PHYS 2380 - Assignment 1

Due February 8, 2017

January 27, 2017

1 Application: The Planck formula

The Planck formula we derived in class is

$$u(\nu) = \frac{8\pi h\nu^3}{c^3\left(e^{\frac{h\nu}{k_b T}} - 1\right)}. \tag{1}$$

Write down the corresponding relationship that depends on wavelength, $u(\lambda)$.

This is a variation of question 3-14 in Tipler & Llewellyn.

2 Application: Power in the cosmic microwave background

The oldest radiation in the Universe is the cosmic microwave background. This relic radiation is the afterglow of the big bang, emitted when the Universe was only 3×10^5 years old. At this time all matter was in a compact, hot and dense state. The radiation emitted by this cosmic furnace can still be observed today, and the spectrum of the radiation is very accurately fit by a blackbody with temperature of 2.72548 K.

a.) Using this temperature, find the wavelength at the maximum intensity of the blackbody spectrum.

b.) What is the frequency that corresponds to the wavelength in part a?

c.) Find the total power delivered to the Earth from the cosmic microwave background, given the radius of the Earth, $R_\oplus = 6.38 \times 10^6$ m.

This is question 3-15 in Tipler & Llewellyn.

3 Application: Power radiated by the human body

The body temperature of a healthy human being is 37° C.

a.) Find the power radiated by the human body per unit area.

b.) Besides emitting heat energy, humans also absorb energy from the surrounding environment. If the ambient temperature is 27° C on a nice summer day, find the power absorbed and calculate the net power lost using the result from part a.

c.) If the typical surface area of a human is 1.73 m^2, find the total power radiated per day. Also, convert the energy to food calories if 1 Cal = 4180 J.

4 Theory: Classical mean energy

In this exercise we will derive the relationship discussed in class for the mean energy of an individual object in a large group at thermal equilibrium, $\bar{E} = k_b T$, where k_b is the Boltzmann constant.

a.) The Maxwell-Boltzmann distribution describes how a fixed amount of energy is partitioned among many identical individuals in thermal equilibrium, such as oscillators in the walls of a blackbody cavity, or the modes of the electromagnetic field inside the cavity. This is called the equipartition of energy in classical physics. The Maxwell-Boltzmann distribution is:

$$f(E) = Ae^{-E/k_b T} \tag{2}$$

where A is a constant. Find the value of this constant by calculating

$$\int_0^\infty f(E)dE = 1. \tag{3}$$

b.) Find the mean energy $\bar{E}$ using

$$\bar{E} = \frac{\int_0^\infty Ef(E)dE}{\int_0^\infty f(E)dE}. \tag{4}$$

5 Theory: Quantum mean energy

Now let's evaluate the expression for the mean energy using Planck's formula $E = nh\nu$. We write the quantum version of the mean energy as

$$\bar{E} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-\frac{nh\nu}{k_b T}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{k_b T}}} \tag{5}$$

a.) Simplify the expression for $\bar{E}$ above using the identity

$$\frac{\sum_{n=0}^{\infty} ne^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}} = -\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\ln\left(\frac{1}{1-e^{-\alpha}}\right)\right] \tag{6}$$

You do not have to prove this complicated identity! Just use it to simplify the expression for $\bar{E}$ above.

b.) Evaluate the right hand side of the identity and use the result of part a to show that the quantum estimate is

$$\bar{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{k_b T}} - 1\right)} \tag{7}$$

which was written down in class.