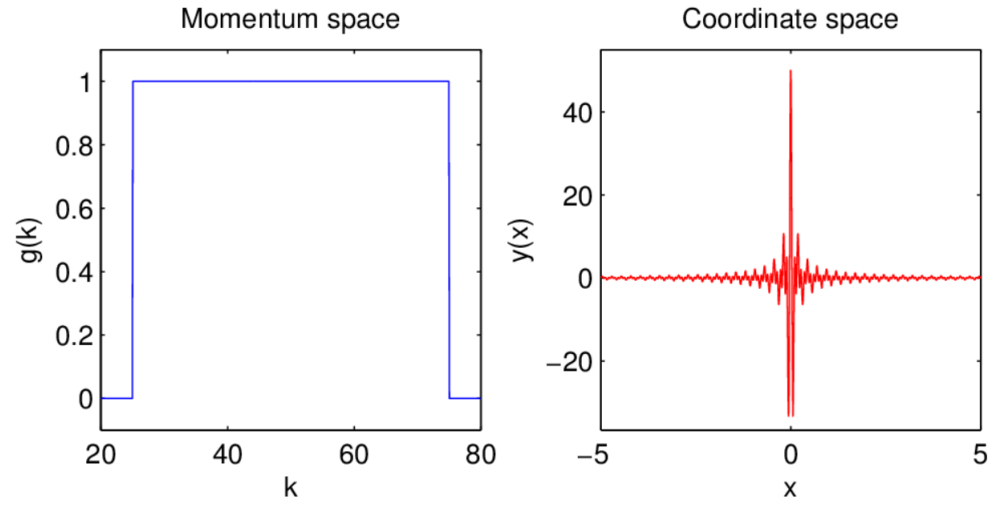
$\Delta k = 2$



$\Delta k = 10$



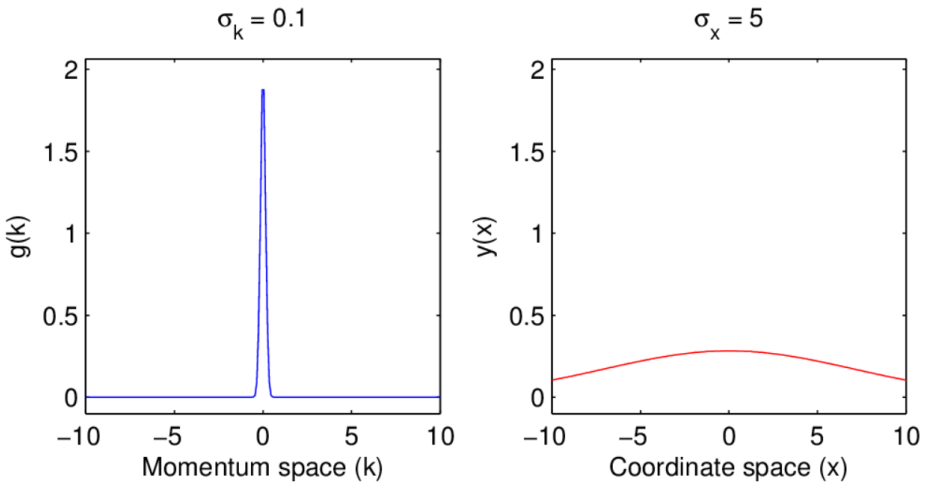
$\Delta k = 50$

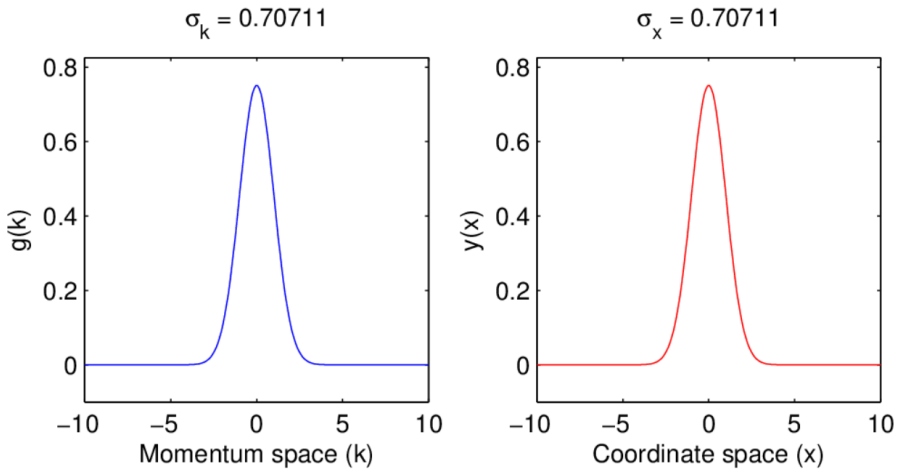


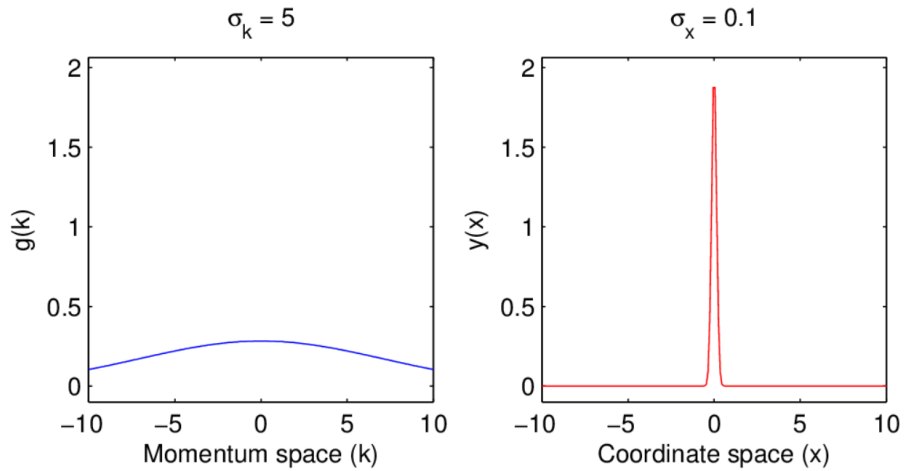
$k_0 = 50$

$$y(x) = A\,\mathrm{sinc}\left(\frac{\Delta k x}{2}\right)\cos(k_0 x)$$

$$\Delta x \Delta k > \frac{1}{2}$$







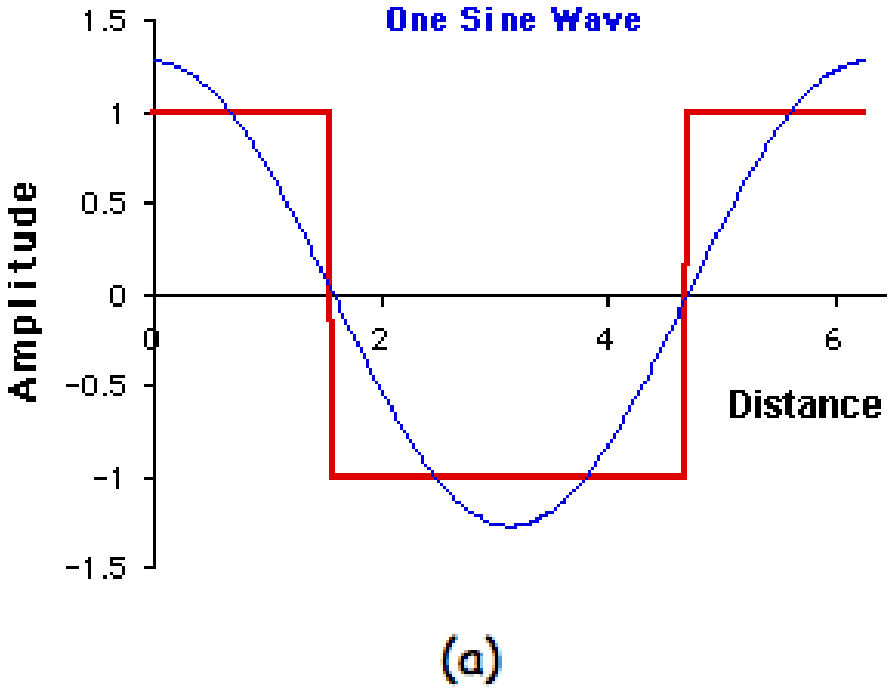
The Fourier transform of a Gaussian is another Gaussian…

