add P_{op}, E_{op} to the formula sheet

PHYS 2380 Quantum Physics I
Term Test March 3rd, 2016, 19:00 - 21:00 hrs
519 Allen Bldg.

1. A certain star has a surface temperature of *4,900 K* and a radius of $6.96x10^9$ *m*. Assuming that it radiates like a black body:

   a. What is the total power radiated by this object? *(3 marks)*

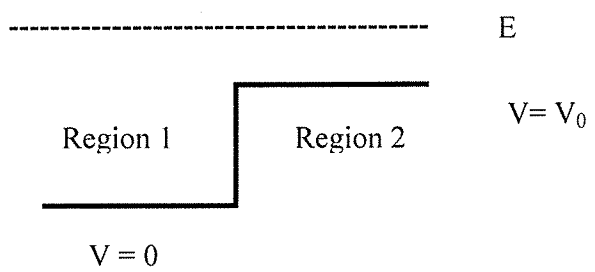
   b. At what wavelength does the spectral distribution have its maximum value? *(3 marks)*

2. A electron is confined to a region of space that is $2.0\times10^{-10}\,m$ wide.

   a. What is the minimum uncertainty in its momentum? *(3 marks)*

   b. What is the minimum kinetic energy, in *eV*, that this particle can have? *(3 marks)*

3. Particles with energy E, are incident from the left, on the step-potential of height V_0 as shown:



   a. If the wave numbers in the two regions are related by $k_1 = 3k_2$, what is the value of the energy *E* in terms of V_0? *(3 marks)*

   b. Write down the most general solutions for the Schrodinger Equation in both regions? Identify, with justification, if any of the coefficients used in your wave-functions are zero. *(3 marks)*

   c. Write down the equations that result for applying the boundary conditions for the wave functions at the step. Solve these equations to express each of the coefficients in terms of one of them. *(3 marks)*

   d. Derive expressions for the probability density in each region. *(3 marks)*

   e. Sketch a plot of the probability density, and identify maximum and minimum values for the probability density and their locations on the x-axis. *(3 marks)*

4. A particle, confined by an infinite square well to the region $0 \le x \le L$, is described by the wave function $\psi(x) = \sqrt{2/L}\,\sin(\pi x/L)\,e^{-iEt/\hbar}$ $\Psi(x,t)$, where *A* is a constant, in this region. The wave function is zero everywhere else.

   a. What is the average value for the position, <*x*>, and the square of the position, <x^2> of the particle? *(4 marks)*

   b. What is the average value for the momentum, <*p*>, and the square of the momentum, <p^2> of the particle? *(4 marks)*

   c. Use the results from (a) and *(b)* to determine the uncertainty in the position of the particle, and ~~use the results from (c) to determine~~ the uncertainty in the momentum of the particle? *(4 Marks)*

   d. Show that the results of ~~(d) and~~ (~~e~~ c) satisfy the uncertainty principle. *(2 marks)*

The End

Appendix: Some information from the text and lectures:

Special Relativity:

Relativistic momentum and energy:

$$\vec{p} = \gamma m \vec{v}$$
$$E = \gamma mc^2 = mc^2 + K$$
$$E^2 = c^2 p^2 + m^2 c^4$$

Electromagnetic radiation:

Power received by a detector from a wave:

$$P = \left(\frac{1}{\mu_0 c}\right) E_0^2 A \sin^2(kz - \omega t + \phi)$$

$$P_{ave} = \frac{1}{T_0}\int_0^{T_0} P\,dt = \frac{E_0^2 A}{2\mu_0 c}, \quad I = \frac{P_{ave}}{A} = \frac{E_0^2}{2\mu_0 c}$$

Where P is the instantaneous power, P_{ave} is the average power delivered to a detector of area A and I is the intensity of the light.

Interference and diffraction:

Pattern Type	Bright Fringes	Dark Fringes
Single slit (width *w*)	$\frac{w}{2}\sin\theta = m\lambda$	$\frac{w}{2}\sin\theta = \left(m + \frac{1}{2}\right)\lambda$
Double slit (spacing *d*)	$d\sin\theta = m\lambda$	
Grating (lines spaced *d* apart)	$d\sin\theta = m\lambda$	
Bragg (layers of atoms *d* apart)	$2d\sin\theta = m\lambda$	
Circular object		First fringe at $1.22\lambda/d$

Photons and light:

$$\lambda \nu = c$$
$$E_{ph} = h\nu = cp_{ph}$$
$$p_{ph} = h/\lambda$$

Photoelectric effect: $K = h\nu - \phi = eV_s$

Where K is the kinetic energy of the emitted electrons, ϕ is the work function of the material and V_s is the stopping potential.

