PHYS 2380 - Assignment 3

Due March 1, 2017

February 17, 2017

Note: an electron volt (eV) is equivalent to 1.60×10^{-19} J. It may also be convenient to use the combination $hc = 1240$ eV·nm.

1

Recalling that an object smaller than the wavelength illuminating it cannot be "seen", what is the minimum kinetic energy of electrons needed in an electron microscope in order to "see" an atom whose diameter is 0.1 nm, about the size of a silicon atom?

This is question 5-41 in Tipler & Llewellyn.

2

The wavelength of light emitted by a ruby laser is 694.3 nm. Assuming that the emission of a photon of this wavelength accompanies the transititon of an electrion from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute L for the well.

This is question 6-16 in Tipler & Llewellyn.

3

An electron moving in a one-dimensional infinite square well is trapped in the $n = 5$ state.

a.) Show that the probability of finding the electron between $x = 0.2L$ and $x = 0.4L$ is $1/5$.

b.) Compute the probability of finding the electron within the "volume" $\Delta x = 0.01L$ at $x = L/2$.

This is question 6-20 in Tipler & Llewellyn.

4

a.) Find the wavefunction $\psi(x)$ by computing the Fourier transform of the function

$$g(k) = Ae^{\frac{-|k|}{a}} \tag{1}$$

with A and a constant. You can deal with the absolute value by splitting the domain of integration in half, taking $g(k) = Ae^{\frac{k}{a}}$ for $k < 0$, and $g(k) = Ae^{\frac{-k}{a}}$ for $k > 0$.

b.) Normalize the resulting wavefunction to find the constant A.

5

In class we calculated the Fourier transform of a Gaussian centered on the origin. Now let us consider a Gaussian centered at an arbitrary wave number k_0, such that

$$g(k) = A \exp\left(-\frac{(k-k_0)^2}{4\sigma_k^2}\right). \tag{2}$$

a.) Calculate the resulting wavefunction. What is the difference in the wavefunction between the two cases, and is this difference observable?

b.) Does the uncertainty relationship we derived in class change if the Gaussian is offset from the origin in momentum space?

c.) Normalize the wavefunction to determine the constant A.

d.) For the Gaussian centered at 0, use the wavefunction we derived in class to find the expectation value of the momentum, $< p >$.

e.) For the Gaussian centered at k_0, derived in part a, calculate the expectation value of the momentum, $< p >$.

f.) Compare your answers in d and e. What does the expectation value tell us about the momentum distribution $g(k)$?