$$g(k) = \frac{1}{\sqrt{2\,\pi}\,\sigma_k}\exp\left(\frac{-k^2}{2\,{\sigma_k}^2}\right)$$

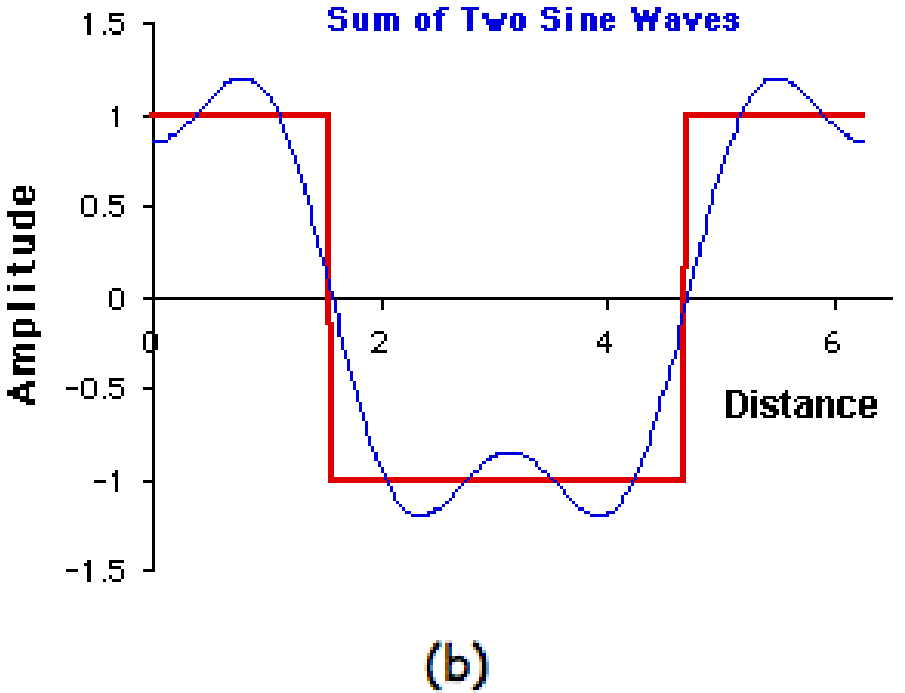
$$y(x) = \frac{1}{\sqrt{2\,\pi}\,\sigma_x}\exp\left(\frac{-x^2}{2\,{\sigma_x}^2}\right)$$

$$\sigma_x\sigma_k = \frac{1}{2}$$

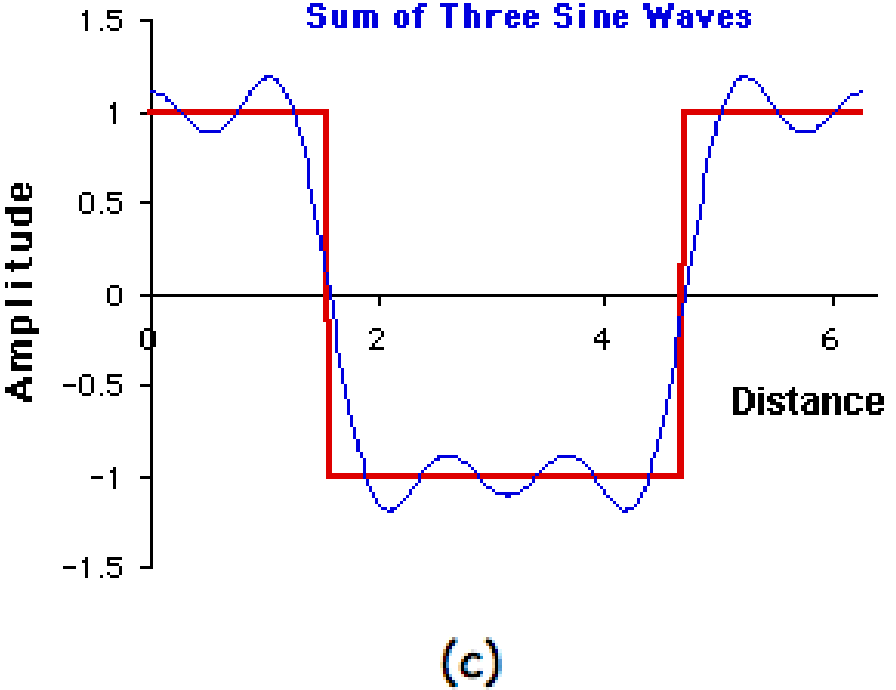
$$\Delta x\Delta k > \frac{1}{2}$$


One Sine Wave
Amplitude
Distance
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6

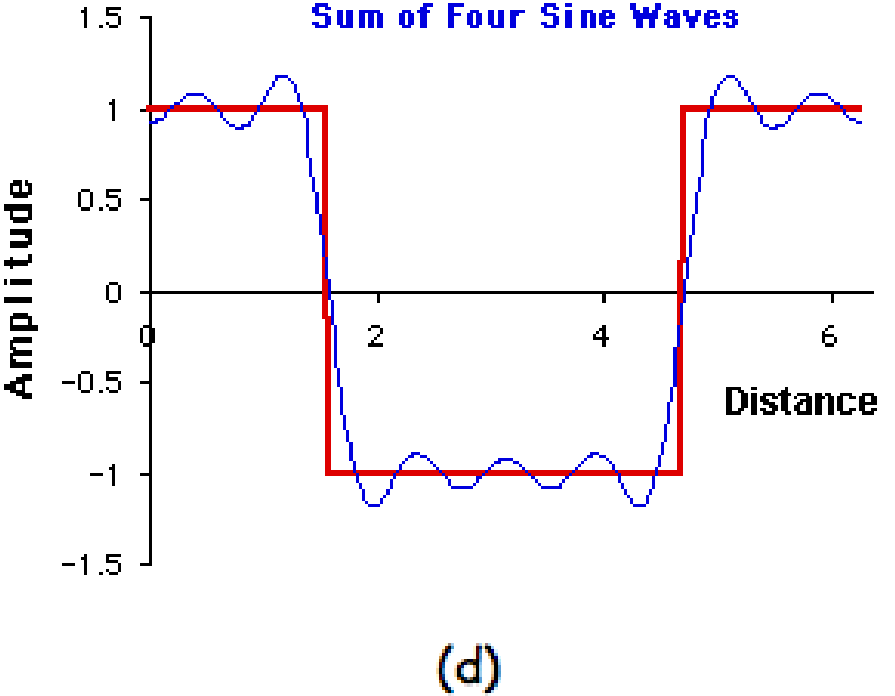
(a)