Black body radiation:

$$I = \sigma T^4$$
$$\lambda_{max} T = 2.898 \times 10^{-3}\, m \bullet K$$
$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$
$$R(\lambda) = \frac{c}{4}u(\lambda)$$
$$dI = R(\lambda)d\lambda$$

Compton Scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Bremsstrahlung: $\lambda_{min} = \frac{hc}{eV}$

Wavelike properties of particles:

De Broglie wavelength: $\lambda = h/p$

Heisenberg uncert. relationships:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Wave packets:

$$p = h/\lambda = \hbar k$$

$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

Time independent: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$

Probability Current: $S(x,t) = \frac{i\hbar}{2m}\left\{\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x}\right\}$

Normalization (1-D): $\int_{-\infty}^{+\infty} \Psi^* \Psi dx = 1$

Rutherford Scattering:	**Bohr model:**
$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$ $\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{b^2 v^2}{r_{\min}^2}\right) + \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{r_{\min}}$ $d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$	$E_n = -\frac{Z_{eff}^2 me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6 Z_{eff}^2}{n^2} eV$ $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ $R = R_\infty\left(\frac{1}{1+m/M}\right)$ $R_\infty = \frac{mk^2e^4}{4\pi c\hbar^3}$ - *Where* R_∞ *= 1.0973732 X* 10^7 m^{-1} *is the Rydberg constant*

X-rays:

K-series: $E_{photon} = (13.6eV)\left(\frac{1}{1^2} - \frac{1}{n^2}\right)(Z-1)^2$

L-series: $E_{photon} = (13.6eV)\left(\frac{1}{2^2} - \frac{1}{n^2}\right)(Z-3)^2$

M-series: $E_{photon} = (13.6eV)\left(\frac{1}{3^2} - \frac{1}{n^2}\right)(Z-5)^2$

Some useful mathematical relations:

$$\sqrt{\left(1-\frac{u^2}{c^2}\right)} \approx 1-\frac{1}{2}\frac{u^2}{c^2}$$

$$\frac{1}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}} \approx 1+\frac{1}{2}\frac{u^2}{c^2}$$

For $u^2/c^2 \ll 1$

$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\sin(2A)=2\sin(A)\cos(A)$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Some useful Integrals:

$$\int \sin^2(u)\,du = \frac{1}{2}(u-\sin u\cos u)$$

$$\int \sin u\cos u\,du = \frac{1}{2}\sin^2 u$$

$$\int \cos^2 u\,du = \frac{1}{2}(u+\sin u\cos u)$$

$$\int u\sin^2 u\,du = \frac{u^2}{4}-\frac{u\sin 2u}{4}-\frac{\cos 2u}{8}$$

$$\int u\cos^2 u\,du = \frac{u^2}{4}+\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}$$

$$\int u^2\sin^2 u\,du = \frac{u^3}{6}-\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u-\frac{u\cos 2u}{4}$$

$$\int u^2\cos^2 u\,du = \frac{u^3}{6}+\left(\frac{u^2}{4}-\frac{1}{8}\right)\sin 2u+\frac{u\cos 2u}{4}$$

$$\int_0^\infty u^n e^{-u}\,du = n! \quad for\ n>0$$

$$\int \cos^n u\sin u\,du = -\frac{\cos^{n+1} u}{n+1} \quad for\ n>0$$

$$\int \sin^n u\cos u\,du = \frac{\sin^{n+1} u}{n+1} \quad for\ n>0$$

Constants:

Constant	Standard value	Alternate units
Speed of light	$c = 2.998\times10^{8}\, m/s$	
Electronic charge	$e = 1.602\times10^{-19}\, C$	
Boltzmann constant	$k = 1.381\times10^{-23}\, J/K$	$8.617\times10^{-5}\, eV/K$
Planck's constant	$h = 6.626\times10^{-34}\, J\cdot s$	$4.136\times10^{-15}\, eV\cdot s$
	$\hbar = 1.055\times10^{-34}\, J\cdot s$	$0.652\times10^{-15}\, eV\cdot s$
Avogadro's constant	$N_A = 6.022\times10^{23}\, mole^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67\times10^{-8}\, W/m^2\cdot K^4$	
Electron mass	$m_e = 5.49\times10^{-4}\, u$ or $9.11\times10^{-31}\, kg$	$0.511 MeV/c^2$
Proton mass	$1.007276\, u$ or $1.67262171\times10^{-27}\, kg$	$938.3 MeV/c^2$
Neutron mass	$1.008665\, u$ or $1.67492728\times10^{-27}\, kg$	$939.6 MeV/c^2$
Mass of ^{4}He	4.002603 u	
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2 / m_e e^2 = 0.0529 nm$	
Hydrogen ionization energy	$13.6 eV$	
	$hc = 1240 ev\cdot nm$	
	$1 eV = 1.602\times10^{-19}\, J$	
Atomic mass unit (dalton)	$1u = 931.5 MeV/c^2$	$1.661\times10^{-27}\, kg$
	$kT = 0.02525 eV \approx \frac{1}{40} eV$ at T=293 K	
Coulomb constant	$\frac{1}{4\pi\varepsilon_0} = 8.988\times10^{9}\, N\cdot m^2\cdot C^{-2}$	
	$\frac{e^2}{4\pi\varepsilon_0} = 1.44 eV\cdot nm$	