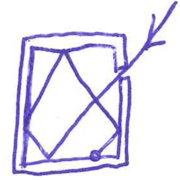
$R(\nu)d\nu = \frac{c}{4}u(\nu)d\nu$, $u(\nu)d\nu =$



E_{0x}, E_{0y}, E_{0z} constant ①

n_x, n_y, n_z integer ≥ 0

Wave modes for Cavity radiation

$$E_x = E_{0x} \cos\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right) \sin(2\pi\nu t)$$

$$E_y = E_{0y} \sin\left(\frac{n_x \pi x}{L}\right) \cos\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right) \sin(2\pi\nu t)$$

$$E_z = E_{0z} \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \cos\left(\frac{n_z \pi z}{L}\right) \sin(2\pi\nu t)$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}$$

Any of the components give the condition for standing waves:

$$-\left[\left(\frac{n_x \pi}{L}\right)^2 + \left(\frac{n_y \pi}{L}\right)^2 + \left(\frac{n_z \pi}{L}\right)^2\right] E_x = -\frac{(2\pi\nu)^2}{c^2} E_x$$

$$n_r = (n_x^2 + n_y^2 + n_z^2)^{1/2} = \frac{2L\nu}{c}$$

How many wave modes meet this condition? How to count themodes?

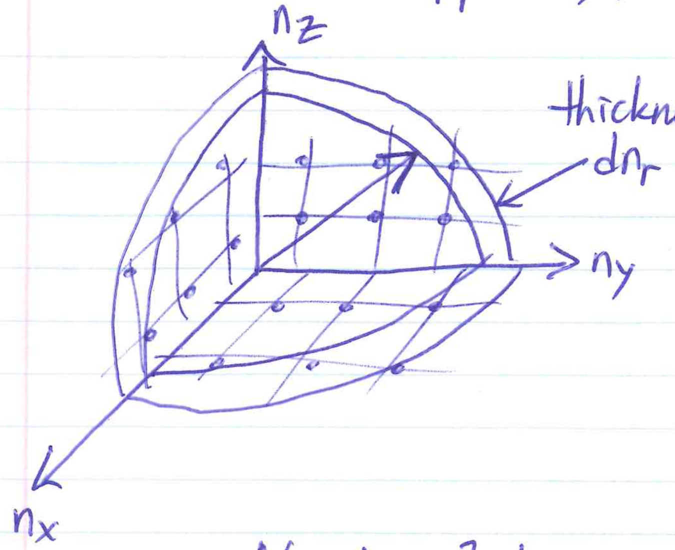
n_x, n_y, n_z integer and positive $\rightarrow$

n_r is the equation of a sphere!

Consider a spherical shell with thickness dn_r

*Note $\langle E\rangle = \langle E\rangle = \bar{E}$ = mean energy

$$\vec{n}_r = n_x \hat{x} + n_y \hat{y} + n_z \hat{z}$$



Number of states per unit volume = 1

Positive part of sphere:

$\frac{V_r}{8} \cdot 2$ ← polarization modes

$$N = \frac{4\pi n_r^2 \, dn_r}{4} \qquad \frac{dn_r}{d\nu} = \frac{2L}{c}$$

$$= \pi \left(\frac{4L^2\nu^2}{c}\right)\left(\frac{2L\,d\nu}{c}\right)$$

$$N = \frac{8\pi \nu^2 L^3 d\nu}{c^3} \qquad V_{box} = L^3$$

$$u(\nu)d\nu = \frac{N}{V}\langle E\rangle \qquad \langle E\rangle = kT \text{ from classical physics}$$

$$u(\nu)d\nu = \frac{8\pi\nu^2 kT\,d\nu}{c^3}$$

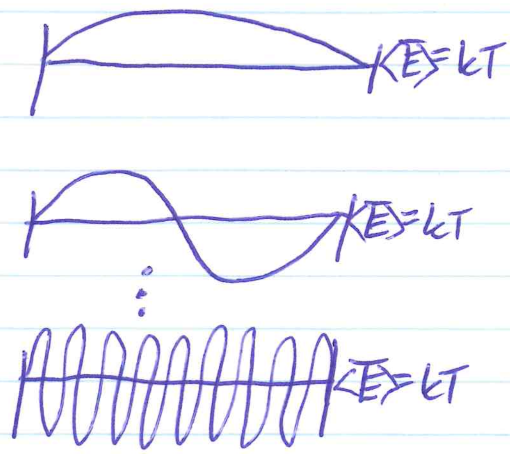
$$R(\nu)d\nu = \frac{c}{4}u(\nu)d\nu = \frac{2\pi\nu^2}{c^2}kT\,d\nu$$

"Rayleigh Jeans Law"

$$R = \int_0^\infty R(\nu)\,d\nu \longrightarrow \infty \, !$$

ULTRAVIOLET CATASTROPHE!

The problem is with the average energy per wave mode, $kT = \langle E \rangle$



From Maxwell-Boltzmann distribution

$$f(E) = \frac{e^{-E/k_bT}}{k_bT}$$

$$k_b = 1.381 \times 10^{-23} \text{ J/K}$$

Maxwell-Boltzmann distribution describes how energy is distributed among a large number of objects (oscillators in BB walls).

$$E = \frac{\int_0^\infty E f(E)\,dE}{\int_0^\infty f(E)\,dE} = k_bT$$

Planck's solution:

E IS NOT CONTINUOUS

Instead, let $E = nh\nu$ h = constant, n = + integer, $n \geq 0$

Then $$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

After some algebra...

$$\langle E \rangle = \frac{h\nu}{(e^{h\nu/kT} - 1)}$$

Previously, we had: $u(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\langle E \rangle d\nu$

Include Planck formula:

$$\boxed{u(\nu)\,d\nu = \frac{8\pi h \nu^3}{c^3\left(e^{h\nu/kT}-1\right)}\,d\nu}$$

Planck distribution function

For small ν set $x = \dfrac{h\nu}{kT} \ll 1$

Expand $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots$

$$e^x - 1 \approx x$$

$$u(\nu)\,d\nu \approx \frac{8\pi h \nu^3}{c^3\left(\frac{h\nu}{kT}\right)}\,d\nu \approx \frac{8\pi \nu^2}{c^3}\,kT\,d\nu$$

Rayleigh-Jeans Law!

when ν large,

$$u(\nu)\,d\nu \approx \frac{8\pi h \nu^3}{c^3 e^{h\nu/kT}} \to 0 \quad \text{as } \nu \to \infty.$$

As expected!

In order to get here, we needed Plancks suggestion

$$E = nh\nu \qquad n = 0, 1, 2 \ldots$$

Energy is continuous on large scales but on small scales "grainy"...

$$h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}.$$