Sum of Two Sine Waves
Amplitude
Distance
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6

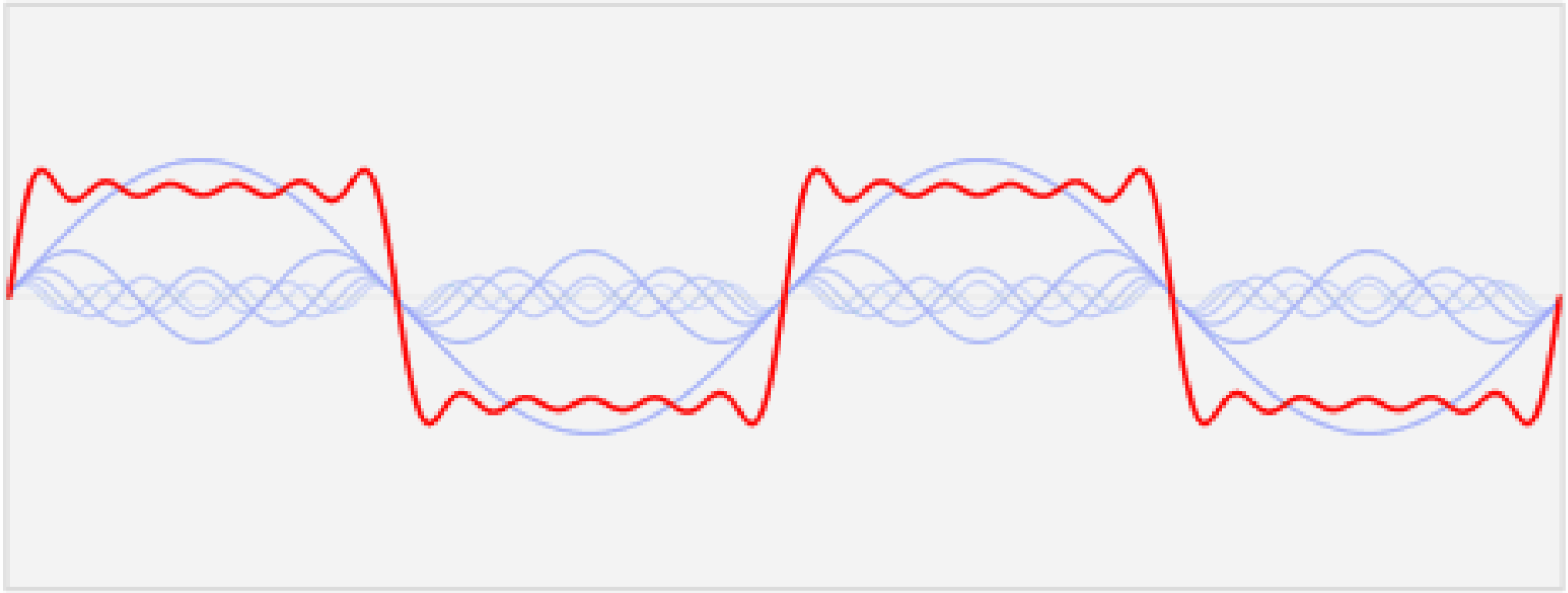
(b)

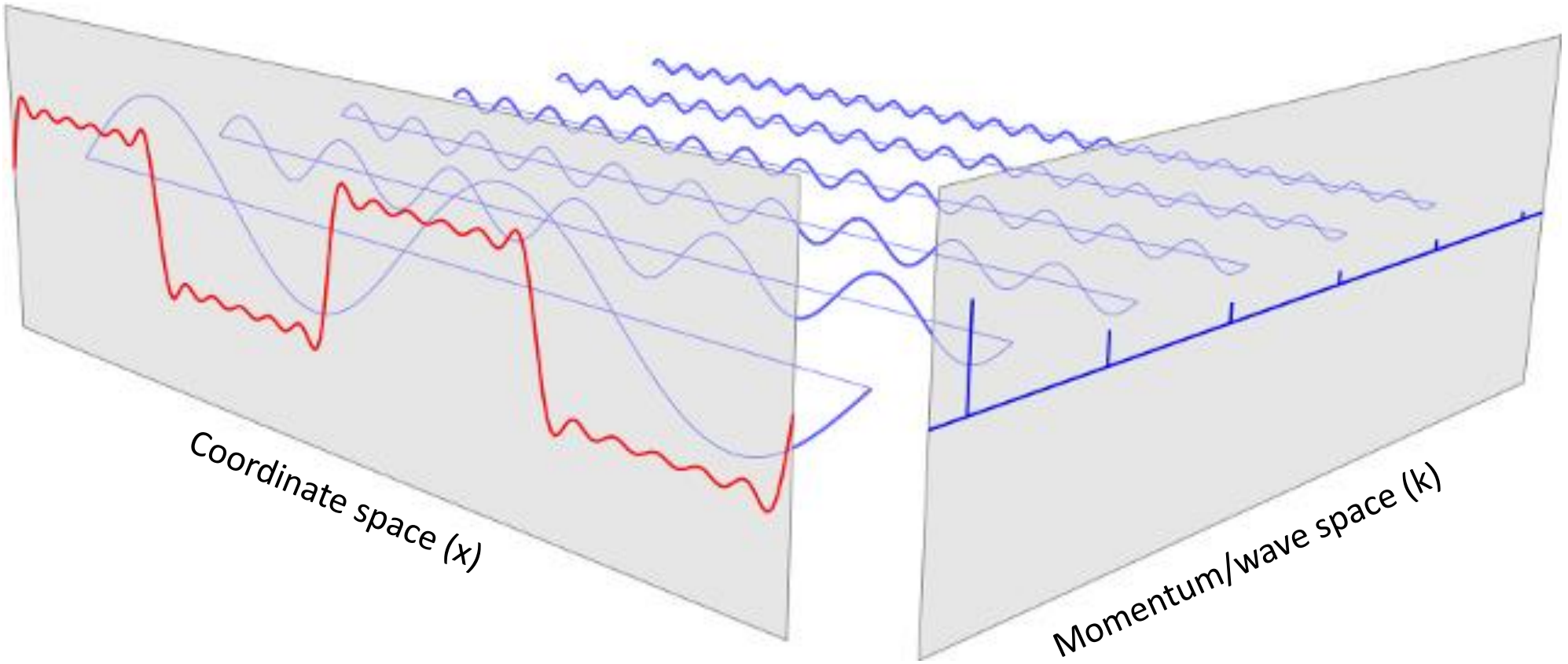

Sum of Three Sine Waves
Amplitude
Distance
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6

