PHYS 2380 - Assignment 4

Due April 5, 2017

March 22, 2017

1.) Let a particle with $E < V_0$ be incident on a potential step with $V(x) = 0$ when $x < a$, and $V(x) = V_0$ in the region with $x \geq a$.

a.) Consider the wavefunction in the region $x < a$. Use the wavefunction in this region to calculate the probability current $J(x,t)$. Your answer will be in terms of the arbitrary coefficients A and B, which represent forward and reverse current components.

b.) In the class notes, we used the boundary conditions for the potential step case to find the coefficients

$$A = \frac{D}{2}\left(1 + i\frac{k_2}{k_1}\right) \tag{1}$$

$$B = \frac{D}{2}\left(1 - i\frac{k_2}{k_1}\right) \tag{2}$$

in terms of the arbitrary amplitude D in the region defined by $x \geq a$. With these expressions show that the probability current vanishes. This is equivalent to showing the reflection coefficient $R = 1$, as it must be for a potential step. Physically this says that there is no flow of probability from region 1 to region 2, and thus the wavefunction must be entirely reflected from the step.

2.) The ground state $n = 0$ and the first two excited states ($n = 1, 2$) of the quantum simple harmonic oscillator (SHO) are

$$\psi_0(u) = A_0 e^{-u^2/2} \tag{3}$$

$$\psi_1(u) = A_1 u e^{-u^2/2} \tag{4}$$

$$\psi_2(u) = A_2\left(1 - 2u^2\right)e^{-u^2/2} \tag{5}$$

a.) Show that these eigenfunctions obey the condition

$$\int_{-\infty}^{\infty} \psi_q(u)\psi_s(u)du = 0 \tag{6}$$

when $q \neq s$. Functions that obey the above condition are said to be *orthogonal*.

b.) Find the values of the normalization constants A_0, A_1 and A_2.

c.) Suppose the oscillator is in a superposition of states, such that

$$\Psi = \sqrt{\frac{3}{8}}\psi_0 + \sqrt{\frac{3}{8}}\psi_1 + \frac{1}{2}\psi_2. \tag{7}$$

Use the orthogonality property to show that if the wavefunctions are normalized (as in part b), then the superposition above is also normalized. There is no need to substitute the SHO wavefunctions to solve this problem.

d.) In general, suppose a superposition is weighted by the arbitrary coefficients c_0, c_1 and c_2, such that

$$\Psi = c_0\psi_0 + c_1\psi_1 + c_2\psi_2. \tag{8}$$

What property must the weighting coefficients satisfy for Ψ to also be normalized? Do the weighted coefficients from part c satisfy this condition?

e.) Physically interpret the results of part c and d. What do the coefficients tell us about the state of the SHO, and what might measurements of the state show?

3.) Consider the $n = 1$ state of the SHO.

a.) Calculate the position of the classical turning points of the potential, x_T, by setting the energy of the SHO equal to the potential and solving for x_T.

b.) What is the probability of finding the particle outside the classical limits for an SHO in the $n = 1$ state?

4.) For this problem, consider the ground state of the SHO.

a.) Calculate $< x >$, $< x^2 >$ and Δx for an oscillator in the ground state.

b.) Calculate $< p >$, $< p^2 >$ and Δp for an oscillator in the ground state.

c.) What is the product of the uncertainties from parts a and b?

d.) Using the uncertainty in momentum Δp, find the uncertainty in kinetic energy of the SHO, and compare it to the ground state energy.