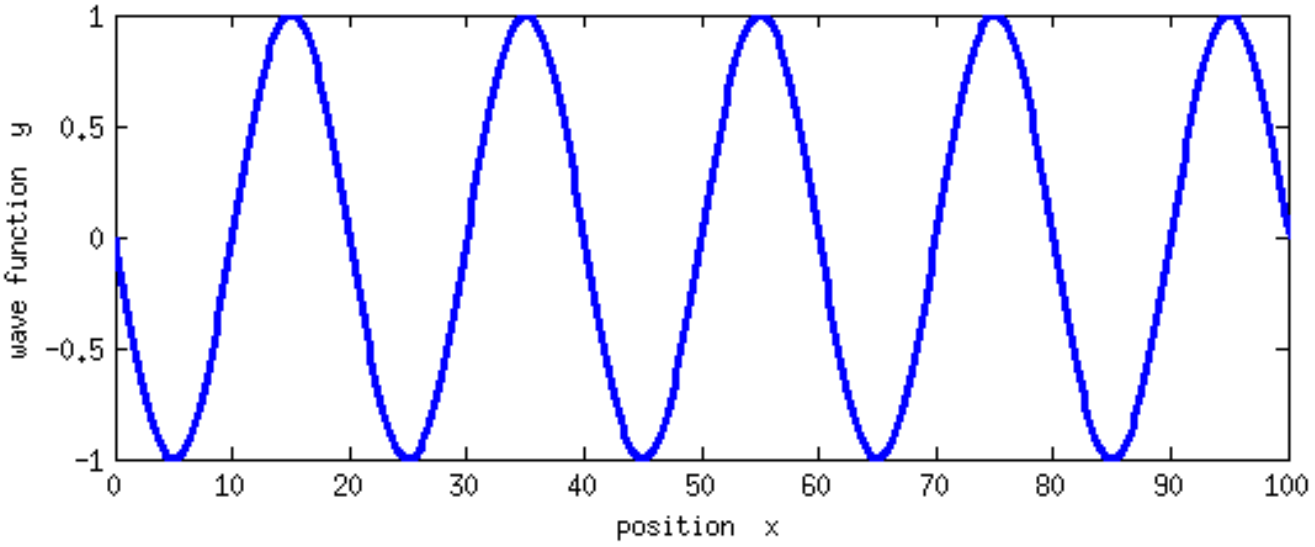
(c)

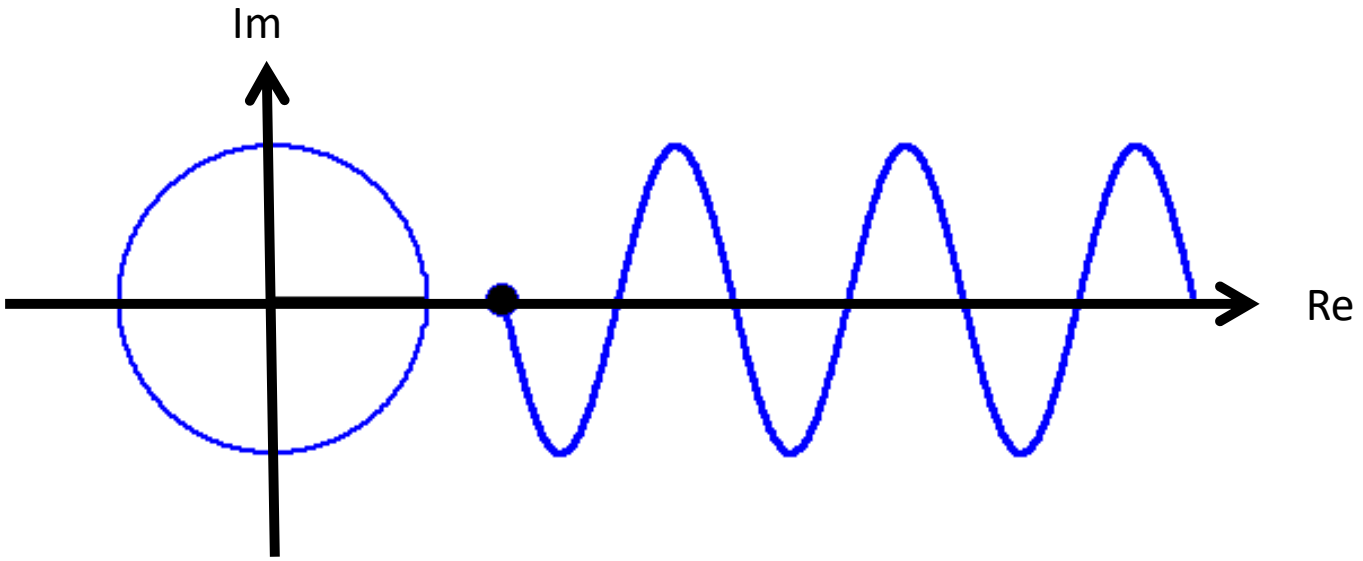

Sum of Four Sine Waves
Amplitude
Distance
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6

(d)

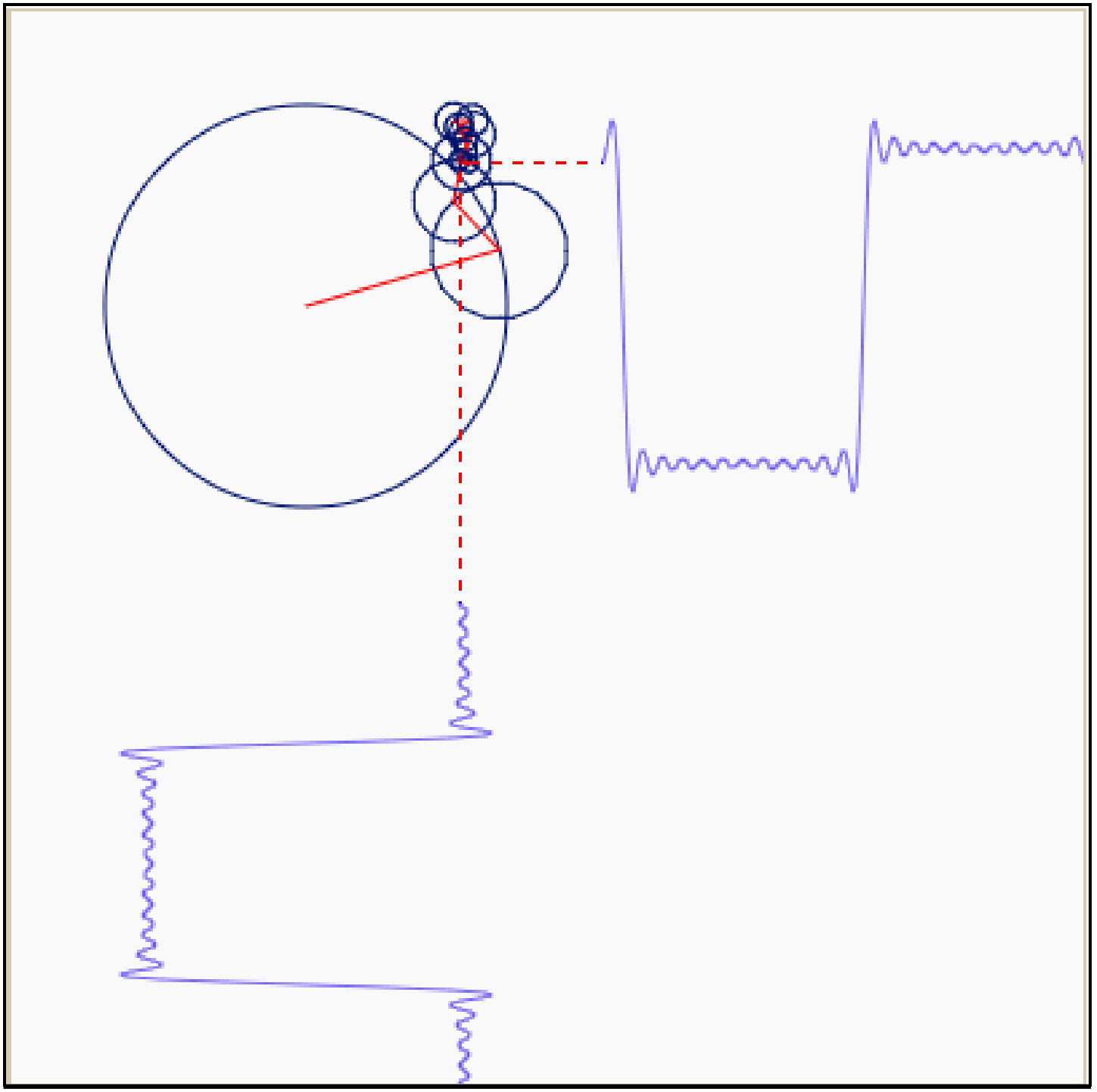



Coordinate space (x)
Momentum/wave space (k)





$$\exp(i\theta) = \cos(\theta) + i\,\sin(\theta)$$



Copenhagen Interpretation of Quantum Mechanics

1.) A wave function represents the state of a system. It contains all information about anything that can be known about a system before an observation. The wavefunction evolves smoothly in time.

2.) Certain properties cannot be known simultaneously for a given system (Heisenberg's Uncertainty Principle). Momentum is meaningless for a perfectly localized particle.

3.) During an observation, the system must interact with a laboratory device. When that device makes a measurement, the wavefunction of the system collapses. Collapse of the wavefunction means a wavefunction irreversibly reduces to an eigenfunction corresponding to a particular eigenvalue.

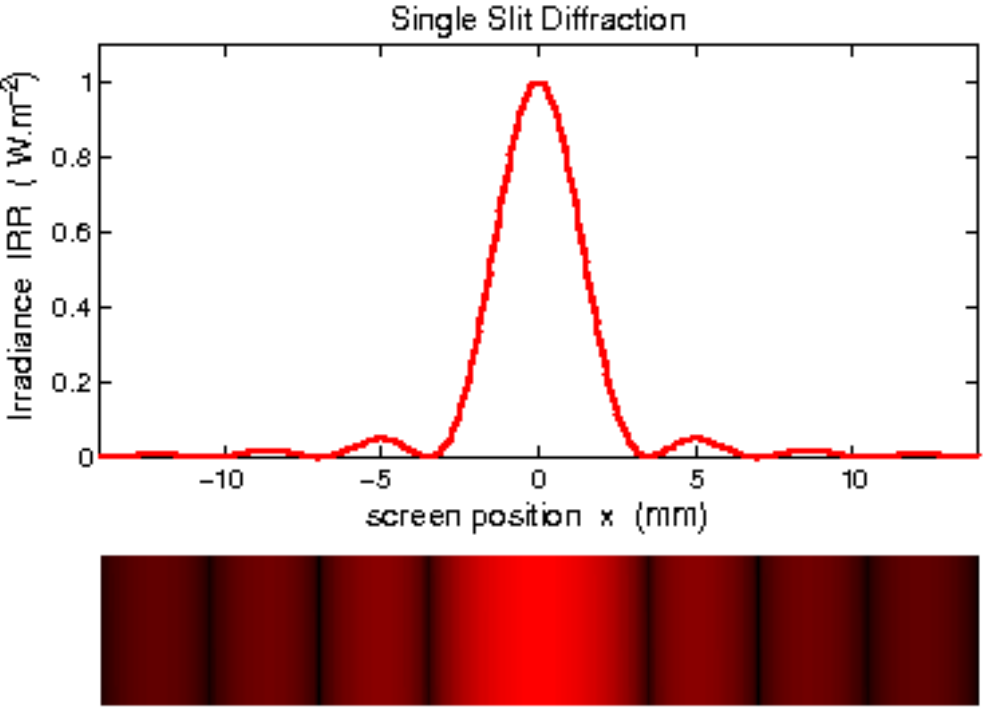
4.) The results provided by measuring devices are classical, and must be stated in classical terms.

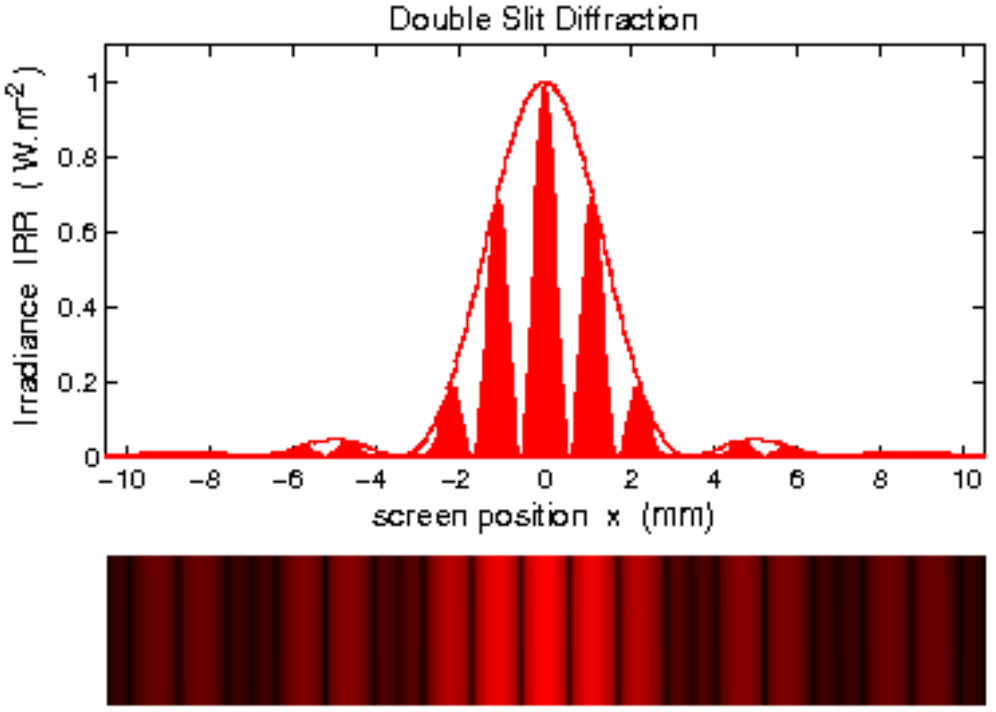
5.) The Born rule: Wavefunctions give a probabilistic description of an experimental outcome.

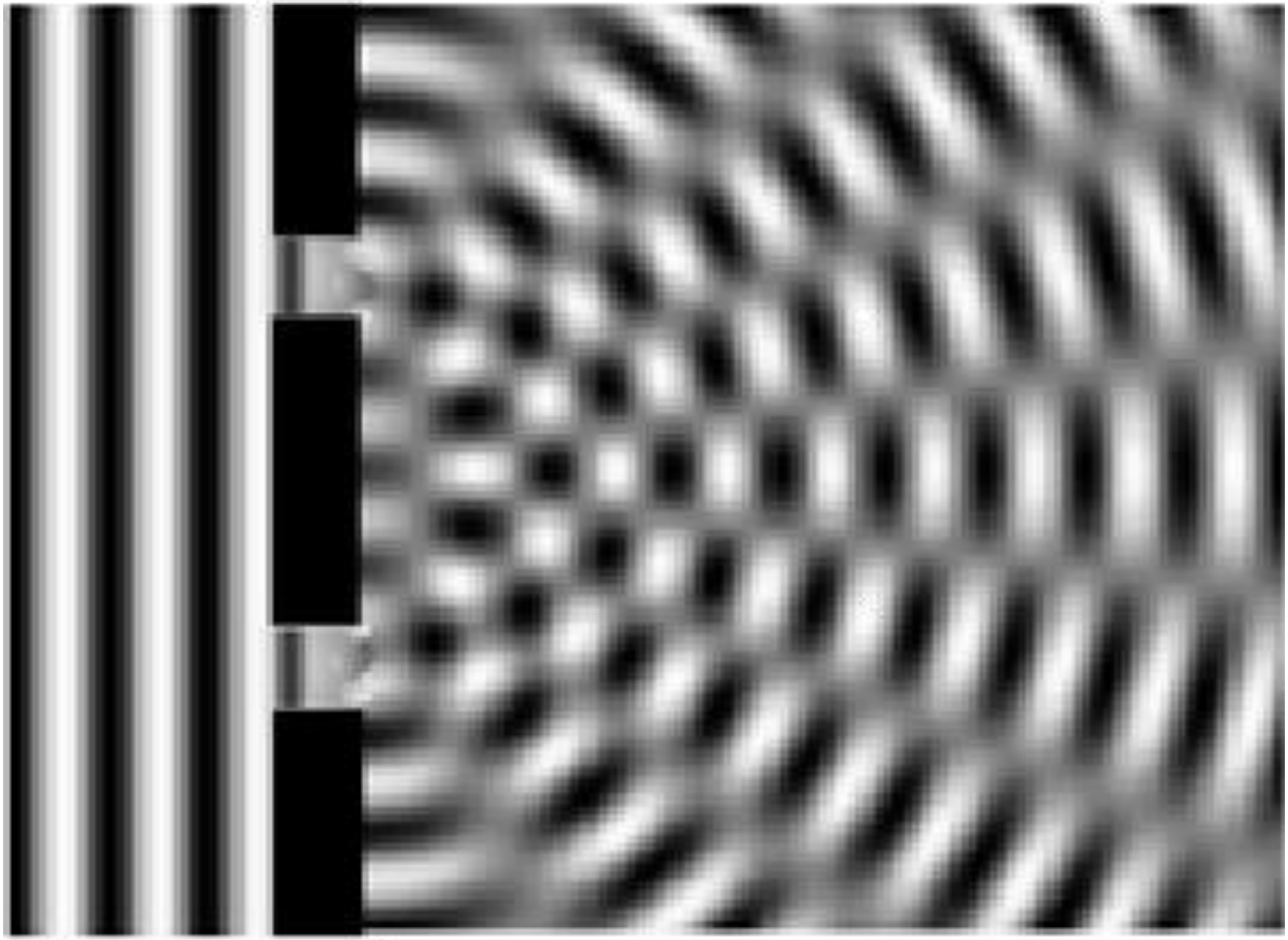
6.) Complementarity rule (Bohr): The wavefunction demonstrates a fundamental wave-particle duality that is a necessary feature of nature.

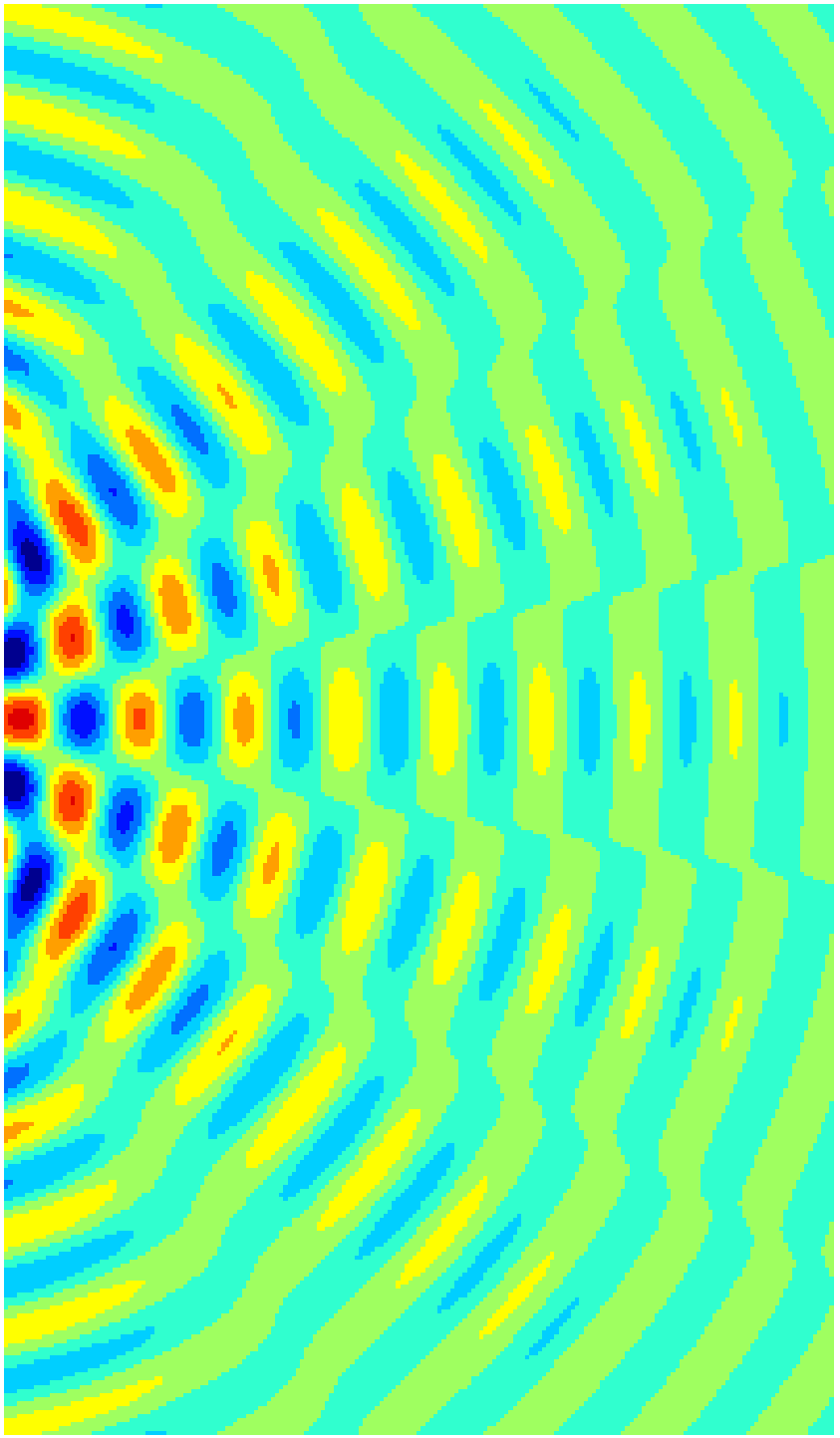
7.) Correspondence rule (Bohr and Heisenberg): When quantum numbers are large, the behaviour of a system reproduces classical predictions.

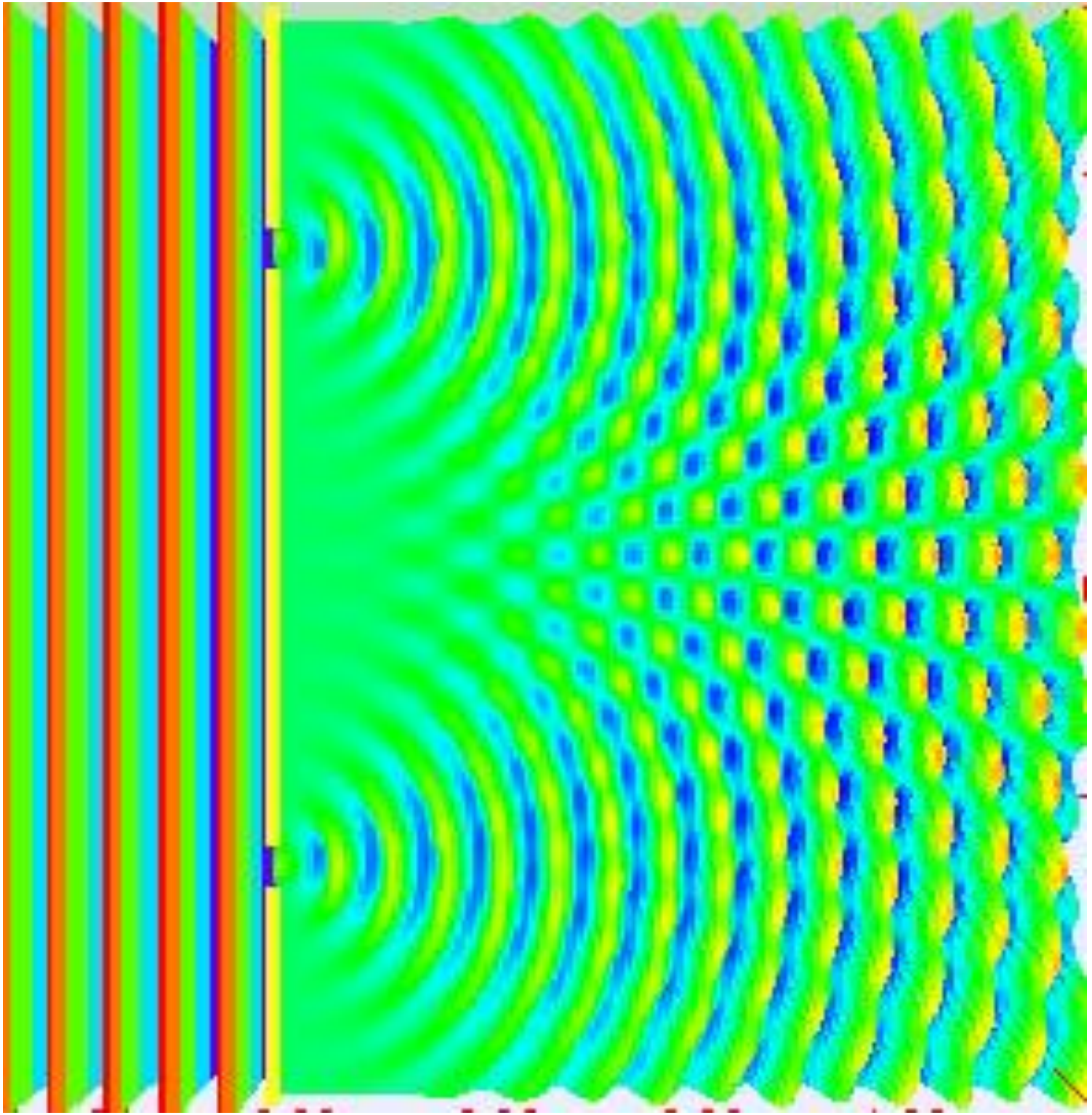
The double-slit experiment for particles

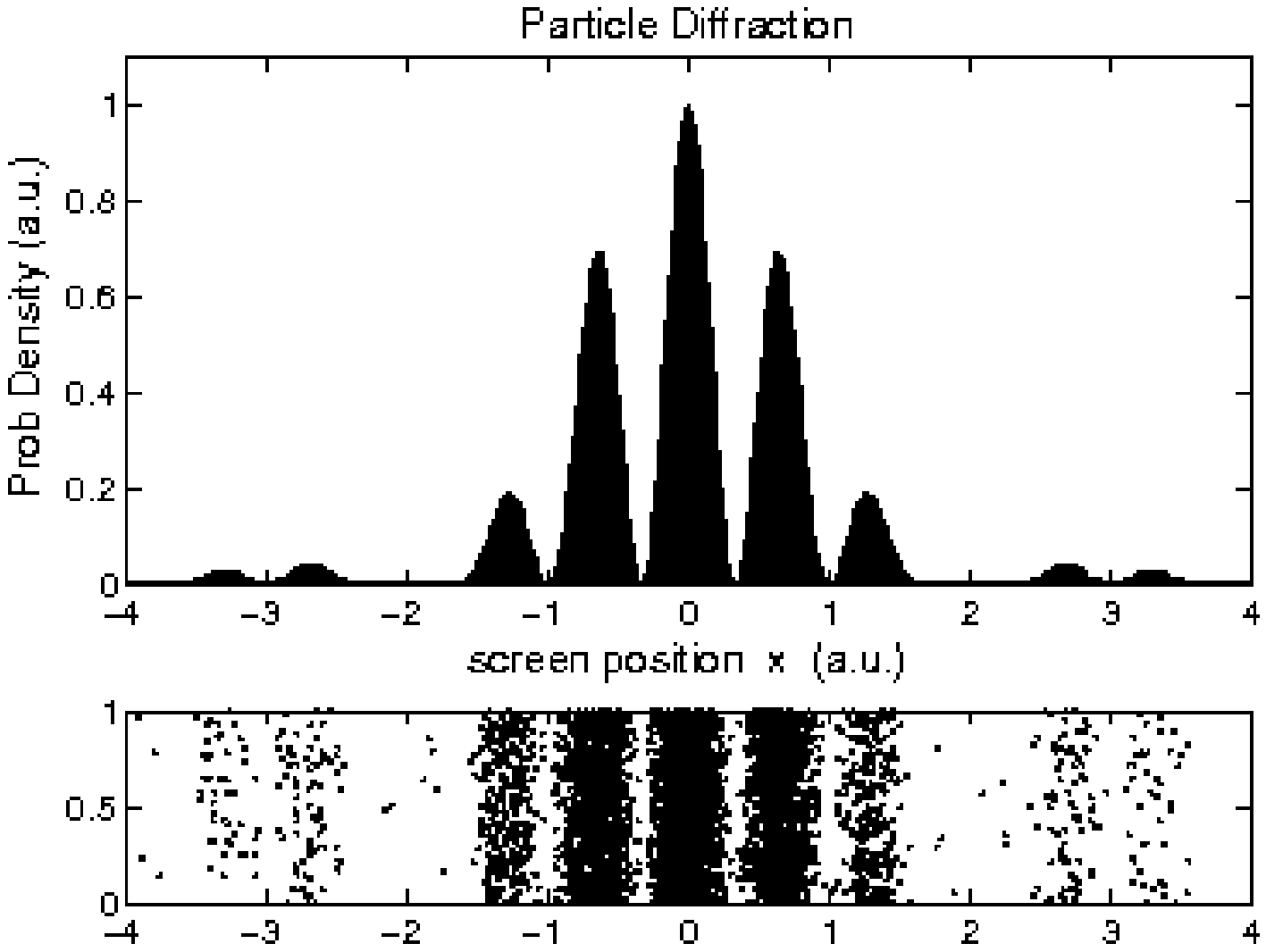




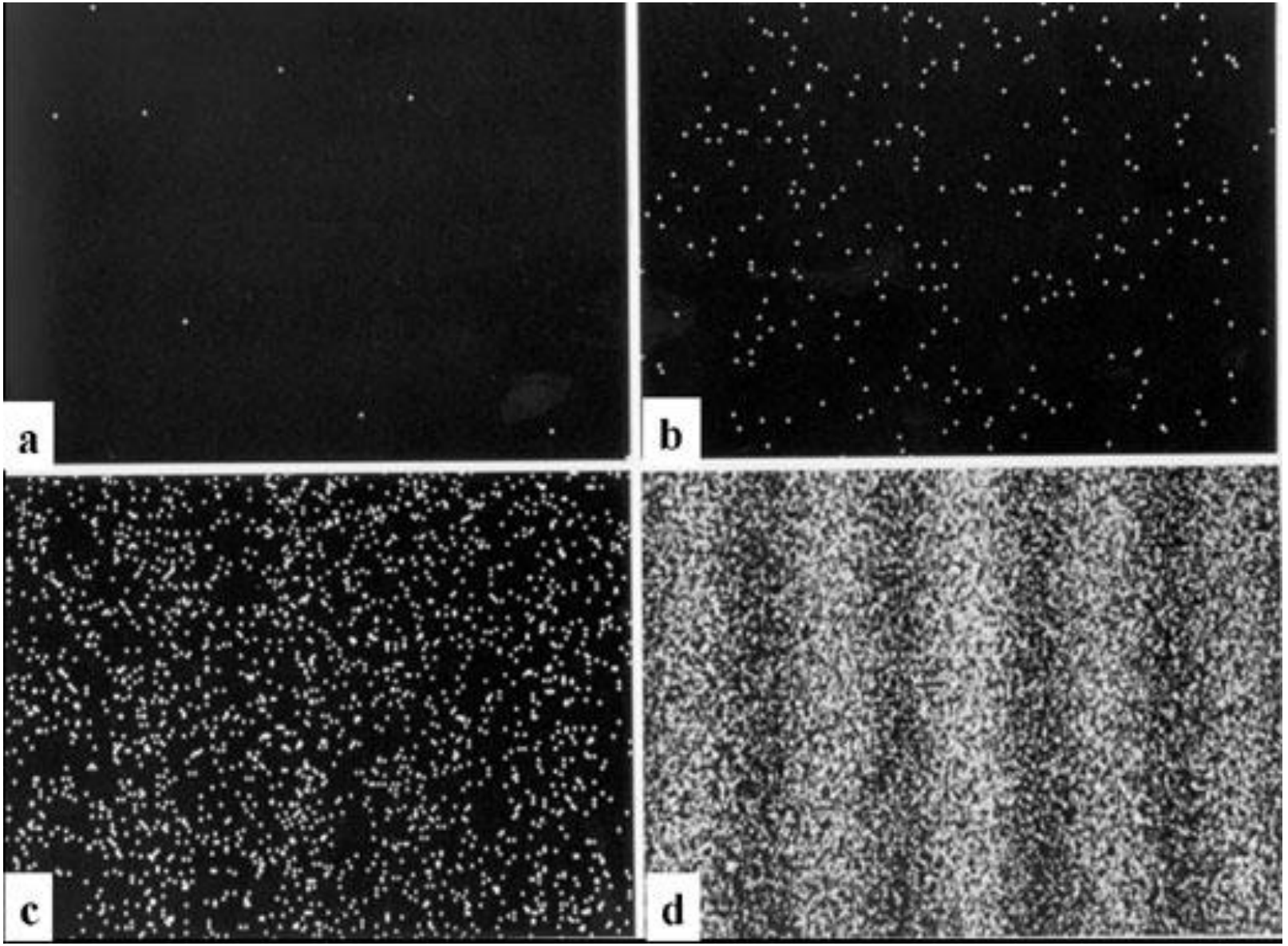







Particle Diffraction
Prob Density (a.u.)
0
0.2
0.4
0.6
0.8
1
−4
−3
−2
−1
0
1
2
3
4
screen position x (a.u.)
0
0.5
1

Single electron diffraction pattern (20 minutes duration)



a) 8 electrons
b) 270 electrons
c) 2000 electrons
d) 160,000 electrons